

Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/1.2.4.2-d-x^m-a-x^q+b-xⁿ+c-x^{-2-n-q}-p

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3.86	$\int \frac{1}{ax+bx^3+cx^5} dx$	462

3.87	$\int \frac{1}{x(ax+bx^3+cx^5)} dx$	467
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3.99	$\int \frac{x}{(ax+bx^3+cx^5)^2} dx$	536
3.100	$\int \frac{1}{(ax+bx^3+cx^5)^2} dx$	542
3.101	$\int \frac{1}{x(ax+bx^3+cx^5)^2} dx$	549
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3.104	$\int \frac{x}{\sqrt{ax+bx^3+cx^5}} dx$	570
3.105	$\int x^{3/2} \sqrt{ax+bx^3+cx^5} dx$	574
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3.109	$\int x^{3/2} (ax+bx^3+cx^5)^{3/2} dx$	594
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3.112	$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}} dx$	612
3.113	$\int \frac{1}{\sqrt{ax+bx^3+cx^5}} dx$	618
3.114	$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} dx$	622
3.115	$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}} dx$	626
3.116	$\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}} dx$	630
3.117	$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}} dx$	635
3.118	$\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}} dx$	640
3.119	$\int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}} dx$	644
3.120	$\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}} dx$	650
3.121	$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx$	655
3.122	$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}} dx$	658
3.123	$\int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx$	663
3.124	$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$	667
3.125	$\int \frac{1}{\sqrt{1-(1-x^2)^3}} dx$	671
3.126	$\int \sqrt{3x^2-3x^4+x^6} dx$	675
3.127	$\int \sqrt{x^2(3-3x^2+x^4)} dx$	679
3.128	$\int \sqrt{1-(1-x^2)^3} dx$	683
3.129	$\int \frac{1}{x\sqrt{a+bx+cx^2}} dx$	687
3.130	$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx$	691
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3.132	$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx$	699
3.133	$\int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx$	703
3.134	$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx$	707
3.135	$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} dx$	711
3.136	$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx$	715

3.137	$\int \frac{1}{x\sqrt{3-3x^2+x^4}} dx$	719
3.138	$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$	723
3.139	$\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}} dx$	727
3.140	$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}} dx$	731
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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (140)	% 0. (0)
Mathematica	% 99.29 (139)	% 0.71 (1)
Maple	% 97.14 (136)	% 2.86 (4)
Maxima	% 15.71 (22)	% 84.29 (118)
Fricas	% 92.14 (129)	% 7.86 (11)
Sympy	% 47.86 (67)	% 52.14 (73)
Giac	% 62.14 (87)	% 37.86 (53)

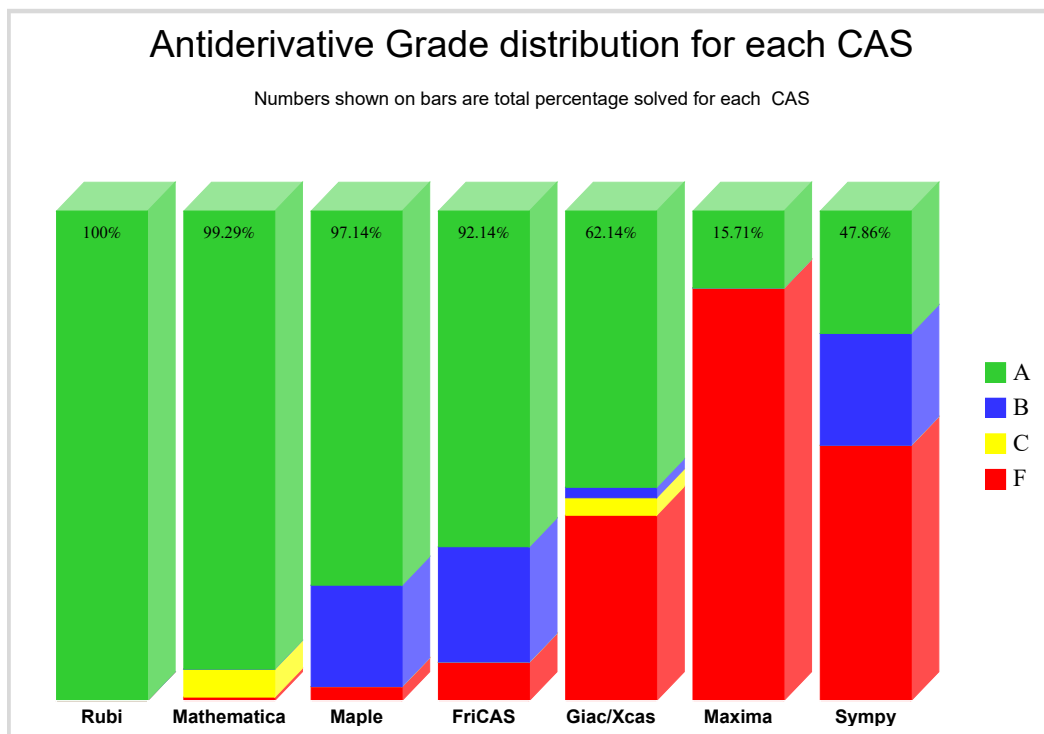
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

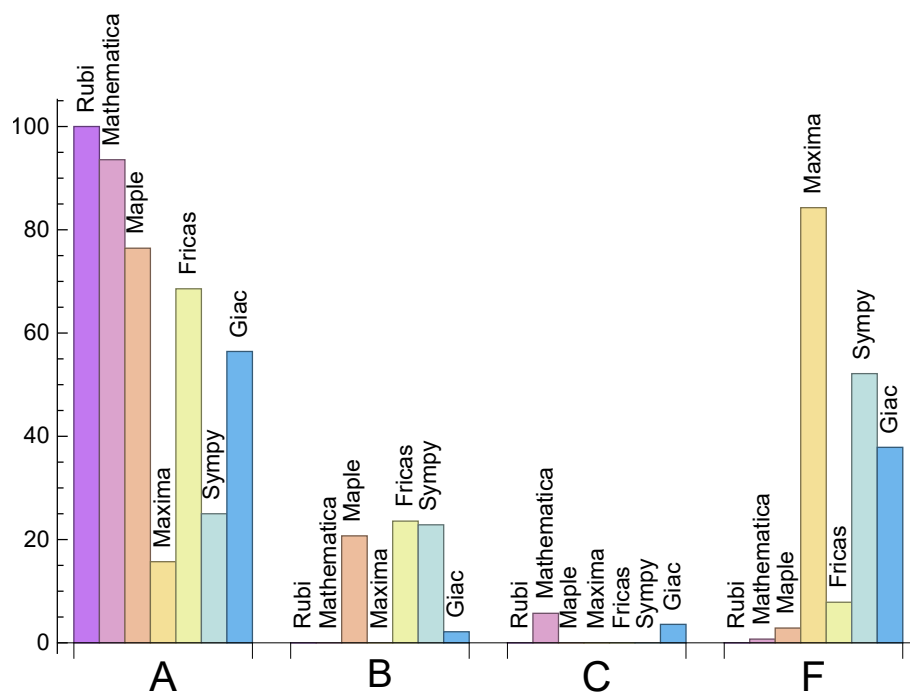
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	93.57	0.	5.71	0.71
Maple	76.43	20.71	0.	2.86
Maxima	15.71	0.	0.	84.29
Fricas	68.57	23.57	0.	7.86
Sympy	25.	22.86	0.	52.14
Giac	56.43	2.14	3.57	37.86

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.2	139.59	1.	103.5	1.
Mathematica	0.23	135.97	1.03	109.	0.99
Maple	0.01	236.36	1.43	143.	1.33
Maxima	1.15	40.73	1.08	26.5	1.09
Fricas	1.73	1036.53	6.84	626.	6.38
Sympy	7.06	613.04	4.55	267.	3.54
Giac	3.46	390.94	2.8	103.	1.35

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {104,122}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

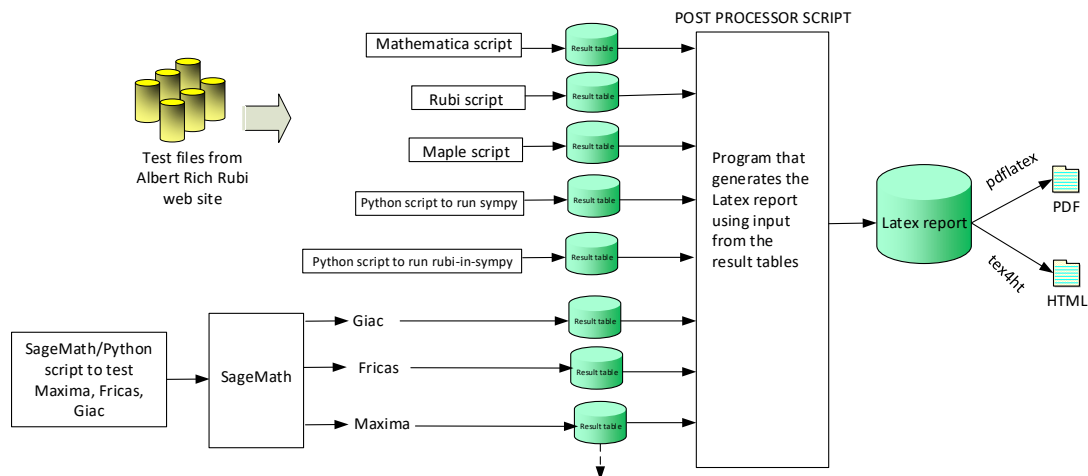
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { }

C grade: { 105, 107, 110, 112, 114, 116, 117, 119 }

F grade: { 140 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 86, 87, 88, 91, 93, 95, 96, 97, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 24, 25, 26, 27, 28, 35, 37, 47, 48, 65, 72, 79, 81, 89, 90, 92, 94, 98, 99, 100, 101, 102, 103, 105, 110, 112, 118, 119 }

C grade: { }

F grade: { 104, 121, 122, 140 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 137 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 72, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 106, 108, 109, 111, 113, 115, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade: { }

F grade: { 104, 105, 107, 110, 112, 114, 116, 117, 119, 122, 140 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102 }

B grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103 }

C grade: { }

F grade: { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 50, 51, 52, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 126, 127, 128, 129, 130, 131, 132, 139 }

B grade: { 65, 72, 111 }

C grade: { 79, 81, 83, 85, 87 }

F grade: { 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 60, 61, 62, 63, 64, 90, 92, 94, 96, 98, 100, 102, 104, 105, 107, 108, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 140 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.099	1.279	0.058	1.099

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.158	1.301	0.058	1.101

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.004	0.	0.	1.076	1.365	0.058	1.081

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.006	0.001	0.001	1.071	1.442	0.057	1.083

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.006	0.001	0.002	1.118	1.468	0.056	1.075

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	116	48	62
normalized size	1	1.	1.	0.83	1.09	2.15	0.89	1.15
time (sec)	N/A	0.054	0.01	0.001	1.165	1.268	0.071	1.071

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	115	49	62
normalized size	1	1.	1.	0.83	1.09	2.13	0.91	1.15
time (sec)	N/A	0.029	0.007	0.	1.127	1.289	0.07	1.081

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	112	48	62
normalized size	1	1.	1.	0.83	1.2	2.07	0.89	1.15
time (sec)	N/A	0.027	0.006	0.	1.337	1.365	0.073	1.096

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	107	49	62
normalized size	1	1.	1.	0.83	1.09	1.98	0.91	1.15
time (sec)	N/A	0.032	0.006	0.002	1.125	1.425	0.07	1.099

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	107	48	62
normalized size	1	1.	1.	0.83	1.09	1.98	0.89	1.15
time (sec)	N/A	0.032	0.007	0.	1.153	1.455	0.072	1.072

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	653	381	116
normalized size	1	1.	0.94	1.48	0.	7.34	4.28	1.3
time (sec)	N/A	0.094	0.11	0.003	0.	1.59	0.784	1.083

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	537	306	90
normalized size	1	1.	1.04	1.44	0.	7.67	4.37	1.29
time (sec)	N/A	0.059	0.064	0.003	0.	1.688	0.647	1.083

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	427	216	74
normalized size	1	1.	1.02	1.	0.	7.62	3.86	1.32
time (sec)	N/A	0.04	0.032	0.001	0.	1.559	0.302	1.105

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	277	124	46
normalized size	1	1.	1.12	1.03	0.	8.15	3.65	1.35
time (sec)	N/A	0.026	0.006	0.002	0.	1.571	0.201	1.113

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	494	564	84
normalized size	1	1.	0.98	1.	0.	7.97	9.1	1.35
time (sec)	N/A	0.047	0.073	0.006	0.	1.762	1.927	1.083

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	626	862	107
normalized size	1	1.	0.95	1.38	0.	7.73	10.64	1.32
time (sec)	N/A	0.098	0.079	0.006	0.	1.776	3.507	1.093

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	798	1525	142
normalized size	1	1.	0.98	1.44	0.	7.67	14.66	1.37
time (sec)	N/A	0.146	0.138	0.007	0.	1.946	5.173	1.132

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	979	2105	184
normalized size	1	1.	0.96	1.56	0.	7.15	15.36	1.34
time (sec)	N/A	0.203	0.114	0.009	0.	2.039	8.804	1.093

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	132	352	0	1760	842	217
normalized size	1	1.	0.88	2.35	0.	11.73	5.61	1.45
time (sec)	N/A	0.159	0.193	0.008	0.	1.598	1.625	1.116

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	109	209	0	1374	729	169
normalized size	1	1.	0.96	1.83	0.	12.05	6.39	1.48
time (sec)	N/A	0.103	0.153	0.009	0.	1.658	1.209	1.097

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	826	280	119
normalized size	1	1.	1.21	1.45	0.	12.33	4.18	1.78
time (sec)	N/A	0.039	0.094	0.006	0.	1.597	0.756	1.117

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	740	252	103
normalized size	1	1.	1.05	1.06	0.	11.21	3.82	1.56
time (sec)	N/A	0.038	0.068	0.002	0.	1.53	0.705	1.14

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	745	265	103
normalized size	1	1.	1.06	1.03	0.	11.29	4.02	1.56
time (sec)	N/A	0.035	0.079	0.003	0.	1.551	0.722	1.106

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	237	0	1685	2236	170
normalized size	1	1.	0.99	2.19	0.	15.6	20.7	1.57
time (sec)	N/A	0.148	0.236	0.012	0.	2.092	7.645	1.131

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	131	328	0	2049	2672	231
normalized size	1	1.	0.89	2.22	0.	13.84	18.05	1.56
time (sec)	N/A	0.198	0.305	0.013	0.	2.612	11.941	1.089

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	175	418	0	2606	4083	309
normalized size	1	1.	0.87	2.07	0.	12.9	20.21	1.53
time (sec)	N/A	0.25	0.428	0.016	0.	3.117	19.174	1.121

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	218	515	0	2989	4774	381
normalized size	1	1.	0.87	2.04	0.	11.86	18.94	1.51
time (sec)	N/A	0.323	0.341	0.016	0.	4.108	31.024	1.107

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	272	619	0	3563	6181	468
normalized size	1	1.	0.86	1.95	0.	11.2	19.44	1.47
time (sec)	N/A	0.392	0.413	0.018	0.	5.725	47.472	1.123

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	180	310	0	898	0	382
normalized size	1	1.	0.7	1.21	0.	3.49	0.	1.49
time (sec)	N/A	0.588	0.241	0.007	0.	1.736	0.	1.182

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	150	265	0	737	0	311
normalized size	1	1.	0.73	1.29	0.	3.6	0.	1.52
time (sec)	N/A	0.37	0.193	0.008	0.	1.646	0.	1.211

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	119	167	0	591	0	224
normalized size	1	1.	0.73	1.02	0.	3.63	0.	1.37
time (sec)	N/A	0.058	0.22	0.006	0.	1.615	0.	1.144

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	100	146	0	502	0	169
normalized size	1	1.	0.84	1.23	0.	4.22	0.	1.42
time (sec)	N/A	0.078	0.146	0.004	0.	1.669	0.	1.166

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	134	126	0	1457	0	0
normalized size	1	1.	0.77	0.73	0.	8.42	0.	0.
time (sec)	N/A	0.126	0.11	0.004	0.	1.842	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	174	0	1489	0	0
normalized size	1	1.	0.76	1.01	0.	8.61	0.	0.
time (sec)	N/A	0.124	0.127	0.006	0.	1.963	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	112	207	0	517	0	0
normalized size	1	1.	0.98	1.82	0.	4.54	0.	0.
time (sec)	N/A	0.148	0.1	0.006	0.	1.692	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	131	234	0	609	0	0
normalized size	1	1.	0.85	1.51	0.	3.93	0.	0.
time (sec)	N/A	0.256	0.144	0.006	0.	1.841	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	160	387	0	757	0	0
normalized size	1	1.	0.78	1.89	0.	3.69	0.	0.
time (sec)	N/A	0.386	0.225	0.006	0.	2.129	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	236	649	0	1600	0	703
normalized size	1	1.	0.56	1.54	0.	3.79	0.	1.67
time (sec)	N/A	1.203	0.426	0.008	0.	2.031	0.	1.294

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	197	479	0	1299	0	579
normalized size	1	1.	0.54	1.32	0.	3.57	0.	1.59
time (sec)	N/A	1.039	0.267	0.009	0.	1.985	0.	1.302

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	180	431	0	1099	0	493
normalized size	1	1.	0.62	1.5	0.	3.82	0.	1.71
time (sec)	N/A	0.519	0.242	0.009	0.	1.835	0.	1.325

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	163	289	0	878	0	383
normalized size	1	1.	0.82	1.46	0.	4.43	0.	1.93
time (sec)	N/A	0.178	0.182	0.006	0.	1.695	0.	1.273

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	132	265	0	725	0	313
normalized size	1	1.	0.8	1.61	0.	4.39	0.	1.9
time (sec)	N/A	0.128	0.064	0.006	0.	1.646	0.	1.28

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	166	222	0	1825	0	0
normalized size	1	1.	0.73	0.98	0.	8.04	0.	0.
time (sec)	N/A	0.255	0.238	0.005	0.	2.562	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	158	254	0	1735	0	0
normalized size	1	1.	0.72	1.16	0.	7.92	0.	0.
time (sec)	N/A	0.242	0.184	0.006	0.	2.218	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	162	338	0	1737	0	0
normalized size	1	1.	0.74	1.54	0.	7.93	0.	0.
time (sec)	N/A	0.238	0.199	0.006	0.	2.242	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	175	435	0	1871	0	0
normalized size	1	1.	0.68	1.69	0.	7.28	0.	0.
time (sec)	N/A	0.351	0.307	0.007	0.	2.5	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	141	501	0	743	0	0
normalized size	1	1.	0.72	2.54	0.	3.77	0.	0.
time (sec)	N/A	0.363	0.117	0.009	0.	2.341	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	177	534	0	898	0	0
normalized size	1	1.	0.71	2.14	0.	3.61	0.	0.
time (sec)	N/A	0.504	0.176	0.008	0.	2.71	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	105	144	0	514	0	0
normalized size	1	1.	0.73	1.01	0.	3.59	0.	0.
time (sec)	N/A	0.174	0.137	0.006	0.	1.569	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	88	0	429	0	146
normalized size	1	1.	0.86	0.85	0.	4.17	0.	1.42
time (sec)	N/A	0.078	0.054	0.006	0.	1.591	0.	1.156

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	65	0	298	0	50
normalized size	1	1.	0.93	0.92	0.	4.2	0.	0.7
time (sec)	N/A	0.037	0.041	0.006	0.	1.546	0.	1.159

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	66	0	300	0	80
normalized size	1	1.	1.56	1.47	0.	6.67	0.	1.78
time (sec)	N/A	0.016	0.021	0.005	0.	1.616	0.	1.157

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	89	88	0	444	0	0
normalized size	1	1.	1.16	1.14	0.	5.77	0.	0.
time (sec)	N/A	0.054	0.056	0.006	0.	1.732	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	112	152	0	527	0	0
normalized size	1	1.	0.94	1.28	0.	4.43	0.	0.
time (sec)	N/A	0.149	0.096	0.006	0.	1.791	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	183	283	0	1316	0	0
normalized size	1	1.	0.7	1.08	0.	5.02	0.	0.
time (sec)	N/A	0.506	0.258	0.009	0.	2.276	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	141	199	0	1026	0	263
normalized size	1	1.	0.7	0.99	0.	5.1	0.	1.31
time (sec)	N/A	0.305	0.159	0.006	0.	2.09	0.	1.17

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	112	166	0	873	0	149
normalized size	1	1.	0.73	1.08	0.	5.71	0.	0.97
time (sec)	N/A	0.175	0.123	0.006	0.	2.159	0.	1.192

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	53	0	150	0	61
normalized size	1	1.	0.92	1.32	0.	3.75	0.	1.52
time (sec)	N/A	0.04	0.074	0.004	0.	1.985	0.	1.192

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	0	151	0	61
normalized size	1	1.	0.92	1.33	0.	3.87	0.	1.56
time (sec)	N/A	0.04	0.024	0.003	0.	1.946	0.	1.165

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	109	164	0	869	0	0
normalized size	1	1.	1.16	1.74	0.	9.24	0.	0.
time (sec)	N/A	0.068	0.123	0.006	0.	2.065	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	138	201	0	1041	0	0
normalized size	1	1.	0.96	1.4	0.	7.23	0.	0.
time (sec)	N/A	0.163	0.106	0.006	0.	1.876	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	181	292	0	1328	0	0
normalized size	1	1.	0.87	1.4	0.	6.35	0.	0.
time (sec)	N/A	0.287	0.167	0.007	0.	2.208	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	225	340	0	1550	0	0
normalized size	1	1.	0.83	1.25	0.	5.72	0.	0.
time (sec)	N/A	0.452	0.265	0.009	0.	2.913	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	272	446	0	1901	0	0
normalized size	1	1.	0.79	1.3	0.	5.54	0.	0.
time (sec)	N/A	0.621	0.287	0.008	0.	4.085	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	77	0	161	280	144
normalized size	1	1.	0.92	2.08	0.	4.35	7.57	3.89
time (sec)	N/A	0.013	0.029	0.003	0.	1.313	1.09	1.112

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.008	0.002	0.002	1.127	1.051	0.056	1.098

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.007	0.002	0.	1.072	1.087	0.056	1.091

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	47	19	26
normalized size	1	1.	1.	0.8	1.04	1.88	0.76	1.04
time (sec)	N/A	0.004	0.	0.	1.114	1.049	0.056	1.1

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	39	15	22
normalized size	1	1.	1.	0.85	1.1	1.95	0.75	1.1
time (sec)	N/A	0.006	0.001	0.	1.131	1.23	0.056	1.1

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	46	17	27
normalized size	1	1.	1.	0.86	1.1	2.19	0.81	1.29
time (sec)	N/A	0.008	0.002	0.003	1.142	1.224	0.089	1.121

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	42	12	22
normalized size	1	1.	1.	0.94	1.22	2.33	0.67	1.22
time (sec)	N/A	0.007	0.002	0.003	1.112	1.191	0.248	1.091

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	300	0	602	1377	539
normalized size	1	1.	0.91	3.95	0.	7.92	18.12	7.09
time (sec)	N/A	0.046	0.079	0.006	0.	1.41	3.99	1.127

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	117	51	62
normalized size	1	1.	1.	0.83	1.09	2.17	0.94	1.15
time (sec)	N/A	0.036	0.007	0.001	1.119	1.015	0.07	1.086

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	116	46	62
normalized size	1	1.	0.89	0.83	1.09	2.15	0.85	1.15
time (sec)	N/A	0.054	0.009	0.001	1.09	1.123	0.07	1.09

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	115	51	62
normalized size	1	1.	1.	0.83	1.2	2.13	0.94	1.15
time (sec)	N/A	0.026	0.008	0.002	1.12	1.118	0.07	1.098

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	109	46	62
normalized size	1	1.	0.89	0.83	1.09	2.02	0.85	1.15
time (sec)	N/A	0.046	0.009	0.002	1.022	1.309	0.069	1.089

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	55	99	48	58
normalized size	1	1.	1.	0.86	1.12	2.02	0.98	1.18
time (sec)	N/A	0.027	0.005	0.001	1.128	1.099	0.072	1.078

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	675	391	124
normalized size	1	1.	0.93	1.42	0.	6.75	3.91	1.24
time (sec)	N/A	0.122	0.094	0.005	0.	1.329	1.892	1.1

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	3217	194	5466
normalized size	1	1.	1.23	2.3	0.	15.85	0.96	26.93
time (sec)	N/A	0.602	0.172	0.023	0.	1.54	2.207	2.487

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	556	316	101
normalized size	1	1.	0.96	1.37	0.	6.86	3.9	1.25
time (sec)	N/A	0.087	0.048	0.003	0.	1.278	1.541	1.095

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	2168	129	4593
normalized size	1	1.	1.13	1.92	0.	12.11	0.72	25.66
time (sec)	N/A	0.227	0.145	0.013	0.	1.412	1.604	2.336

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	443	223	80
normalized size	1	1.	0.98	0.95	0.	7.03	3.54	1.27
time (sec)	N/A	0.067	0.028	0.003	0.	1.316	0.813	1.089

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	1206	75	5350
normalized size	1	1.	1.1	1.39	0.	8.04	0.5	35.67
time (sec)	N/A	0.094	0.098	0.011	0.	1.522	0.768	2.206

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	290	131	47
normalized size	1	1.	1.08	1.	0.	8.06	3.64	1.31
time (sec)	N/A	0.043	0.009	0.	0.	1.539	0.445	1.116

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	1323	87	1365
normalized size	1	1.	0.86	0.77	0.	8.82	0.58	9.1
time (sec)	N/A	0.08	0.083	0.01	0.	1.571	0.896	1.359

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	510	253	92
normalized size	1	1.	1.64	0.96	0.	7.39	3.67	1.33
time (sec)	N/A	0.072	0.073	0.006	0.	1.588	2.868	1.096

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	2279	148	4581
normalized size	1	1.	1.1	1.33	0.	13.1	0.85	26.33
time (sec)	N/A	0.195	0.423	0.019	0.	1.625	1.808	2.354

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	664	345	127
normalized size	1	1.	1.52	1.34	0.	7.46	3.88	1.43
time (sec)	N/A	0.133	0.125	0.006	0.	2.017	7.246	1.095

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	151	383	0	1806	877	217
normalized size	1	1.	0.91	2.31	0.	10.88	5.28	1.31
time (sec)	N/A	0.223	0.209	0.013	0.	1.671	4.751	21.874

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	327	844	0	6311	450	0
normalized size	1	1.	0.99	2.55	0.	19.07	1.36	0.
time (sec)	N/A	0.703	0.713	0.033	0.	2.158	6.661	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	121	222	0	1412	745	205
normalized size	1	1.	0.92	1.68	0.	10.7	5.64	1.55
time (sec)	N/A	0.152	0.186	0.011	0.	1.385	3.505	25.605

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	282	602	0	4806	379	0
normalized size	1	1.	1.04	2.22	0.	17.73	1.4	0.
time (sec)	N/A	0.526	0.572	0.029	0.	1.723	4.632	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	864	282	130
normalized size	1	1.	1.19	1.33	0.	11.08	3.62	1.67
time (sec)	N/A	0.072	0.091	0.009	0.	1.366	1.746	29.874

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	235	452	0	3584	294	0
normalized size	1	1.	0.99	1.91	0.	15.12	1.24	0.
time (sec)	N/A	0.358	0.441	0.026	0.	1.395	3.359	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	778	267	111
normalized size	1	1.	1.05	1.03	0.	10.37	3.56	1.48
time (sec)	N/A	0.069	0.069	0.004	0.	1.282	1.667	25.713

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	3623	298	0
normalized size	1	1.	1.	1.55	0.	16.39	1.35	0.
time (sec)	N/A	0.24	0.462	0.063	0.	1.612	3.489	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	783	267	111
normalized size	1	1.	1.07	1.01	0.	10.58	3.61	1.5
time (sec)	N/A	0.065	0.085	0.004	0.	1.378	1.574	31.512

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	4918	394	0
normalized size	1	1.	0.96	2.91	0.	19.52	1.56	0.
time (sec)	N/A	0.461	0.461	0.044	0.	1.767	4.473	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	207	253	0	1728	772	224
normalized size	1	1.	1.7	2.07	0.	14.16	6.33	1.84
time (sec)	N/A	0.188	0.322	0.014	0.	1.784	40.968	22.145

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	302	712	0	6460	481	0
normalized size	1	1.	0.98	2.31	0.	20.97	1.56	0.
time (sec)	N/A	1.352	0.668	0.03	0.	2.108	7.145	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	248	352	0	2103	906	246
normalized size	1	1.	1.53	2.17	0.	12.98	5.59	1.52
time (sec)	N/A	0.25	0.275	0.017	0.	2.194	72.952	25.533

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	344	913	0	7960	566	0
normalized size	1	1.	0.95	2.53	0.	22.05	1.57	0.
time (sec)	N/A	3.077	0.788	0.036	0.	2.942	12.022	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	328	443	0	2627	1074	370
normalized size	1	1.	1.5	2.02	0.	12.	4.9	1.69
time (sec)	N/A	0.312	0.366	0.019	0.	2.724	131.412	23.764

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.095	0.017	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	486	1042	0	0	0	0
normalized size	1	1.	1.28	2.74	0.	0.	0.	0.
time (sec)	N/A	0.287	1.52	0.069	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	126	157	0	547	0	171
normalized size	1	1.	0.98	1.22	0.	4.24	0.	1.33
time (sec)	N/A	0.093	0.078	0.016	0.	1.397	0.	1.194

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	347	347	452	508	0	0	0	0
normalized size	1	1.	1.3	1.46	0.	0.	0.	0.
time (sec)	N/A	0.224	0.971	0.019	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	155	136	0	1575	0	0
normalized size	1	1.	0.8	0.7	0.	8.12	0.	0.
time (sec)	N/A	0.209	0.066	0.017	0.	1.641	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	192	369	0	921	0	0
normalized size	1	1.	0.79	1.51	0.	3.77	0.	0.
time (sec)	N/A	0.357	0.212	0.026	0.	1.444	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	487	487	609	1878	0	0	0	0
normalized size	1	1.	1.25	3.86	0.	0.	0.	0.
time (sec)	N/A	0.457	2.321	0.024	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	152	295	0	768	0	624
normalized size	1	1.	0.86	1.67	0.	4.34	0.	3.53
time (sec)	N/A	0.138	0.114	0.02	0.	1.511	0.	1.518

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	540	1394	0	0	0	0
normalized size	1	1.	1.27	3.28	0.	0.	0.	0.
time (sec)	N/A	0.449	1.748	0.023	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	72	0	325	0	54
normalized size	1	1.	1.	0.88	0.	3.96	0.	0.66
time (sec)	N/A	0.062	0.018	0.012	0.	1.469	0.	1.314

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	0	0	0
normalized size	1	1.	1.6	1.46	0.	0.	0.	0.
time (sec)	N/A	0.046	0.126	0.016	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	327	0	0
normalized size	1	1.	1.63	1.41	0.	6.41	0.	0.
time (sec)	N/A	0.029	0.019	0.014	0.	1.4	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	303	508	0	0	0	0
normalized size	1	1.	0.92	1.54	0.	0.	0.	0.
time (sec)	N/A	0.184	0.487	0.023	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	0	0	0
normalized size	1	1.	1.18	1.36	0.	0.	0.	0.
time (sec)	N/A	0.248	1.083	0.027	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	126	179	0	918	0	0
normalized size	1	1.	1.22	1.74	0.	8.91	0.	0.
time (sec)	N/A	0.073	0.083	0.019	0.	1.777	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0
normalized size	1	1.	1.11	2.43	0.	0.	0.	0.
time (sec)	N/A	0.409	1.351	0.029	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	160	220	0	1084	0	0
normalized size	1	1.	1.04	1.43	0.	7.04	0.	0.
time (sec)	N/A	0.175	0.082	0.019	0.	1.854	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	186	0	0
normalized size	1	1.	0.9	0.	0.	3.65	0.	0.
time (sec)	N/A	0.05	0.088	0.074	0.	1.657	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	239	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	5.135	0.02	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.009	0.018	0.011	0.	1.268	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.003	0.008	0.	1.37	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.004	0.003	0.	1.284	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.041	0.034	0.007	0.	1.296	0.	1.089

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.042	0.009	0.003	0.	1.288	0.	1.092

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	0	157	0	93
normalized size	1	1.	0.81	0.94	0.	1.83	0.	1.08
time (sec)	N/A	0.041	0.003	0.003	0.	1.293	0.	1.096

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	273	0	47
normalized size	1	1.	0.97	0.92	0.	7.18	0.	1.24
time (sec)	N/A	0.016	0.009	0.003	0.	1.361	0.	1.097

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	70	64	0	300	0	80
normalized size	1	1.	1.56	1.42	0.	6.67	0.	1.78
time (sec)	N/A	0.022	0.023	0.008	0.	1.394	0.	1.104

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	72	64	0	315	0	47
normalized size	1	1.	1.53	1.36	0.	6.7	0.	1.
time (sec)	N/A	0.076	0.033	0.011	0.	1.704	0.	1.145

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	74	66	0	325	0	47
normalized size	1	1.	1.51	1.35	0.	6.63	0.	0.96
time (sec)	N/A	0.09	0.028	0.006	0.	1.707	0.	1.157

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	294	0	0
normalized size	1	1.	1.	0.89	0.	6.68	0.	0.
time (sec)	N/A	0.036	0.012	0.004	0.	1.467	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	81	72	0	311	0	0
normalized size	1	1.	1.65	1.47	0.	6.35	0.	0.
time (sec)	N/A	0.015	0.016	0.005	0.	1.325	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	72	0	327	0	0
normalized size	1	1.	1.63	1.41	0.	6.41	0.	0.
time (sec)	N/A	0.067	0.02	0.008	0.	1.372	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	85	74	0	338	0	0
normalized size	1	1.	1.6	1.4	0.	6.38	0.	0.
time (sec)	N/A	0.077	0.019	0.007	0.	1.362	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	27	131	0	0
normalized size	1	1.	1.	0.78	0.68	3.28	0.	0.
time (sec)	N/A	0.027	0.009	0.003	1.646	1.311	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	58	0	142	0	0
normalized size	1	1.	1.62	1.29	0.	3.16	0.	0.
time (sec)	N/A	0.012	0.008	0.	0.	1.289	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	50	0	132	0	69
normalized size	1	1.	1.44	1.16	0.	3.07	0.	1.6
time (sec)	N/A	0.047	0.02	0.011	0.	1.393	0.	1.144

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.393	0.121	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [0.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	18	0.056
3	A	1	0	1.	16	0.
4	A	2	1	1.	20	0.05
5	A	2	1	1.	20	0.05
6	A	3	2	1.	22	0.091
7	A	3	2	1.	20	0.1
8	A	3	2	1.	18	0.111
9	A	3	2	1.	22	0.091
10	A	3	2	1.	22	0.091
11	A	7	6	1.	22	0.273
12	A	6	6	1.	22	0.273
13	A	5	5	1.	22	0.227
14	A	3	3	1.	22	0.136
15	A	7	7	1.	20	0.35
16	A	8	7	1.	18	0.389
17	A	8	7	1.	22	0.318
18	A	8	7	1.	22	0.318
19	A	8	7	1.	22	0.318
20	A	7	7	1.	22	0.318
21	A	4	4	1.	22	0.182
22	A	4	4	1.	22	0.182
23	A	4	4	1.	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	8	7	1.	22	0.318
25	A	8	7	1.	22	0.318
26	A	8	7	1.	20	0.35
27	A	8	7	1.	18	0.389
28	A	8	7	1.	22	0.318
29	A	8	6	1.	24	0.25
30	A	7	6	1.	22	0.273
31	A	5	5	1.	20	0.25
32	A	4	4	1.	24	0.167
33	A	7	6	1.	24	0.25
34	A	7	6	1.	24	0.25
35	A	5	5	1.	24	0.208
36	A	6	5	1.	24	0.208
37	A	7	5	1.	24	0.208
38	A	10	7	1.	22	0.318
39	A	10	7	1.	20	0.35
40	A	8	7	1.	24	0.292
41	A	6	5	1.	24	0.208
42	A	5	4	1.	24	0.167
43	A	8	7	1.	24	0.292
44	A	8	7	1.	24	0.292
45	A	8	7	1.	24	0.292
46	A	9	8	1.	24	0.333
47	A	7	6	1.	24	0.25
48	A	8	6	1.	24	0.25
49	A	6	6	1.	24	0.25
50	A	4	4	1.	24	0.167
51	A	3	3	1.	22	0.136
52	A	2	2	1.	20	0.1
53	A	3	3	1.	24	0.125
54	A	5	5	1.	24	0.208
55	A	8	6	1.	24	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	7	6	1.	24	0.25
57	A	6	6	1.	24	0.25
58	A	1	1	1.	24	0.042
59	A	1	1	1.	24	0.042
60	A	3	3	1.	24	0.125
61	A	5	5	1.	22	0.227
62	A	6	5	1.	20	0.25
63	A	7	5	1.	24	0.208
64	A	8	5	1.	24	0.208
65	A	2	1	1.	18	0.056
66	A	2	1	1.	18	0.056
67	A	2	1	1.	16	0.062
68	A	1	0	1.	14	0.
69	A	2	1	1.	18	0.056
70	A	2	1	1.	18	0.056
71	A	2	1	1.	18	0.056
72	A	3	2	1.	20	0.1
73	A	3	2	1.	20	0.1
74	A	4	3	1.	18	0.167
75	A	3	2	1.	16	0.125
76	A	4	3	1.	20	0.15
77	A	3	2	1.	20	0.1
78	A	8	7	1.	20	0.35
79	A	6	5	1.	20	0.25
80	A	7	7	1.	20	0.35
81	A	5	4	1.	20	0.2
82	A	6	6	1.	20	0.3
83	A	4	3	1.	20	0.15
84	A	4	4	1.	20	0.2
85	A	4	3	1.	18	0.167
86	A	8	8	1.	16	0.5
87	A	5	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	9	8	1.	20	0.4
89	A	9	8	1.	20	0.4
90	A	7	5	1.	20	0.25
91	A	8	8	1.	20	0.4
92	A	6	5	1.	20	0.25
93	A	5	5	1.	20	0.25
94	A	5	4	1.	20	0.2
95	A	5	5	1.	20	0.25
96	A	5	4	1.	20	0.2
97	A	5	5	1.	20	0.25
98	A	5	4	1.	20	0.2
99	A	9	8	1.	18	0.444
100	A	6	5	1.	16	0.312
101	A	9	8	1.	20	0.4
102	A	7	5	1.	20	0.25
103	A	9	8	1.	20	0.4
104	A	3	3	1.	20	0.15
105	A	5	5	1.	24	0.208
106	A	5	5	1.	24	0.208
107	A	5	5	1.	24	0.208
108	A	8	7	1.	24	0.292
109	A	8	8	1.	24	0.333
110	A	6	6	1.	24	0.25
111	A	6	5	1.	24	0.208
112	A	6	6	1.	24	0.25
113	A	4	4	1.	24	0.167
114	A	2	2	1.	24	0.083
115	A	2	2	1.	24	0.083
116	A	6	6	1.	24	0.25
117	A	5	5	1.	24	0.208
118	A	3	3	1.	24	0.125
119	A	6	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	5	5	1.	24	0.208
121	A	1	1	1.	34	0.029
122	A	7	4	1.	27	0.148
123	A	2	2	1.	18	0.111
124	A	3	3	1.	18	0.167
125	A	3	3	1.	17	0.176
126	A	5	5	1.	18	0.278
127	A	6	6	1.	18	0.333
128	A	6	6	1.	17	0.353
129	A	2	2	1.	18	0.111
130	A	3	3	1.	18	0.167
131	A	3	3	1.	22	0.136
132	A	3	3	1.	24	0.125
133	A	3	3	1.	20	0.15
134	A	3	3	1.	20	0.15
135	A	3	3	1.	24	0.125
136	A	3	3	1.	26	0.115
137	A	3	3	1.	18	0.167
138	A	3	3	1.	18	0.167
139	A	3	3	1.	20	0.15
140	A	2	2	1.	36	0.056

Chapter 3

Listing of integrals

3.1 $\int x^2 (ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7

Rubi [A] time = 0.0074164, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {14}

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7

Rule 14

Int[(u_)*((c_)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)

+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\int x^2 (ax^2 + bx^3 + cx^4) dx = \int (ax^4 + bx^5 + cx^6) dx$$

$$= \frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Mathematica [A] time = 0.0022189, size = 25, normalized size = 1.

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^3+a*x^2),x)

[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7

Maxima [A] time = 1.09851, size = 26, normalized size = 1.04

$$\frac{1}{7} cx^7 + \frac{1}{6} bx^6 + \frac{1}{5} ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] $1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5$

Fricas [A] time = 1.27924, size = 47, normalized size = 1.88

$$\frac{1}{7}x^7c + \frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")`

[Out] $1/7*x^7*c + 1/6*x^6*b + 1/5*x^5*a$

Sympy [A] time = 0.058303, size = 19, normalized size = 0.76

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**4+b*x**3+a*x**2),x)`

[Out] $a*x**5/5 + b*x**6/6 + c*x**7/7$

Giac [A] time = 1.09862, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")`

[Out] $1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5$

3.2 $\int x(ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6

Rubi [A] time = 0.0073601, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3 + c*x^4), x]

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x(ax^2 + bx^3 + cx^4) dx &= \int (ax^3 + bx^4 + cx^5) dx \\ &= \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6} \end{aligned}$$

Mathematica [A] time = 0.0015328, size = 25, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2),x)

[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6

Maxima [A] time = 1.15779, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4

Fricas [A] time = 1.30137, size = 47, normalized size = 1.88

$$\frac{1}{6}x^6c + \frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] $1/6*x^6*c + 1/5*x^5*b + 1/4*x^4*a$

Sympy [A] time = 0.05838, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**4+b*x**3+a*x**2),x)`

[Out] $a*x**4/4 + b*x**5/5 + c*x**6/6$

Giac [A] time = 1.10149, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")`

[Out] $1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4$

3.3 $\int (ax^2 + bx^3 + cx^4) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5

Rubi [A] time = 0.00384, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[a*x^2 + b*x^3 + c*x^4,x]

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5

Rubi steps

$$\int (ax^2 + bx^3 + cx^4) dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Mathematica [A] time = 0.0000454, size = 25, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(c*x^4+b*x^3+a*x^2,x)`

[Out] `1/3*a*x^3+1/4*b*x^4+1/5*c*x^5`

Maxima [A] time = 1.0763, size = 26, normalized size = 1.04

$$\frac{1}{5}cx^5 + \frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")`

[Out] `1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3`

Fricas [A] time = 1.36483, size = 47, normalized size = 1.88

$$\frac{1}{5}x^5c + \frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")`

[Out] `1/5*x^5*c + 1/4*x^4*b + 1/3*x^3*a`

Sympy [A] time = 0.057533, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

Giac [A] time = 1.08135, size = 26, normalized size = 1.04

$$\frac{1}{5}cx^5 + \frac{1}{4}bx^4 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

$$3.4 \quad \int \frac{ax^2+bx^3+cx^4}{x} dx$$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4

Rubi [A] time = 0.0061748, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3 + cx^4}{x} dx &= \int (ax + bx^2 + cx^3) dx \\ &= \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4} \end{aligned}$$

Mathematica [A] time = 0.0014316, size = 25, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)/x,x)

[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4

Maxima [A] time = 1.07064, size = 26, normalized size = 1.04

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2

Fricas [A] time = 1.44192, size = 47, normalized size = 1.88

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")

[Out] $1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2$

Sympy [A] time = 0.057205, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)/x,x)`

[Out] $a*x**2/2 + b*x**3/3 + c*x**4/4$

Giac [A] time = 1.0827, size = 26, normalized size = 1.04

$$\frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")`

[Out] $1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2$

$$3.5 \quad \int \frac{ax^2+bx^3+cx^4}{x^2} dx$$

Optimal. Leaf size=20

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Rubi [A] time = 0.0063249, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {14}

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax^2 + bx^3 + cx^4}{x^2} dx &= \int (a + bx + cx^2) dx \\ &= ax + \frac{bx^2}{2} + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.001181, size = 20, normalized size = 1.

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)/x^2,x)

[Out] a*x+1/2*b*x^2+1/3*c*x^3

Maxima [A] time = 1.11766, size = 22, normalized size = 1.1

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")

[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x

Fricas [A] time = 1.46768, size = 39, normalized size = 1.95

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")

[Out] $1/3*c*x^3 + 1/2*b*x^2 + a*x$

Sympy [A] time = 0.056357, size = 15, normalized size = 0.75

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)/x**2,x)`

[Out] $a*x + b*x**2/2 + c*x**3/3$

Giac [A] time = 1.07503, size = 22, normalized size = 1.1

$$\frac{1}{3}cx^3 + \frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")`

[Out] $1/3*c*x^3 + 1/2*b*x^2 + a*x$

3.6 $\int x^2 (ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^11)/11

Rubi [A] time = 0.053594, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^11)/11

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 698

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned} \int x^2 (ax^2 + bx^3 + cx^4)^2 dx &= \int x^6 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^6 + 2abx^7 + (b^2 + 2ac)x^8 + 2bcx^9 + c^2x^{10}) dx \\ &= \frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11} \end{aligned}$$

Mathematica [A] time = 0.0101222, size = 54, normalized size = 1.

$$\frac{a^2x^7}{7} + \frac{1}{9}x^9(2ac + b^2) + \frac{1}{4}abx^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^11)/11

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{(2ac + b^2)x^9}{9} + \frac{bcx^{10}}{5} + \frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)

[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11

Maxima [A] time = 1.16539, size = 59, normalized size = 1.09

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{4}abx^8 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $1/11*c^2*x^{11} + 1/5*b*c*x^{10} + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^2*x^7$

Fricas [A] time = 1.26837, size = 116, normalized size = 2.15

$$\frac{1}{11}x^{11}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $1/11*x^{11}*c^2 + 1/5*x^{10}*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 1/4*x^8*b*a + 1/7*x^7*a^2$

Sympy [A] time = 0.070734, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{bcx^{10}}{5} + \frac{c^2x^{11}}{11} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b**2/9)$

Giac [A] time = 1.07082, size = 62, normalized size = 1.15

$$\frac{1}{11}c^2x^{11} + \frac{1}{5}bcx^{10} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{1}{4}abx^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $1/11*c^2*x^{11} + 1/5*b*c*x^{10} + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7$

3.7 $\int x (ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*x^10)/10

Rubi [A] time = 0.028877, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1585, 698}

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*x^10)/10

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 698

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned}
 \int x(ax^2 + bx^3 + cx^4)^2 dx &= \int x^5(a + bx + cx^2)^2 dx \\
 &= \int (a^2x^5 + 2abx^6 + (b^2 + 2ac)x^7 + 2bcx^8 + c^2x^9) dx \\
 &= \frac{a^2x^6}{6} + \frac{2}{7}abx^7 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}
 \end{aligned}$$

Mathematica [A] time = 0.0068741, size = 54, normalized size = 1.

$$\frac{a^2x^6}{6} + \frac{1}{8}x^8(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*x^10)/10

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{(2ac + b^2)x^8}{8} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2)^2,x)

[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10

Maxima [A] time = 1.12733, size = 59, normalized size = 1.09

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{2}{7}abx^7 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $1/10*c^2*x^{10} + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2*x^6$

Fricas [A] time = 1.28945, size = 115, normalized size = 2.13

$$\frac{1}{10}x^{10}c^2 + \frac{2}{9}x^9cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{2}{7}x^7ba + \frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $1/10*x^{10}*c^2 + 2/9*x^9*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 2/7*x^7*b*a + 1/6*x^6*a^2$

Sympy [A] time = 0.069998, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b**2/8)$

Giac [A] time = 1.08124, size = 62, normalized size = 1.15

$$\frac{1}{10}c^2x^{10} + \frac{2}{9}bcx^9 + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{2}{7}abx^7 + \frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $1/10*c^2*x^{10} + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6$

3.8 $\int (ax^2 + bx^3 + cx^4)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)/9

Rubi [A] time = 0.0265261, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1594, 698}

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)/9

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned} \int (ax^2 + bx^3 + cx^4)^2 dx &= \int x^4 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^4 + 2abx^5 + (b^2 + 2ac)x^6 + 2bcx^7 + c^2x^8) dx \\ &= \frac{a^2x^5}{5} + \frac{1}{3}abx^6 + \frac{1}{7}(b^2 + 2ac)x^7 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A] time = 0.0060407, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{1}{7}x^7(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{4}bcx^8 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)/9

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{(2ac + b^2)x^7}{7} + \frac{bcx^8}{4} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^2,x)

[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9

Maxima [A] time = 1.33673, size = 65, normalized size = 1.2

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{1}{5}a^2x^5 + \frac{1}{21}(6cx^7 + 7bx^6)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{9}c^2x^9 + \frac{1}{4}b^2cx^8 + \frac{1}{7}b^2x^7 + \frac{1}{5}a^2x^5 + \frac{1}{21}(6cx^7 + 7b^2x^6)a$

Fricas [A] time = 1.36451, size = 112, normalized size = 2.07

$$\frac{1}{9}x^9c^2 + \frac{1}{4}x^8cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{1}{3}x^6ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] $\frac{1}{9}x^9c^2 + \frac{1}{4}x^8c^2b + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7c^2a + \frac{1}{3}x^6b^2a + \frac{1}{5}x^5a^2$

Sympy [A] time = 0.073278, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**4+b*x**3+a*x**2)**2,x)`

[Out] $a^2x^5/5 + abx^6/3 + bcx^8/4 + c^2x^9/9 + x^7(2ac/7 + b^2/7)$

Giac [A] time = 1.09585, size = 62, normalized size = 1.15

$$\frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{1}{3}abx^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")`

[Out] $\frac{1}{9}c^2x^9 + \frac{1}{4}b^2cx^8 + \frac{1}{7}b^2x^7 + \frac{2}{7}a^2cx^7 + \frac{1}{3}a^2bx^6 + \frac{1}{5}a^2x^5$

$$3.9 \quad \int \frac{(ax^2+bx^3+cx^4)^2}{x} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*x^8)/8

Rubi [A] time = 0.0320555, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*x^8)/8

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^2}{x} dx &= \int x^3 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^3 + 2abx^4 + (b^2 + 2ac)x^5 + 2bcx^6 + c^2x^7) dx \\ &= \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8} \end{aligned}$$

Mathematica [A] time = 0.0062127, size = 54, normalized size = 1.

$$\frac{a^2x^4}{4} + \frac{1}{6}x^6(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]

[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*x^8)/8

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{(2ac + b^2)x^6}{6} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^2/x,x)

[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8

Maxima [A] time = 1.12473, size = 59, normalized size = 1.09

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")

[Out] $\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$

Fricas [A] time = 1.42506, size = 107, normalized size = 1.98

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")

[Out] $\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{2}{5}abx^5 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}a^2x^4$

Sympy [A] time = 0.070307, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)

[Out] $a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**2/6)$

Giac [A] time = 1.09907, size = 62, normalized size = 1.15

$$\frac{1}{8}c^2x^8 + \frac{2}{7}bcx^7 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")

[Out] $\frac{1}{8}c^2x^8 + \frac{2}{7}b^2cx^7 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{2}{5}abx^5 + \frac{1}{4}a^2x^4$

$$3.10 \quad \int \frac{(ax^2+bx^3+cx^4)^2}{x^2} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

Rubi [A] time = 0.0324322, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1585, 698}

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 698

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2} dx &= \int x^2 (a + bx + cx^2)^2 dx \\ &= \int (a^2x^2 + 2abx^3 + (b^2 + 2ac)x^4 + 2bcx^5 + c^2x^6) dx \\ &= \frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7} \end{aligned}$$

Mathematica [A] time = 0.0071731, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)/7

Maple [A] time = 0., size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{(2ac + b^2)x^5}{5} + \frac{bcx^6}{3} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x)

[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7

Maxima [A] time = 1.15267, size = 59, normalized size = 1.09

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")

[Out] $\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$

Fricas [A] time = 1.45505, size = 107, normalized size = 1.98

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")

[Out] $\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{2}abx^4 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{1}{3}a^2x^3$

Sympy [A] time = 0.072056, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)

[Out] $a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/5)$

Giac [A] time = 1.07228, size = 62, normalized size = 1.15

$$\frac{1}{7}c^2x^7 + \frac{1}{3}bcx^6 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")

[Out] $\frac{1}{7}c^2x^7 + \frac{1}{3}b^2cx^6 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=89

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

[Out] $-\left(\frac{b*x}{c^2}\right) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^3)$

Rubi [A] time = 0.093805, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1585, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} - \frac{bx}{c^2} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]

[Out] $-\left(\frac{b*x}{c^2}\right) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^3)$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 701

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x^3}{a + bx + cx^2} dx \\
&= \int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab + (b^2 - ac)x}{c^2(a + bx + cx^2)} \right) dx \\
&= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{\int \frac{ab + (b^2 - ac)x}{a + bx + cx^2} dx}{c^2} \\
&= -\frac{bx}{c^2} + \frac{x^2}{2c} - \frac{(b(b^2 - 3ac)) \int \frac{1}{a + bx + cx^2} dx}{2c^3} + \frac{(b^2 - ac) \int \frac{b + 2cx}{a + bx + cx^2} dx}{2c^3} \\
&= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3} + \frac{(b(b^2 - 3ac)) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^3} \\
&= -\frac{bx}{c^2} + \frac{x^2}{2c} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx + cx^2)}{2c^3}
\end{aligned}$$

Mathematica [A] time = 0.109711, size = 84, normalized size = 0.94

$$\frac{(b^2 - ac) \log(a + x(b + cx)) - \frac{2b(b^2 - 3ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + cx(cx - 2b)}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4), x]

[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)]/(2*c^3)

Maple [A] time = 0.003, size = 132, normalized size = 1.5

$$\frac{x^2}{2c} - \frac{bx}{c^2} - \frac{\ln(cx^2 + bx + a)a}{2c^2} + \frac{\ln(cx^2 + bx + a)b^2}{2c^3} + 3 \frac{ab}{c^2 \sqrt{4ac - b^2}} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) - \frac{b^3}{c^3} \arctan\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^4+b*x^3+a*x^2), x)

[Out] 1/2*x^2/c-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+a)*b^2+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.59039, size = 653, normalized size = 7.34

$$\left[\frac{(b^2c^2 - 4ac^3)x^2 - (b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - 2(b^3c - 4abc^2)x + (b^4 - 5ab^2c + 4a^2c^2)}{2(b^2c^3 - 4ac^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]

Sympy [B] time = 0.783942, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac + b^2}(3ac - b^2)}{2c^3(4ac - b^2)} - \frac{ac - b^2}{2c^3} \right) \log \left(x + \frac{2a^2c - ab^2 + 4ac^3 \left(-\frac{b\sqrt{-4ac + b^2}(3ac - b^2)}{2c^3(4ac - b^2)} - \frac{ac - b^2}{2c^3} \right) - b^2c^2 \left(-\frac{b\sqrt{-4ac + b^2}(3ac - b^2)}{2c^3(4ac - b^2)} - \frac{ac - b^2}{2c^3} \right)}{3abc - b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)

[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + x**2/(2*c)

Giac [A] time = 1.08281, size = 116, normalized size = 1.3

$$\frac{cx^2 - 2bx}{2c^2} + \frac{(b^2 - ac) \log(cx^2 + bx + a)}{2c^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=70

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

Rubi [A] time = 0.0587906, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1585, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4), x]

[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol]
:> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x]
;/; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_)^(m_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x]
;/; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x^2}{a + bx + cx^2} dx \\
&= \frac{x}{c} + \frac{\int \frac{-a-bx}{a+bx+cx^2} dx}{c} \\
&= \frac{x}{c} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} + \frac{(b^2 - 2ac) \int \frac{1}{a+bx+cx^2} dx}{2c^2} \\
&= \frac{x}{c} - \frac{b \log(a + bx + cx^2)}{2c^2} - \frac{(b^2 - 2ac) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c^2} \\
&= \frac{x}{c} - \frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx + cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.0636759, size = 73, normalized size = 1.04

$$\frac{(b^2 - 2ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) - \frac{b \log(a + bx + cx^2)}{2c^2} + \frac{x}{c}}{c^2 \sqrt{4ac - b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]

[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)

Maple [A] time = 0.003, size = 101, normalized size = 1.4

$$\frac{x}{c} - \frac{b \ln(cx^2 + bx + a)}{2c^2} - 2 \frac{a}{c\sqrt{4ac - b^2}} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) + \frac{b^2}{c^2} \arctan\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^3+a*x^2),x)

[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.6885, size = 537, normalized size = 7.67

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - 2(b^2c - 4ac^2)x + (b^3 - 4abc) \log(cx^2 + bx + a)}{2(b^2c^2 - 4ac^3)}, - \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]

Sympy [B] time = 0.647319, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) \log\left(x + \frac{-ab - 4ac^2\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right) + b^2c\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{2c^2(4ac - b^2)}\right)}{2ac - b^2}\right) + \left(-\frac{b}{2c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**3+a*x**2),x)

[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))/(2*a*c - b**2)) + x/c

Giac [A] time = 1.08301, size = 90, normalized size = 1.29

$$\frac{x}{c} - \frac{b \log(cx^2 + bx + a)}{2c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)

3.13

$$\int \frac{x^3}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=56

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x + c*x^2]/(2*c)

Rubi [A] time = 0.0395897, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1585, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x + c*x^2]/(2*c)

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax^2 + bx^3 + cx^4} dx &= \int \frac{x}{a + bx + cx^2} dx \\ &= \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2c} \\ &= \frac{\log(a + bx + cx^2)}{2c} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{c} \\ &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a + bx + cx^2)}{2c} \end{aligned}$$

Mathematica [A] time = 0.0318506, size = 57, normalized size = 1.02

$$\frac{\log(a + x(b + cx)) - \frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4), x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.001, size = 56, normalized size = 1.

$$\frac{\ln(cx^2 + bx + a)}{2c} - \frac{b}{c} \arctan\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c*x^4+b*x^3+a*x^2),x)`

[Out] `1/2*ln(c*x^2+b*x+a)/c-1/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.55918, size = 427, normalized size = 7.62

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^2 - 4ac) \log(cx^2 + bx + a)}{2(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} b \arctan\left(-\frac{\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right)}{2(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")`

[Out] `[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c`

- 4*a*c^2)]

Sympy [B] time = 0.301536, size = 216, normalized size = 3.86

$$\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right) \log \left(x + \frac{-4ac \left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right) + 2a + b^2 \left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right)}{b} \right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right) \log \left(x + \frac{-4ac \left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right) + 2a + b^2 \left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)} + \frac{1}{2c} \right)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)

[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b)

Giac [A] time = 1.10495, size = 74, normalized size = 1.32

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \frac{\log(cx^2+bx+a)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(c*x^2 + b*x + a)/c

$$3.14 \quad \int \frac{x^2}{ax^2+bx^3+cx^4} dx$$

Optimal. Leaf size=34

$$-\frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

[Out] $(-2*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/\text{Sqrt}[b^2 - 4*a*c]$

Rubi [A] time = 0.0259065, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1585, 618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(a*x^2 + b*x^3 + c*x^4), x]$

[Out] $(-2*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/\text{Sqrt}[b^2 - 4*a*c]$

Rule 1585

$\text{Int}[(u_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)} + (c_.)*(x_.)^{(r_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(m+n*p)}*(a + b*x^{(q-p)} + c*x^{(r-p)})^n, x] /;$ FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 618

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{(-1)}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{a + bx + cx^2} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)\right) \\ &= -\frac{2 \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}} \end{aligned}$$

Mathematica [A] time = 0.0063149, size = 38, normalized size = 1.12

$$\frac{2 \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4), x]

[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]

Maple [A] time = 0.002, size = 35, normalized size = 1.

$$2 \frac{1}{\sqrt{4ac-b^2}} \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^3+a*x^2), x)

[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.57138, size = 277, normalized size = 8.15

$$\left[\frac{\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, -\frac{2\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{b^2-4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")`

[Out] `[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]`

Sympy [B] time = 0.201467, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}} + b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}} \log\left(x + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}} - b^2\sqrt{-\frac{1}{4ac-b^2}} + b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**4+b*x**3+a*x**2),x)`

[Out] `-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))`

Giac [A] time = 1.11321, size = 46, normalized size = 1.35

$$\frac{2 \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```


3.15 $\int \frac{x}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=62

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x + c*x^2]/(2*a)

Rubi [A] time = 0.0469107, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1585, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} - \frac{\log(a+bx+cx^2)}{2a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4), x]

[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x + c*x^2]/(2*a)

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 705

Int[1/(((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 634

`Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 206

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 628

`Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{x(a + bx + cx^2)} dx \\
 &= \frac{\int \frac{1}{x} dx}{a} + \frac{\int \frac{-b-cx}{a+bx+cx^2} dx}{a} \\
 &= \frac{\log(x)}{a} - \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2a} - \frac{b \int \frac{1}{a+bx+cx^2} dx}{2a} \\
 &= \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a} + \frac{b \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{a} \\
 &= \frac{b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx + cx^2)}{2a}
 \end{aligned}$$

Mathematica [A] time = 0.0729725, size = 61, normalized size = 0.98

$$\frac{2b \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \log(a + x(b + cx)) - 2 \log(x)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]

[Out] -((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*Log[x] + Log[a + x*(b + c*x)])/(2*a)

Maple [A] time = 0.006, size = 62, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln(cx^2 + bx + a)}{2a} - \frac{b}{a} \arctan\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^4+b*x^3+a*x^2),x)

[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.76169, size = 494, normalized size = 7.97

$$\left[\frac{\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - (b^2 - 4ac) \log(cx^2 + bx + a) + 2(b^2 - 4ac) \log(x) \sqrt{-b^2 + 4acb}}{2(ab^2 - 4a^2c)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")

[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

Sympy [B] time = 1.92687, size = 564, normalized size = 9.1

$$\left(-\frac{b\sqrt{-4ac + b^2}}{2a(4ac - b^2)} - \frac{1}{2a} \right) \log \left(x + \frac{24a^4c^2 \left(-\frac{b\sqrt{-4ac + b^2}}{2a(4ac - b^2)} - \frac{1}{2a} \right)^2 - 14a^3b^2c \left(-\frac{b\sqrt{-4ac + b^2}}{2a(4ac - b^2)} - \frac{1}{2a} \right)^2 - 12a^3c^2 \left(-\frac{b\sqrt{-4ac + b^2}}{2a(4ac - b^2)} - \frac{1}{2a} \right) + 2a}{9abc^2 - 2b^3c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)

[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c))

*3*c)) + log(x)/a

Giac [A] time = 1.08312, size = 84, normalized size = 1.35

$$-\frac{b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - \frac{\log(cx^2+bx+a)}{2a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(c*x^2 + b*x + a)/a + log(abs(x))/a

3.16 $\int \frac{1}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

[Out] $-(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)$

Rubi [A] time = 0.0976829, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{b \log(x)}{a^2} - \frac{1}{ax}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3 + c*x^4)^{-1}, x]$

[Out] $-(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)$

Rule 1594

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)} + c*x^{(r-p)})^n, x] /;$ FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 709

$\text{Int}[(d_.) + (e_.)*(x_)^{(m_.)}/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{(m+1})/((m+1)*(c*d^2 - b*d*e + a*e^2))), x] + \text{Dist}[1/(c*d^2 - b*d*e + a*e^2), \text{Int}[(d + e*x)^{(m+1)}*\text{Simp}[c*d - b*e - c*e*x, x]/(a + b*x + c*x^2), x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax^2 + bx^3 + cx^4} dx &= \int \frac{1}{x^2(a + bx + cx^2)} dx \\
&= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx}{a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^2} + \frac{(b^2 - 2ac) \int \frac{1}{a+bx+cx^2} dx}{2a^2} \\
&= -\frac{1}{ax} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx \right)}{a^2} \\
&= -\frac{1}{ax} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{a^2 \sqrt{b^2 - 4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx + cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.0789887, size = 77, normalized size = 0.95

$$\frac{2(b^2-2ac) \tan^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right) + b \log(a + x(b + cx)) - \frac{2a}{x} - 2b \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]

[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)

Maple [A] time = 0.006, size = 112, normalized size = 1.4

$$-\frac{1}{ax} - \frac{b \ln(x)}{a^2} + \frac{b \ln(cx^2 + bx + a)}{2a^2} - 2 \frac{c}{a\sqrt{4ac - b^2}} \arctan \left(\frac{2cx + b}{\sqrt{4ac - b^2}} \right) + \frac{b^2}{a^2} \arctan \left((2cx + b) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+b*x^3+a*x^2),x)`

[Out]
$$-1/a/x - b \ln(x)/a^2 + 1/2 * b \ln(c*x^2 + b*x + a)/a^2 - 2/a / (4*a*c - b^2)^{(1/2)} * \arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)}) * c + 1/a^2 / (4*a*c - b^2)^{(1/2)} * \arctan((2*c*x + b)/(4*a*c - b^2)^{(1/2)}) * b^2$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.77597, size = 626, normalized size = 7.73

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac}x \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2ab^2 - 8a^2c - (b^3 - 4abc)x \log(cx^2 + bx + a) + 2(b^3 - 4abc)x}{2(a^2b^2 - 4a^3c)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")`

[Out]
$$\left[-1/2 * ((b^2 - 2*a*c) * \sqrt{b^2 - 4*a*c}) * x * \log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c} * (2*c*x + b)) / (c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c) * x * \log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c) * x * \log(x) / ((a^2*b^2 - 4*a^3*c) * x), -1/2 * (2*(b^2 - 2*a*c) * \sqrt{-b^2 + 4*a*c}) * x * \arctan(-\sqrt{-b^2 + 4*a*c} * (2*c*x + b) / (b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a*b*c) * x * \log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c) * x * \log(x) / ((a^2*b^2 - 4*a^3*c) * x) \right]$$

Sympy [B] time = 3.50723, size = 862, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)

[Out] (b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))))**2 - 4*a**5*c**3*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))))**2 + 15*a**5*b**3*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))))**2 - 4*a**5*c**3*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2 + 2*b**6*c)) - 1/(a*x) - b*log(x)/a**2

Giac [A] time = 1.09282, size = 107, normalized size = 1.32

$$\frac{b \log(cx^2 + bx + a)}{2a^2} - \frac{b \log(|x|)}{a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^2} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)

$$3.17 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)} dx$$

Optimal. Leaf size=104

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)(b^2-ac)}{a^3} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)$

Rubi [A] time = 0.145913, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{\log(x)(b^2-ac)}{a^3} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{b}{a^2x} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] $-1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 709

Int[((d_)+(e_)*(x_))^(m_)/((a_)+(b_)*(x_)+(c_)*(x_)^2), x_Symbol] := Simp[(e*(d+e*x)^(m+1))/((m+1)*(c*d^2-b*d*e+a*e^2)), x] + Dist[1/(c*d^2-b*d*e+a*e^2), Int[((d+e*x)^(m+1)*Simp[c*d-b*e-c*e*x, x])/(a+b*x+c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2-

$4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{LtQ}[m, -1]$

Rule 800

$\text{Int}[(((d_.) + (e_.)*(x_))^{(m_)}*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \text{IntegerQ}[m]$

Rule 634

$\text{Int}[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 206

$\text{Int}[((a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3 + cx^4)} dx &= \int \frac{1}{x^3(a + bx + cx^2)} dx \\
&= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2} dx}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac)) \int \frac{1}{a+bx+cx^2} dx}{2a^3} - \frac{(b^2-ac) \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^2-3ac)) \text{Subst}\left(\int \frac{1}{a^2}\right)}{a^3} \\
&= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.137503, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2} + 2\log(x)(b^2-ac) + (ac-b^2)\log(a+x(b+cx)) - \frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] $(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c))*\text{ArcTan}[(b + 2*c*x)/\text{Sqrt}[-b^2 + 4*a*c]])/\text{Sqrt}[-b^2 + 4*a*c] + 2*(b^2 - a*c)*\text{Log}[x] + (-b^2 + a*c)*\text{Log}[a + x*(b + c*x)]/(2*a^3)$

Maple [A] time = 0.007, size = 150, normalized size = 1.4

$$-\frac{1}{2ax^2} - \frac{\ln(x)c}{a^2} + \frac{\ln(x)b^2}{a^3} + \frac{b}{xa^2} + \frac{c\ln(cx^2 + bx + a)}{2a^2} - \frac{\ln(cx^2 + bx + a)b^2}{2a^3} + 3\frac{bc}{a^2\sqrt{4ac-b^2}} \arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^4+b*x^3+a*x^2),x)`

[Out]
$$-1/2/a/x^2-1/a^2*\ln(x)*c+1/a^3*\ln(x)*b^2+b/a^2/x+1/2/a^2*c*\ln(c*x^2+b*x+a)-1/2/a^3*\ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 1.94557, size = 798, normalized size = 7.67

$$\left[\frac{(b^3 - 3abc)\sqrt{b^2 - 4ac}x^2 \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + a^2b^2 - 4a^3c + (b^4 - 5ab^2c + 4a^2c^2)x^2 \log(cx^2 + bx + a)}{2(a^3b^2 - 4a^4c)x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")`

[Out]
$$\left[-1/2*((b^3 - 3*a*b*c)*\text{sqrt}(b^2 - 4*a*c))*x^2*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - \text{sqrt}(b^2 - 4*a*c))*(2*c*x + b))/(c*x^2 + b*x + a) + a^2*b^2 - 4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(c*x^2 + b*x + a) - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*\text{sqrt}(-b^2 + 4*a*c))*x^2*\arctan(-\text{sqrt}(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*\log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2) \right]$$

Sympy [B] time = 5.17258, size = 1525, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)

[Out]
$$\begin{aligned} & (-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) \cdot \log(x + (24a^9c^3(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 - 42a^8b^2c^2(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 + 17a^7b^4c(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 + 12a^7c^4(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) - 2a^6b^6(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 - 15a^6b^2c^3(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) + 7a^5b^4c^2(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) - 12a^5c^5 - a^4b^6c(-b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) + 63a^4b^2c^4 - 103a^3b^4c^3 + 70a^2b^6c^2 - 20ab^8c + 2b^{10})/(27a^4b^5c^5 - 63a^3b^3c^4 + 54a^2b^5c^3 - 18ab^7c^2 + 2b^9c) + (b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) \cdot \log(x + (24a^9c^3(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 - 42a^8b^2c^2(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 + 17a^7b^4c(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 + 12a^7c^4(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) - 2a^6b^6(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3))^2 - 15a^6b^2c^3(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) + 7a^5b^4c^2(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) - 12a^5c^5 - a^4b^6c(b\sqrt{-4ac + b^2})(3ac - b^2)/(2a^3(4ac - b^2)) + (ac - b^2)/(2a^3) + 63a^4b^2c^4 - 103a^3b^4c^3 + 70a^2b^6c^2 - 20ab^8c + 2b^{10})/(27a^4b^5c^5 - 63a^3b^3c^4 + 54a^2b^5c^3 - 18ab^7c^2 + 2b^9c) + (-a + 2bx)/(2a^2x^2) - (ac - b^2) \cdot \log(x + (-12a^5c^5 + 63a^4b^2c^4 - 12a^4c^4(ac - b^2) - 103a^3b^4c^3 + 15a^3b^2c^3(ac - b^2) + 24a^3c^3(ac - b^2)^2 + 70a^2b^6c^2 - 7a^2b^4c^2(ac - b^2) - 42a^2b^2c^2(ac - b^2)^2 - 20ab^8c + ab^6c(ac - b^2) + 17ab^4c(ac - b^2)^2 + 2b^{10} - 2b^6(ac - b^2)^2)/(27a^4b^5c^5 - 63a^3b^3c^4 + 54a^2b^5c^3 - 18ab^7c^2 + 2b^9c) \end{aligned}$$

$$c^{**3} - 18*a*b^{**7}*c^{**2} + 2*b^{**9}*c)/a^{**3}$$

Giac [A] time = 1.13193, size = 142, normalized size = 1.37

$$-\frac{(b^2 - ac) \log(cx^2 + bx + a)}{2a^3} + \frac{(b^2 - ac) \log(|x|)}{a^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^3} + \frac{2abx - a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)

$$3.18 \quad \int \frac{1}{x^2(ax^2+bx^3+cx^4)} dx$$

Optimal. Leaf size=137

$$\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} + \frac{b(b^2-2ac) \log(a+bx+cx^2)}{2a^4} - \frac{b^2-ac}{a^3x} - \frac{b \log(x)(b^2-2ac)}{a^4} + \frac{b}{2a^2x^2} - \frac{3}{3}$$

[Out] $-1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)$

Rubi [A] time = 0.202787, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 709, 800, 634, 618, 206, 628}

$$\frac{(2a^2c^2 - 4ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} + \frac{b(b^2-2ac) \log(a+bx+cx^2)}{2a^4} - \frac{b^2-ac}{a^3x} - \frac{b \log(x)(b^2-2ac)}{a^4} + \frac{b}{2a^2x^2} - \frac{3}{3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] $-1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^ (n_.), x_Symbol] :> Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d+e*x)^(m+1))/((m+1)*(c*d^2-b*d*e+a*e^2)), x] + Dis

```
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax^2 + bx^3 + cx^4)} dx &= \int \frac{1}{x^4(a + bx + cx^2)} dx \\
&= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)} dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)} \right) dx}{a} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2} dx}{a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac)) \int \frac{b+2cx}{a+bx+cx^2} dx}{2a^4} + \frac{(b^4-4a^2c^2)\log(a+bx+cx^2)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4} - \frac{(b^4-4a^2c^2)\log(a+bx+cx^2)}{2a^4} \\
&= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(x)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.114359, size = 131, normalized size = 0.96

$$\frac{6(2a^2c^2-4ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) + \frac{3a^2b}{x^2} - \frac{2a^3}{x^3} + \frac{6a(ac-b^2)}{x} - 6\log(x)(b^3-2abc) + 3(b^3-2abc)\log(a+x(b+cx))}{6a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)]/(6*a^4)

Maple [A] time = 0.009, size = 214, normalized size = 1.6

$$-\frac{1}{3ax^3} + \frac{c}{xa^2} - \frac{b^2}{a^3x} + 2\frac{b\ln(x)c}{a^3} - \frac{b^3\ln(x)}{a^4} + \frac{b}{2a^2x^2} - \frac{c\ln(cx^2 + bx + a)b}{a^3} + \frac{\ln(cx^2 + bx + a)b^3}{2a^4} + 2\frac{c^2}{a^2\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/2*b/a^2/x^
2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3+2/a^2/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-4/a^3/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03884, size = 979, normalized size = 7.15

$$\frac{3(b^4 - 4ab^2c + 2a^2c^2)\sqrt{b^2 - 4ac}x^3 \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) - 2a^3b^2 + 8a^4c + 3(b^5 - 6ab^3c + 8a^2bc^2)x^3}{6(a^4b^2 - 4a^5c)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
- 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
```

$$8*a^2*b*c^2*x^3*\log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]$$

Sympy [B] time = 8.80404, size = 2105, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)

[Out]
$$\begin{aligned} & (-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 9*a**6*b**6*c**2*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 - 8*a**6*b*c**6 - a**5*b**8*c*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 + 312*a**3*b**7*c**3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c) + (-b*(2*a*c - b**2)/(2*a**4) + \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*\log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) - \sqrt{-4*a*c + b**2}*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))))**2 \end{aligned}$$

```

*4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
- b**2))) - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b
**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b*
*6*c**2*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**8*c*(-b
*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c +
b**4)/(2*a**4*(4*a*c - b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 +
312*a**3*b**7*c**3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c
**7 + 60*a**5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**
2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c)) + (-2*a**2 + 3*a*b*x + x**2*(6*
a*c - 6*b**2))/(6*a**3*x**3) + b*(2*a*c - b**2)*log(x + (-8*a**6*b*c**6 + 1
66*a**5*b**3*c**5 + 4*a**5*b*c**5*(2*a*c - b**2) - 361*a**4*b**5*c**4 + 23*
a**4*b**3*c**4*(2*a*c - b**2) + 312*a**3*b**7*c**3 - 26*a**3*b**5*c**3*(2*a
*c - b**2) - 52*a**3*b**3*c**3*(2*a*c - b**2)**2 - 130*a**2*b**9*c**2 + 9*a
**2*b**7*c**2*(2*a*c - b**2) + 57*a**2*b**5*c**2*(2*a*c - b**2)**2 + 26*a*b
**11*c - a*b**9*c*(2*a*c - b**2) - 19*a*b**7*c*(2*a*c - b**2)**2 - 2*b**13
+ 2*b**9*(2*a*c - b**2)**2)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 207*a**4*b**
4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**1
2*c))/a**4

```

Giac [A] time = 1.09323, size = 184, normalized size = 1.34

$$\frac{(b^3 - 2abc) \log(cx^2 + bx + a)}{2a^4} - \frac{(b^3 - 2abc) \log(|x|)}{a^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}a^4} + \frac{3a^2bx - 2a^3 - 6(ab^2 - a^2c)}{6a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")

[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/ (sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/(a^4*x^3)

$$3.19 \quad \int \frac{x^8}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=150

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{bx^2}{c(b^2-4ac)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3

Rubi [A] time = 0.159277, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 738, 800, 634, 618, 206, 628}

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{2x(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{bx^2}{c(b^2-4ac)} - \frac{b \log(a+bx+cx^2)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 738

Int[((d_)+(e_)*(x_))^(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d+e*x)^(m-1)*(d*b-2*a*e+(2*c*d-b*e)*x)*(a+b*x

```
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[(((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[(((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[(((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^4}{(a + bx + cx^2)^2} dx \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)} \right) dx}{-b^2 + 4ac} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2 \int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2} dx}{c^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \int \frac{b+2cx}{a+bx+cx^2} dx}{c^3} + \frac{(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right)}{c^3} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{b \log(a + bx + cx^2)}{c^3} - \frac{(2(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right))}{c^3} \\
&= \frac{2(b^2 - 3ac)x}{c^2(b^2 - 4ac)} - \frac{bx^2}{c(b^2 - 4ac)} + \frac{x^3(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right)}{c^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.192539, size = 132, normalized size = 0.88

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right) + \frac{a^2c(3b-2cx) - ab^2(b-4cx) + b^4(-x)}{(b^2-4ac)(a+x(b+cx))} - b \log(a + x(b + cx)) + cx}{c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)]/c^3

Maple [B] time = 0.008, size = 352, normalized size = 2.4

$$\frac{x}{c^2} + 2 \frac{xa^2}{c(cx^2 + bx + a)(4ac - b^2)} - 4 \frac{axb^2}{c^2(cx^2 + bx + a)(4ac - b^2)} + \frac{xb^4}{c^3(cx^2 + bx + a)(4ac - b^2)} - 3 \frac{a^2b}{c^2(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)`

[Out] `x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x^2+b*x+a)*b*a^2/(4*a*c-b^2)+1/c^3/(c*x^2+b*x+a)*b^3*a/(4*a*c-b^2)-4/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2+12/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.59795, size = 1760, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")`

[Out] `[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2`

```

*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
- 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]

```

Sympy [B] time = 1.62455, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)

```

[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
))/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4

```

$(a^2b^2c + b^4)/(4a^2c^4 - a^2b^2c^3 + x^2(4ac^5 - b^2c^4) + x(4ab^2c^4 - b^3c^3)) + x/c^2$

Giac [A] time = 1.11596, size = 217, normalized size = 1.45

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}} + \frac{x}{c^2} - \frac{b \log(cx^2 + bx + a)}{c^3} - \frac{\frac{(b^4-4ab^2c+2a^2c^2)x}{c} + \frac{ab^3-3a^2bc}{c}}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2*c^3 - 4*a*c^4)*\sqrt{-b^2 + 4*a*c}) + x/c^2 - b*\log(c*x^2 + b*x + a)/c^3 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)$

$$3.20 \quad \int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=114

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)

Rubi [A] time = 0.103282, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{bx}{c(b^2 - 4ac)} + \frac{\log(a + bx + cx^2)}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 738

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)

```
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] := Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x]/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^3}{(a + bx + cx^2)^2} dx \\
&= \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2} dx}{-b^2 + 4ac} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2} dx}{c(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2} dx}{2c^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a+bx+cx^2} dx}{2c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(a + bx + cx^2)}{2c^2} + \frac{(b(b^2 - 6ac)) \text{Subst}\left(\int \frac{1}{a+bx+cx^2} dx\right)}{c^2(b^2 - 4ac)} \\
&= -\frac{bx}{c(b^2 - 4ac)} + \frac{x^2(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2 - 4ac)^{3/2}} + \frac{\log(a + bx + cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A] time = 0.153361, size = 109, normalized size = 0.96

$$\frac{2(-2a^2c+ab(b-3cx)+b^3x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)

Maple [A] time = 0.009, size = 209, normalized size = 1.8

$$\frac{1}{cx^2 + bx + a} \left(\frac{b(3ac - b^2)x}{c^2(4ac - b^2)} + \frac{a(2ac - b^2)}{c^2(4ac - b^2)} \right) + 2 \frac{\ln(cx^2 + bx + a)a}{c(4ac - b^2)} - \frac{\ln(cx^2 + bx + a)b^2}{2c^2(4ac - b^2)} - 6 \frac{ab}{c(4ac - b^2)^{3/2}} \arctan\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^7/(c*x^4+b*x^3+a*x^2)^2,x)$

[Out] $(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+2/c/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*a-1/2/c^2/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^2-6/c/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*a*b+1/c^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7/(c*x^4+b*x^3+a*x^2)^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 1.65751, size = 1374, normalized size = 12.05

$$\frac{2ab^4 - 12a^2b^2c + 16a^3c^2 + (ab^3 - 6a^2bc + (b^3c - 6abc^2)x^2 + (b^4 - 6ab^2c)x)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}}{cx^2 + bx + a}\right)}{2(ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4 + (b^4c^3 - 8a^2b^2c^4 + 16a^2c^5)x^2 + (b^5c^2 - 8a^2b^3c^3 + 16a^2b^2c^4)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^7/(c*x^4+b*x^3+a*x^2)^2,x, \text{algorithm}="fricas")$

[Out] $[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \text{sqrt}(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^2*b^2*c^4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-\text{sqrt}(-b^2$

$$+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]$$

Sympy [B] time = 1.20897, size = 729, normalized size = 6.39

$$\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2} \right) \log \left(x + \frac{-16a^2c^3 \left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{2c^2} \right) + 8a^2c + 8ab^2c^2}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] $(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*\log(x + (-16*a**2*c**3*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*\log(x + (-16*a**2*c**3*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))$

Giac [A] time = 1.09743, size = 169, normalized size = 1.48

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2c^2 - 4ac^3)\sqrt{-b^2+4ac}} + \frac{\log(cx^2 + bx + a)}{2c^2} + \frac{ab^2 - 2a^2c + (b^3 - 3abc)x}{(cx^2 + bx + a)(b^2 - 4ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)

$$3.21 \quad \int \frac{x^6}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rubi [A] time = 0.0391678, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 722, 618, 206}

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 722

Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &

& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^6}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x^2}{(a + bx + cx^2)^2} dx \\
 &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2a) \int \frac{1}{a+bx+cx^2} dx}{b^2 - 4ac} \\
 &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4a) \operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\
 &= \frac{x(2a + bx)}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{4a \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0937459, size = 81, normalized size = 1.21

$$\frac{a(b - 2cx) + b^2x}{c(4ac - b^2)(a + x(b + cx))} + \frac{4a \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] $(b^2x + a(b - 2cx))/(c(-b^2 + 4ac)(a + x(b + cx))) + (4a \operatorname{ArcTan}[(b + 2cx)/\sqrt{-b^2 + 4ac}]) / (-b^2 + 4ac)^{3/2}$

Maple [A] time = 0.006, size = 97, normalized size = 1.5

$$\frac{1}{cx^2 + bx + a} \left(-\frac{(2ac - b^2)x}{c(4ac - b^2)} + \frac{ab}{c(4ac - b^2)} \right) + 4 \frac{a}{(4ac - b^2)^{3/2}} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)`

[Out] $(-(2ac - b^2)/c / (4ac - b^2) * x + a * b / c / (4ac - b^2)) / (cx^2 + bx + a) + 4a / (4ac - b^2)^{3/2} * \arctan((2cx + b) / (4ac - b^2)^{1/2})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.59667, size = 826, normalized size = 12.33

$$\left[\frac{ab^3 - 4a^2bc + 2(ac^2x^2 + abcx + a^2c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^4 - 6ab^2c + 8a^2c^2)x}{ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^2 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x} \right], -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")`

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c))*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] time = 0.755759, size = 280, normalized size = 4.18

$$-2a \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x + \frac{-32a^3c^2 \sqrt{-\frac{1}{(4ac - b^2)^3}} + 16a^2b^2c \sqrt{-\frac{1}{(4ac - b^2)^3}} - 2ab^4 \sqrt{-\frac{1}{(4ac - b^2)^3}} + 2ab}{4ac} \right) + 2a \sqrt{\frac{1}{(4ac - b^2)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3*2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) - (-a*b + x*(2*a*c - b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**3*c))
```

Giac [A] time = 1.11736, size = 119, normalized size = 1.78

$$\frac{4a \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{b^2x-2acx+ab}{(b^2c-4ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

$$3.22 \quad \int \frac{x^5}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=66

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rubi [A] time = 0.0378451, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 638, 618, 206}

$$\frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&

NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^5}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{x}{(a + bx + cx^2)^2} dx \\
 &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{b \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\
 &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2b) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\
 &= \frac{2a + bx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{2b \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0677842, size = 69, normalized size = 1.05

$$\frac{2a + bx}{(b^2 - 4ac)(a + x(b + cx))} - \frac{2b \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

Maple [A] time = 0.002, size = 70, normalized size = 1.1

$$\frac{-bx - 2a}{(4ac - b^2)(cx^2 + bx + a)} - 2 \frac{b}{(4ac - b^2)^{3/2}} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.53045, size = 740, normalized size = 11.21

$$\left[\frac{2ab^2 - 8a^2c - (bcx^2 + b^2x + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + (b^3 - 4abc)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, \frac{2ab^2 - 8a^2c - 2(bc^2x^2 + 2b^2cx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b))}{ab^4 - 8a^2b^2c + 16a^3c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -

$8a^2c - 2*(b^2cx^2 + b^2x + ab)*\sqrt{-b^2 + 4ac}*\arctan(-\sqrt{-b^2 + 4ac}*(2cx + b)/(b^2 - 4ac)) + (b^3 - 4abc)*x/(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)*x^2 + (b^5 - 8ab^3c + 16a^2bc^2)*x]$

Sympy [B] time = 0.705141, size = 252, normalized size = 3.82

$$b \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x + \frac{-16a^2bc^2 \sqrt{\frac{1}{(4ac - b^2)^3}} + 8ab^3c \sqrt{\frac{1}{(4ac - b^2)^3}} - b^5 \sqrt{\frac{1}{(4ac - b^2)^3}} + b^2}{2bc} \right) - b \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] $b*\sqrt{-1/(4*a*c - b**2)**3}*\log(x + (-16*a**2*b*c**2*\sqrt{-1/(4*a*c - b**2)**3} + 8*a*b**3*c*\sqrt{-1/(4*a*c - b**2)**3} - b**5*\sqrt{-1/(4*a*c - b**2)**3} + b**2)/(2*b*c)) - b*\sqrt{-1/(4*a*c - b**2)**3}*\log(x + (16*a**2*b*c**2*\sqrt{-1/(4*a*c - b**2)**3} - 8*a*b**3*c*\sqrt{-1/(4*a*c - b**2)**3} + b**5*\sqrt{-1/(4*a*c - b**2)**3} + b**2)/(2*b*c)) - (2*a + b*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))$

Giac [A] time = 1.14039, size = 103, normalized size = 1.56

$$\frac{2b \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} + \frac{bx + 2a}{(cx^2 + bx + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $2*b*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c})/((b^2 - 4*a*c)*\sqrt{-b^2 + 4*a*c}) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))$

$$3.23 \quad \int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=66

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rubi [A] time = 0.0345985, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1585, 614, 618, 206}

$$\frac{4c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2, x]

[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int

egerQ[4*p]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{(a + bx + cx^2)^2} dx \\
 &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} - \frac{(2c) \int \frac{1}{a + bx + cx^2} dx}{b^2 - 4ac} \\
 &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{(4c) \operatorname{Subst}\left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx\right)}{b^2 - 4ac} \\
 &= -\frac{b + 2cx}{(b^2 - 4ac)(a + bx + cx^2)} + \frac{4c \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.0794278, size = 70, normalized size = 1.06

$$-\frac{\frac{4c \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{\sqrt{4ac - b^2}} + \frac{b + 2cx}{a + x(b + cx)}}{b^2 - 4ac}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))

Maple [A] time = 0.003, size = 68, normalized size = 1.

$$\frac{2cx + b}{(4ac - b^2)(cx^2 + bx + a)} + 4 \frac{c}{(4ac - b^2)^{3/2}} \arctan\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.55073, size = 745, normalized size = 11.29

$$\left[\frac{b^3 - 4abc + 2(c^2x^2 + bcx + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac - \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x}, -\frac{b^3 - 4abc - 4(c^2x^2 + bcx + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^2 + 2bcx + b^2 - 2ac + \sqrt{b^2 - 4ac}(2cx + b)}{cx^2 + bx + a}\right) + 2(b^2c - 4ac^2)x}{ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^2 + (b^5 - 8ab^3c + 16a^2bc^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -

$4ac - 4(c^2x^2 + bcx + a)\sqrt{-b^2 + 4ac}\arctan\left(\frac{-\sqrt{-b^2 + 4ac}(2cx + b)}{b^2 - 4ac}\right) + 2(b^2c - 4ac^2)x/(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8a^2b^2c^2 + 16a^2c^3)x^2 + (b^5 - 8a^2b^3c + 16a^2bc^2)x)$

Sympy [B] time = 0.721547, size = 265, normalized size = 4.02

$$-2c \sqrt{-\frac{1}{(4ac - b^2)^3}} \log \left(x + \frac{-32a^2c^3 \sqrt{-\frac{1}{(4ac - b^2)^3}} + 16ab^2c^2 \sqrt{-\frac{1}{(4ac - b^2)^3}} - 2b^4c \sqrt{-\frac{1}{(4ac - b^2)^3}} + 2bc}{4c^2} \right) + 2c \sqrt{-\frac{1}{(4ac - b^2)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] $-2c\sqrt{-1/(4ac - b^2)^3}\log(x + (-32a^2c^3\sqrt{-1/(4ac - b^2)^3} + 16ab^2c^2\sqrt{-1/(4ac - b^2)^3} - 2b^4c\sqrt{-1/(4ac - b^2)^3} + 2bc)/(4c^2)) + 2c\sqrt{-1/(4ac - b^2)^3}\log(x + (32a^2c^3\sqrt{-1/(4ac - b^2)^3} - 16ab^2c^2\sqrt{-1/(4ac - b^2)^3} + 2b^4c\sqrt{-1/(4ac - b^2)^3} + 2bc)/(4c^2)) + (b + 2cx)/(4a^2c - ab^2 + x^2(4ac - b^2) + x(4abc - b^3))$

Giac [A] time = 1.10605, size = 103, normalized size = 1.56

$$-\frac{4c \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(b^2 - 4ac)\sqrt{-b^2 + 4ac}} - \frac{2cx + b}{(cx^2 + bx + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-4c\arctan((2cx + b)/\sqrt{-b^2 + 4ac})/((b^2 - 4ac)\sqrt{-b^2 + 4ac}) - (2cx + b)/((cx^2 + bx + a)(b^2 - 4ac))$

$$3.24 \quad \int \frac{x^3}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=108

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)

Rubi [A] time = 0.148149, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx}{a(b^2 - 4ac)(a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e


```

^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

```

Rule 800

```

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

```

Rule 634

```

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 628

```

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x(a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \frac{-b^2 + 4ac - bcx}{x(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} - \frac{\int \left(\frac{-b^2 + 4ac}{ax} + \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a + bx + cx^2} dx}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b + 2cx}{a + bx + cx^2} dx}{2a^2} - \frac{(b(b^2 - 6ac)) \int \frac{1}{a + bx + cx^2} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx + cx^2)}{2a^2} + \frac{(b(b^2 - 6ac)) \text{Subst}\left(\int \frac{1}{b^2 - 4ac - x}\right)}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)(a + bx + cx^2)} + \frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{a^2(b^2 - 4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a + bx + cx^2)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.236263, size = 107, normalized size = 0.99

$$\frac{\frac{2a(-2ac + b^2 + bcx)}{(b^2 - 4ac)(a + x(b + cx))} + \frac{2b(b^2 - 6ac) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}} - \log(a + x(b + cx)) + 2 \log(x)}{2a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)

Maple [B] time = 0.012, size = 237, normalized size = 2.2

$$\frac{\ln(x)}{a^2} - \frac{bcx}{a(cx^2 + bx + a)(4ac - b^2)} + 2 \frac{c}{(cx^2 + bx + a)(4ac - b^2)} - \frac{b^2}{a(cx^2 + bx + a)(4ac - b^2)} - 2 \frac{c \ln(cx^2 + bx + a)}{a(4ac - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)

[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*a*c-b^2)*c-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)+1/2/a^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c+1/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.09242, size = 1685, normalized size = 15.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2

$$\begin{aligned}
& + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + \\
& 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 - 8* \\
& a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^ \\
& 2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^ \\
& 3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)* \\
& x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) \\
& + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c \\
& - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(\\
& c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c \\
& ^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*\log(x))/(a^3*b^4 \\
& - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 \\
& + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
\end{aligned}$$

Sympy [B] time = 7.64518, size = 2236, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)

[Out]
$$\begin{aligned}
& (-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2 \\
& *b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*\log(x + (1536*a**9*c**5*(-b \\
& *\sqrt{-(4*a*c - b**2)**3}*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b \\
& *2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 2112*a**8*b**2*c**4*(-b*s \\
& qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2 \\
& *c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c**3*(-b*sqr \\
& t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c \\
& **2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a**7*c**5*(-b*\sqrt{-(4*a* \\
& c - b**2)**3}*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12 \\
& *a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**2*(-b*\sqrt{-(4*a*c - b \\
& *2)**3}*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b** \\
& 4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(-b*\sqrt{-(4*a*c - b**2) \\
& **3}*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c \\
& - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a* \\
& c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) \\
& - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - \\
& b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1 \\
& / (2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a \\
& *c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) \\
& - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(-b*\sqrt{-(4*a*c - b**2)**3}*(6*a*c - \\
& b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1
\end{aligned}$$

```

/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(-b*sqrt(-(4*a*c - b**2)**3)
*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c
+ 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36
*a*b**7*c**2 + 2*b**9*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*
log(x + (1536*a**9*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 -
2112*a**8*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 11
36*a**7*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a
**7*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a**6*b**6*c**
2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(b*sqrt(-(4*a*c -
b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)) - 1/(2*a**2)) - 768*a**5*c**5 - 2*a**4*b**10*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - 1/(2*a**2)) - 952*a**3*b**4*c**3 + 277*a**2*b**6*c
**2 - 38*a*b**8*c + 2*b**10)/(864*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**
2*b**5*c**3 - 36*a*b**7*c**2 + 2*b**9*c)) - (-2*a*c + b**2 + b*c*x)/(4*a**3
*c - a**2*b**2 + x**2*(4*a**2*c**2 - a*b**2*c) + x*(4*a**2*b*c - a*b**3)) +
log(x)/a**2

```

Giac [A] time = 1.13067, size = 170, normalized size = 1.57

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^2b^2 - 4a^3c)\sqrt{-b^2+4ac}} - \frac{\log(cx^2 + bx + a)}{2a^2} + \frac{\log(|x|)}{a^2} + \frac{abcx + ab^2 - 2a^2c}{(cx^2 + bx + a)(b^2 - 4ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

$$3.25 \quad \int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=148

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2(b^2-3ac)}{a^2x(b^2-4ac)} + \frac{b \log(a+bx+cx^2)}{a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac + b^2 + bcx}{ax(b^2-4ac)(a+bx)}$$

[Out] $(-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3$

Rubi [A] time = 0.198026, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2(b^2-3ac)}{a^2x(b^2-4ac)} + \frac{b \log(a+bx+cx^2)}{a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac + b^2 + bcx}{ax(b^2-4ac)(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] $(-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 740

Int[((d_)+(e_)*(x_))^(m_)*((a_)+(b_)*(x_)+(c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d+e*x)^(m+1)*(b*c*d-b^2*e+2*a*c*e+c*(2*c*d-b*e

```
)x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^2 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \frac{-2(b^2 - 3ac) - 2bcx}{x^2(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{\int \left(\frac{2(-b^2 + 3ac)}{ax^2} - \frac{2b(-b^2 + 4ac)}{a^2x} + \frac{2(-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac)x)}{a^2(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} - \frac{2 \int \frac{-b^4 + 5ab^2c - 3a^2c^2 - bc(b^2 - 4ac)x}{a + bx + cx^2}}{a^3(b^2 - 4ac)} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \int \frac{b + 2cx}{a + bx + cx^2} dx}{a^3} + \frac{(b^4 - 6ab^2c)}{a^3} \\
&= -\frac{2(b^2 - 3ac)}{a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x(a + bx + cx^2)} - \frac{2b \log(x)}{a^3} + \frac{b \log(a + bx + cx^2)}{a^3} - \frac{(2(b^4 - 6ab^2c + 6a^2c^2)) \tanh^{-1}\left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}}\right)}{a^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.305082, size = 131, normalized size = 0.89

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right) + \frac{a(-3abc - 2ac^2x + b^2cx + b^3)}{(b^2 - 4ac)(a + x(b + cx))} - b \log(a + x(b + cx)) + \frac{a}{x} + 2b \log(x)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3

Maple [B] time = 0.013, size = 328, normalized size = 2.2

$$-\frac{1}{xa^2} - 2\frac{b \ln(x)}{a^3} - 2\frac{c^2x}{a(cx^2 + bx + a)(4ac - b^2)} + \frac{cxb^2}{a^2(cx^2 + bx + a)(4ac - b^2)} - 3\frac{bc}{a(cx^2 + bx + a)(4ac - b^2)} + \frac{bc}{a^2(cx^2 + bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)

[Out]
$$-1/a^2/x - 2*b*ln(x)/a^3 - 2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x + 1/a^2/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2 - 3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c + 1/a^2/(c*x^2+b*x+a)*b^3/(4*a*c-b^2) + 4/a^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*b - 1/a^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3 - 12/a/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*c^2 + 12/a^2/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^2*c - 2/a^3/(4*a*c-b^2)^{(3/2)}*arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^4$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.61153, size = 2049, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2$$

$$\begin{aligned}
& + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*\log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*\log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*\log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x)]
\end{aligned}$$

Sympy [B] time = 11.9409, size = 2672, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)

[Out] $(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))*\log(x + (-1728*a**11*b*c**5*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))**2 + 2*256*a**10*b**3*c**4*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))**2 - 1172*a**9*b**5*c**3*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))**2 - 288*a**9*c**6*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 303*a**8*b**7*c**2*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))**2 - 432*a**8*b**2*c**5*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 39*a**7*b**9*c*(b/a**3 - \sqrt{-(4*a*c - b**2)**3}*(6*a**2*c**2 - 6*a*b**2*c + b**4))/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2$

$$\begin{aligned}
& + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 - \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 - \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 - \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b**8*c**2*(b/a**3 - \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a**4*b**10*c*(b/a**3 - \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b**12*c)) + (b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 2256*a**10*b**3*c**4*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 1172*a**9*b**5*c**3*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 288*a**9*c**6*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a**6*b**11*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 + 34*a**5*b**8*c**2*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a**4*b**10*c*(b/a**3 + \sqrt{-(4*a*c - b**2)**3})*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 - 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b**12*c)) - (4*a**2*c - a*b**2 + x**2*(6*a*c**2 - 2*b**2*c) + x*(7*
\end{aligned}$$

$$a*b*c - 2*b**3)/(x**3*(4*a**3*c**2 - a**2*b**2*c) + x**2*(4*a**3*b*c - a**2*b**3) + x*(4*a**4*c - a**3*b**2)) - 2*b*log(x)/a**3$$

Giac [A] time = 1.08899, size = 231, normalized size = 1.56

$$\frac{2(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(a^3b^2 - 4a^4c)\sqrt{-b^2+4ac}} - \frac{2b^2cx^2 - 6ac^2x^2 + 2b^3x - 7abcx + ab^2 - 4a^2c}{(a^2b^2 - 4a^3c)(cx^3 + bx^2 + ax)} + \frac{b \log(cx^2 + bx + a)}{a^3} - 2b \log(x)/a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3

$$3.26 \quad \int \frac{x}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=202

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} - \frac{3b^2-8ac}{2a^2x^2(b^2-4ac)} - \frac{(3b^2-2ac) \log(a+bx+cx^2)}{2a^4} + \frac{b(3b^2-11ac)}{a^3x(b^2-4ac)} + \frac{\log\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4}$$

[Out] $-(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)$

Rubi [A] time = 0.250297, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} - \frac{3b^2-8ac}{2a^2x^2(b^2-4ac)} - \frac{(3b^2-2ac) \log(a+bx+cx^2)}{2a^4} + \frac{b(3b^2-11ac)}{a^3x(b^2-4ac)} + \frac{\log\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2, x]

[Out] $-(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*
(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
+ Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) -
2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1]
&& IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol]
:> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c),
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol]
:> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;
FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /;
FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^3 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} - \frac{\int \frac{-3b^2 + 8ac - 3bcx}{x^3(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} - \frac{\int \left(\frac{-3b^2 + 8ac}{ax^3} + \frac{3b^3 - 11abc}{a^2x^2} + \frac{(b^2 - 4ac)(-3b^2 + 2ac)}{a^3x} + \frac{b(3b^4 - 17ab^2c + 19a^2c^2)}{a^3(a + bx + cx^2)} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)\log(x)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)\log(x)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{(3b^2 - 2ac)\log(x)}{a^4} \\
&= -\frac{3b^2 - 8ac}{2a^2(b^2 - 4ac)x^2} + \frac{b(3b^2 - 11ac)}{a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^2(a + bx + cx^2)} + \frac{b(3b^4 - 20ab^2c + 30a^2c^2)}{a^4(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.427668, size = 175, normalized size = 0.87

$$\frac{2a(2a^2c^2 - 4ab^2c - 3abc^2x + b^3cx + b^4)}{(b^2 - 4ac)(a + x(b + cx))} + \frac{2b(30a^2c^2 - 20ab^2c + 3b^4) \tan^{-1}\left(\frac{b + 2cx}{\sqrt{4ac - b^2}}\right)}{(4ac - b^2)^{3/2}} - \frac{a^2}{x^2} + 2 \log(x) (3b^2 - 2ac) + (2ac - 3b^2) \log(a + x(b + cx))$$

$$2a^4$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]

[Out] $(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)]/(2*a^4)$

Maple [B] time = 0.016, size = 418, normalized size = 2.1

$$-\frac{1}{2a^2x^2} - 2\frac{\ln(x)c}{a^3} + 3\frac{\ln(x)b^2}{a^4} + 2\frac{b}{a^3x} + 3\frac{c^2bx}{a^2(cx^2+bx+a)(4ac-b^2)} - \frac{b^3cx}{a^3(cx^2+bx+a)(4ac-b^2)} - 2\frac{1}{a(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x)

[Out]
$$-1/2/a^2/x^2 - 2/a^3*\ln(x)*c + 3/a^4*\ln(x)*b^2 + 2/a^3*b/x + 3/a^2/(c*x^2+b*x+a)*b*c^2/(4*a*c-b^2)*x - 1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)*x - 2/a/(c*x^2+b*x+a)/(4*a*c-b^2)*c^2 + 4/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c - 1/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4 + 4/a^2/(4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a) - 7/a^3/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^2 + 3/2/a^4/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^4 + 30/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b*c^2 - 20/a^3/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^3*c + 3/a^4/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x+b)/(4*a*c-b^2)^{(1/2)})*b^5$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.11668, size = 2606, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)$$

$$\begin{aligned}
& *b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} - 4116*a^{**11}*b^{**6}*c^{**3}*(-b \\
& *sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64 \\
& *a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2 \\
& *a^{**4}))^{**2} + 3072*a^{**11}*c^{**7}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 2 \\
& 0*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4} \\
& *c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4})) + 987*a^{**10}*b^{**8}*c^{**2}*(-b*sqrt(-(4* \\
& a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} \\
& - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} \\
& - 7536*a^{**10}*b^{**2}*c^{**6}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b \\
& **2*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b \\
& **6)) + (2*a*c - 3*b^{**2})/(2*a^{**4})) - 121*a^{**9}*b^{**10}*c*(-b*sqrt(-(4*a*c - b* \\
& **2))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a* \\
& **2*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} + 8152* \\
& a^{**9}*b^{**4}*c^{**5}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3 \\
& *b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (\\
& 2*a*c - 3*b^{**2})/(2*a^{**4})) + 6*a^{**8}*b^{**12}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a \\
& **2*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} \\
& + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} - 4343*a^{**8}*b^{**6}*c^{** \\
& 4}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{** \\
& 4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{** \\
& 2})/(2*a^{**4})) - 6144*a^{**8}*c^{**8} + 1198*a^{**7}*b^{**8}*c^{**3}*(-b*sqrt(-(4*a*c - b^{**2} \\
&)^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2} \\
& *b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4})) + 50208*a^{**7} \\
& *b^{**2}*c^{**7} - 165*a^{**6}*b^{**10}*c^{**2}*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} \\
& - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b \\
& **4*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4})) - 137792*a^{**6}*b^{**4}*c^{**6} + 9*a^{** \\
& 5}*b^{**12}*c*(-b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4} \\
&)/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c \\
& - 3*b^{**2})/(2*a^{**4})) + 176474*a^{**5}*b^{**6}*c^{**5} - 119275*a^{**4}*b^{**8}*c^{**4} + 4544 \\
& 8*a^{**3}*b^{**10}*c^{**3} - 9846*a^{**2}*b^{**12}*c^{**2} + 1134*a*b^{**14}*c - 54*b^{**16}))/ (1728 \\
& 0*a^{**7}*b*c^{**8} - 69570*a^{**6}*b^{**3}*c^{**7} + 112428*a^{**5}*b^{**5}*c^{**6} - 88605*a^{**4}*b \\
& **7*c^{**5} + 37600*a^{**3}*b^{**9}*c^{**4} - 8820*a^{**2}*b^{**11}*c^{**3} + 1080*a*b^{**13}*c^{**2} \\
& - 54*b^{**15}*c)) + (b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + \\
& 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + \\
& (2*a*c - 3*b^{**2})/(2*a^{**4}))*log(x + (3072*a^{**14}*c^{**6}*(b*sqrt(-(4*a*c - b^{**2} \\
&)^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2} \\
& *b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} - 9408*a^{** \\
& 13}*b^{**2}*c^{**5}*(b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b* \\
& **4))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a \\
& *c - 3*b^{**2})/(2*a^{**4}))^{**2} + 9040*a^{**12}*b^{**4}*c^{**4}*(b*sqrt(-(4*a*c - b^{**2}))^{**3} \\
&)*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4}))/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{** \\
& 2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c - 3*b^{**2})/(2*a^{**4}))^{**2} - 4116*a^{**11} \\
& *b^{**6}*c^{**3}*(b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a^{**2}*c^{**2} - 20*a*b^{**2}*c + 3*b^{**4} \\
&)/ (2*a^{**4}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})) + (2*a*c \\
& - 3*b^{**2})/(2*a^{**4}))^{**2} + 3072*a^{**11}*c^{**7}*(b*sqrt(-(4*a*c - b^{**2}))^{**3}*(30*a*
\end{aligned}$$

$$\begin{aligned}
& *2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + \\
& 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(b \\
& *sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64 \\
& *a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2 \\
& *a**4))**2 - 7536*a**10*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 \\
& - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b \\
& **4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(b*sqrt(-(4* \\
& a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 \\
& - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 \\
& + 8152*a**9*b**4*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b** \\
& 2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b** \\
& 6)) + (2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(b*sqrt(-(4*a*c - b**2)**3) \\
& *(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2 \\
& *c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**8*b* \\
& *6*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(\\
& 2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - \\
& 3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(b*sqrt(-(4*a*c - \\
& b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48* \\
& a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208* \\
& a**7*b**2*c**7 - 165*a**6*b**10*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c \\
& **2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12* \\
& a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9* \\
& a**5*b**12*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b* \\
& **4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a \\
& *c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45 \\
& 448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(17 \\
& 280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88605*a**4 \\
& *b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3 + 1080*a*b**13*c** \\
& 2 - 54*b**15*c)) + (-4*a**3*c + a**2*b**2 + x**3*(22*a*b*c**2 - 6*b**3*c) + \\
& x**2*(-8*a**2*c**2 + 25*a*b**2*c - 6*b**4) + x*(12*a**2*b*c - 3*a*b**3))/(\\
& x**4*(8*a**4*c**2 - 2*a**3*b**2*c) + x**3*(8*a**4*b*c - 2*a**3*b**3) + x**2 \\
& *(8*a**5*c - 2*a**4*b**2)) - (2*a*c - 3*b**2)*log(x + (-6144*a**8*c**8 + 50 \\
& 208*a**7*b**2*c**7 - 3072*a**7*c**7*(2*a*c - 3*b**2) - 137792*a**6*b**4*c** \\
& 6 + 7536*a**6*b**2*c**6*(2*a*c - 3*b**2) + 3072*a**6*c**6*(2*a*c - 3*b**2)* \\
& *2 + 176474*a**5*b**6*c**5 - 8152*a**5*b**4*c**5*(2*a*c - 3*b**2) - 9408*a* \\
& *5*b**2*c**5*(2*a*c - 3*b**2)**2 - 119275*a**4*b**8*c**4 + 4343*a**4*b**6*c \\
& **4*(2*a*c - 3*b**2) + 9040*a**4*b**4*c**4*(2*a*c - 3*b**2)**2 + 45448*a**3 \\
& *b**10*c**3 - 1198*a**3*b**8*c**3*(2*a*c - 3*b**2) - 4116*a**3*b**6*c**3*(2 \\
& *a*c - 3*b**2)**2 - 9846*a**2*b**12*c**2 + 165*a**2*b**10*c**2*(2*a*c - 3*b \\
& **2) + 987*a**2*b**8*c**2*(2*a*c - 3*b**2)**2 + 1134*a*b**14*c - 9*a*b**12* \\
& c*(2*a*c - 3*b**2) - 121*a*b**10*c*(2*a*c - 3*b**2)**2 - 54*b**16 + 6*b**12 \\
& *(2*a*c - 3*b**2)**2)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a* \\
& *5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**1 \\
& 1*c**3 + 1080*a*b**13*c**2 - 54*b**15*c))/a**4
\end{aligned}$$

Giac [A] time = 1.12117, size = 309, normalized size = 1.53

$$\frac{(3b^5 - 20ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - \frac{(3b^2 - 2ac) \log(cx^2 + bx + a)}{2a^4} + \frac{(3b^2 - 2ac) \log(|x|)}{a^4} - \frac{a^3b^2 - 4a^4}{(a^4b^2 - 4a^5c)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*\arctan((2*c*x + b)/\sqrt{-b^2 + 4*a*c}) / ((a^4*b^2 - 4*a^5*c)*\sqrt{-b^2 + 4*a*c}) - 1/2*(3*b^2 - 2*a*c)*\log(c*x^2 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*\log(\text{abs}(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x) / ((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)$

$$3.27 \quad \int \frac{1}{(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=252

$$-\frac{2(5a^2c^2 - 9ab^2c + 2b^4)}{a^4x(b^2 - 4ac)} - \frac{2(30a^2b^2c^2 - 10a^3c^3 - 15ab^4c + 2b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2 - 4ac)^{3/2}} + \frac{b(2b^2 - 7ac)}{a^3x^2(b^2 - 4ac)} - \frac{2(2b^2 - 5ac)}{3a^2x^3(b^2 - 4ac)}$$

[Out] $(-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 + (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5$

Rubi [A] time = 0.323337, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1594, 740, 800, 634, 618, 206, 628}

$$-\frac{2(5a^2c^2 - 9ab^2c + 2b^4)}{a^4x(b^2 - 4ac)} - \frac{2(30a^2b^2c^2 - 10a^3c^3 - 15ab^4c + 2b^6) \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2 - 4ac)^{3/2}} + \frac{b(2b^2 - 7ac)}{a^3x^2(b^2 - 4ac)} - \frac{2(2b^2 - 5ac)}{3a^2x^3(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2), x]

[Out] $(-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 + (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5$

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^4 (a + bx + cx^2)^2} dx \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} - \frac{\int \frac{-2(2b^2 - 5ac) - 4bcx}{x^4 (a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} - \frac{\int \left(\frac{2(-2b^2 + 5ac)}{ax^4} - \frac{2(-2b^3 + 7abc)}{a^2x^3} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^3x^2} + \frac{2b(b^2 - 4ac)(2b^2 - 5ac)}{a^4x} \right) dx}{a(b^2 - 4ac)} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2(b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3(b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2(b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3(b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2(b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3(b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)} \\
&= -\frac{2(2b^2 - 5ac)}{3a^2(b^2 - 4ac)x^3} + \frac{b(2b^2 - 7ac)}{a^3(b^2 - 4ac)x^2} - \frac{2(2b^4 - 9ab^2c + 5a^2c^2)}{a^4(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^3 (a + bx + cx^2)}
\end{aligned}$$

Mathematica [A] time = 0.341258, size = 218, normalized size = 0.87

$$\frac{3a(5a^2bc^2 + 2a^2c^3x - 4ab^2c^2x - 5ab^3c + b^4cx + b^5)}{(b^2 - 4ac)(a + x(b + cx))} - \frac{6(30a^2b^2c^2 - 10a^3c^3 - 15ab^4c + 2b^6) \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}} + \frac{3a^2b}{x^2} - \frac{a^3}{x^3} + \frac{3a(2ac-3b^2)}{x} + 6 \log(x) (3abc - \dots)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2), x]

[Out] $(-a^3/x^3) + (3a^2b)/x^2 + (3a*(-3b^2 + 2a*c))/x - (3a*(b^5 - 5a*b^3*c + 5a^2*b*c^2 + b^4*c*x - 4a*b^2*c^2*x + 2a^2*c^3*x))/((b^2 - 4a*c)*(a + x*(b + c*x))) - (6*(2*b^6 - 15a*b^4*c + 30a^2*b^2*c^2 - 10a^3*c^3)*\text{ArcTan}[(b + 2*c*x)/\text{Sqrt}[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^{(3/2)} + 6*(-2*b^3 + 3a*b*c)*\text{Log}[x] + 3*(2*b^3 - 3a*b*c)*\text{Log}[a + x*(b + c*x)]/(3a^5)$

Maple [B] time = 0.016, size = 515, normalized size = 2.

$$-\frac{1}{3a^2x^3} + 2\frac{c}{a^3x} - 3\frac{b^2}{a^4x} + \frac{b}{x^2a^3} + 6\frac{b\ln(x)c}{a^4} - 4\frac{b^3\ln(x)}{a^5} + 2\frac{c^3x}{a^2(cx^2+bx+a)(4ac-b^2)} - 4\frac{c^2xb^2}{a^3(cx^2+bx+a)(4ac-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x)

[Out]
$$-1/3/a^2/x^3+2/a^3/x*c-3/a^4/x*b^2+1/a^3*b/x^2+6*b/a^4*\ln(x)*c-4*b^3/a^5*\ln(x)+2/a^2/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x-4/a^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2+1/a^4/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^4+5/a^2/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c^2-5/a^3/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)*c+1/a^4/(c*x^2+b*x+a)*b^5/(4*a*c-b^2)-12/a^3/(4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a)*b+11/a^4/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^3-2/a^5/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^5+20/a^2/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3-60/a^3/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2+30/a^4/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*c-4/a^5/(4*a*c-b^2)^(3/2)*\arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 4.10804, size = 2989, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 +
41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*
c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*
a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^
5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 1
5*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^
5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b
^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^
2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a
^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*
x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -
1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 4
1*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^
2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^
5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5
+ (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*
a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)
*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^
7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a)
+ 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8
- 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^
5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c
^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^
4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```

Sympy [B] time = 31.0241, size = 4774, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**
2*b**2*c**2 + 15*a*b**4*c - 2*b**6))/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))*log(x + (-4928*a**16*b*c**6*(-b*(3*a*c - 2*b**2)/a
```

$$\begin{aligned}
& **5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) \\
&)**2 + 10032*a**15*b**3*c**5*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 7980*a**14*b**5*c**4*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 3249*a**13*b**7*c**3*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 800*a**13*c**8*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 723*a**12*b**9*c**2*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 6704*a**12*b**2*c**7*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 84*a**11*b**11*c*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 15182*a**11*b**4*c**6*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 4*a**10*b**13*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 12844*a**10*b**6*c**5*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 5546*a**9*b**8*c**4*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 4800*a**9*b*c**9 + 1306*a**8*b**10*c**3*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 140384*a**8*b**3*c**8 - 160*a**7*b**12*c**2*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 479788*a**7*b**5*c**7 + 8*a**6*b**14*c*(-b*(3*a*c - 2*b**2)/a**5 - \text{sqrt}(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 709872*a**6*b**7*c**6 - 575864*a**5*b**9*c**5 + 279640*a**4*b**11*c**4 - 83528*a**3*b**13*c**3 + 15056*a**2*b**15*c**2 - 1504*a*b**17*c + 64*b**19)/(1000*a**9*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b**6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**12*c**4 + 13680*a**2*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c)) + (-b*(3*a*c -
\end{aligned}$$

$$\begin{aligned}
& 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + \\
& 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c \\
& - b^{**6})))*\log(x + (-4928*a^{**16}*b^{**6}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})* \\
& (10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6}) \\
& / (a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))^{**2} + 10032 \\
& *a^{**15}*b^{**3}*c^{**5}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a \\
& **3*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - \\
& 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))^{**2} - 7980*a^{**14}*b^{**5}*c^{**4}*(-b*(3* \\
& a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}* \\
& c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a \\
& *b^{**4}*c - b^{**6})))^{**2} + 3249*a^{**13}*b^{**7}*c^{**3}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt} \\
& t(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b \\
& **6)/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))^{**2} + 8 \\
& 00*a^{**13}*c^{**8}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3} \\
& *c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48* \\
& a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) - 723*a^{**12}*b^{**9}*c^{**2}*(-b*(3*a*c - 2 \\
& *b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + \\
& 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c \\
& - b^{**6})))^{**2} + 6704*a^{**12}*b^{**2}*c^{**7}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a \\
& *c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a \\
& **5*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) + 84*a^{**11}*b \\
& *11*c*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - \\
& 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b \\
& **2*c^{**2} + 12*a*b^{**4}*c - b^{**6})))^{**2} - 15182*a^{**11}*b^{**4}*c^{**6}*(-b*(3*a*c - 2*b \\
& *2)/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15* \\
& a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - \\
& b^{**6}))) - 4*a^{**10}*b^{**13}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3} \\
&)*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}* \\
& c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6})))^{**2} + 12844*a^{**10}*b^{**6}*c^{**5} \\
& *(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a* \\
& **2*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{** \\
& 2 + 12*a*b^{**4}*c - b^{**6}))) - 5546*a^{**9}*b^{**8}*c^{**4}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \\
& \text{sqrt}(-(4*a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - \\
& 2*b^{**6})/(a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) - \\
& 4800*a^{**9}*b*c^{**9} + 1306*a^{**8}*b^{**10}*c^{**3}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4 \\
& *a*c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6}) \\
& / (a^{**5}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) + 140384*a \\
& **8*b^{**3}*c^{**8} - 160*a^{**7}*b^{**12}*c^{**2}*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a* \\
& c - b^{**2})^{**3})*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a \\
& *5*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) - 479788*a^{**7}* \\
& b^{**5}*c^{**7} + 8*a^{**6}*b^{**14}*c*(-b*(3*a*c - 2*b^{**2})/a^{**5} + \text{sqrt}(-(4*a*c - b^{**2}) \\
& **3)*(10*a^{**3}*c^{**3} - 30*a^{**2}*b^{**2}*c^{**2} + 15*a*b^{**4}*c - 2*b^{**6})/(a^{**5}*(64*a* \\
& **3*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) + 709872*a^{**6}*b^{**7}*c^{**6} \\
& - 575864*a^{**5}*b^{**9}*c^{**5} + 279640*a^{**4}*b^{**11}*c^{**4} - 83528*a^{**3}*b^{**13}*c^{**3} + \\
& 15056*a^{**2}*b^{**15}*c^{**2} - 1504*a*b^{**17}*c + 64*b^{**19})/(1000*a^{**9}*c^{**10} + 4284
\end{aligned}$$

```

0*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b**6*c**7 - 406368*a
**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**12*c**4 + 13680*a**2
*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c)) + (-4*a**4*c + a**3*b**2 + x
**4*(30*a**2*c**3 - 54*a*b**2*c**2 + 12*b**4*c) + x**3*(51*a**2*b*c**2 - 60
*a*b**3*c + 12*b**5) + x**2*(20*a**3*c**2 - 29*a**2*b**2*c + 6*a*b**4) + x
(8*a**3*b*c - 2*a**2*b**3))/(x**5*(12*a**5*c**2 - 3*a**4*b**2*c) + x**4*(12
*a**5*b*c - 3*a**4*b**3) + x**3*(12*a**6*c - 3*a**5*b**2)) + 2*b*(3*a*c - 2
*b**2)*log(x + (-4800*a**9*b*c**9 + 140384*a**8*b**3*c**8 + 1600*a**8*b*c**
8*(3*a*c - 2*b**2) - 479788*a**7*b**5*c**7 + 13408*a**7*b**3*c**7*(3*a*c -
2*b**2) + 709872*a**6*b**7*c**6 - 30364*a**6*b**5*c**6*(3*a*c - 2*b**2) - 1
9712*a**6*b**3*c**6*(3*a*c - 2*b**2)**2 - 575864*a**5*b**9*c**5 + 25688*a**
5*b**7*c**5*(3*a*c - 2*b**2) + 40128*a**5*b**5*c**5*(3*a*c - 2*b**2)**2 + 2
79640*a**4*b**11*c**4 - 11092*a**4*b**9*c**4*(3*a*c - 2*b**2) - 31920*a**4*
b**7*c**4*(3*a*c - 2*b**2)**2 - 83528*a**3*b**13*c**3 + 2612*a**3*b**11*c**
3*(3*a*c - 2*b**2) + 12996*a**3*b**9*c**3*(3*a*c - 2*b**2)**2 + 15056*a**2*
b**15*c**2 - 320*a**2*b**13*c**2*(3*a*c - 2*b**2) - 2892*a**2*b**11*c**2*(3
*a*c - 2*b**2)**2 - 1504*a*b**17*c + 16*a*b**15*c*(3*a*c - 2*b**2) + 336*a*
b**13*c*(3*a*c - 2*b**2)**2 + 64*b**19 - 16*b**15*(3*a*c - 2*b**2)**2)/(100
0*a**9*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b
**6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*a**3*b**1
2*c**4 + 13680*a**2*b**14*c**3 - 1440*a*b**16*c**2 + 64*b**18*c))/a**5

```

Giac [A] time = 1.10674, size = 381, normalized size = 1.51

$$\frac{2(2b^6 - 15ab^4c + 30a^2b^2c^2 - 10a^3c^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) + (2b^3 - 3abc) \log(cx^2 + bx + a) - 2(2b^3 - 3abc) \log(|x|)}{(a^5b^2 - 4a^6c)\sqrt{-b^2 + 4ac}} + \frac{(2b^3 - 3abc) \log(cx^2 + bx + a)}{a^5} - \frac{2(2b^3 - 3abc) \log(|x|)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*b*c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*(a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x^3)

$$3.28 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)^2} dx$$

Optimal. Leaf size=318

$$-\frac{12a^2c^2 - 22ab^2c + 5b^4}{2a^4x^2(b^2 - 4ac)} - \frac{(3a^2c^2 - 12ab^2c + 5b^4) \log(a + bx + cx^2)}{2a^6} + \frac{b(29a^2c^2 - 27ab^2c + 5b^4)}{a^5x(b^2 - 4ac)} + \frac{\log(x)(3a^2c^2 - 12ab^2c + 5b^4)}{a^6}$$

[Out] $-(5b^2 - 12ac)/(4a^2(b^2 - 4ac)x^4) + (b(5b^2 - 17ac))/(3a^3(b^2 - 4ac)x^3) - (5b^4 - 22ab^2c + 12a^2c^2)/(2a^4(b^2 - 4ac)x^2) + (b(5b^4 - 27ab^2c + 29a^2c^2))/(a^5(b^2 - 4ac)x) + (b^2 - 2ac + bcx)/(a(b^2 - 4ac)x^4(a + bx + cx^2)) + (b(5b^6 - 42ab^4c + 105a^2b^2c^2 - 70a^3c^3) \operatorname{ArcTanh}[(b + 2cx)/\sqrt{b^2 - 4ac}])/(a^6(b^2 - 4ac)^{3/2}) + ((5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[x])/a^6 - ((5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[a + bx + cx^2])/(2a^6)$

Rubi [A] time = 0.391815, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1585, 740, 800, 634, 618, 206, 628}

$$-\frac{12a^2c^2 - 22ab^2c + 5b^4}{2a^4x^2(b^2 - 4ac)} - \frac{(3a^2c^2 - 12ab^2c + 5b^4) \log(a + bx + cx^2)}{2a^6} + \frac{b(29a^2c^2 - 27ab^2c + 5b^4)}{a^5x(b^2 - 4ac)} + \frac{\log(x)(3a^2c^2 - 12ab^2c + 5b^4)}{a^6}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]

[Out] $-(5b^2 - 12ac)/(4a^2(b^2 - 4ac)x^4) + (b(5b^2 - 17ac))/(3a^3(b^2 - 4ac)x^3) - (5b^4 - 22ab^2c + 12a^2c^2)/(2a^4(b^2 - 4ac)x^2) + (b(5b^4 - 27ab^2c + 29a^2c^2))/(a^5(b^2 - 4ac)x) + (b^2 - 2ac + bcx)/(a(b^2 - 4ac)x^4(a + bx + cx^2)) + (b(5b^6 - 42ab^4c + 105a^2b^2c^2 - 70a^3c^3) \operatorname{ArcTanh}[(b + 2cx)/\sqrt{b^2 - 4ac}])/(a^6(b^2 - 4ac)^{3/2}) + ((5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[x])/a^6 - ((5b^4 - 12ab^2c + 3a^2c^2) \operatorname{Log}[a + bx + cx^2])/(2a^6)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,

$x]$ /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 740

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 800

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},

e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(ax^2 + bx^3 + cx^4)^2} dx &= \int \frac{1}{x^5(a + bx + cx^2)^2} dx \\
 &= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^4(a + bx + cx^2)} - \frac{\int \frac{-5b^2 + 12ac - 5bcx}{x^5(a + bx + cx^2)} dx}{a(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bcx}{a(b^2 - 4ac)x^4(a + bx + cx^2)} - \frac{\int \left(\frac{-5b^2 + 12ac}{ax^5} + \frac{5b^3 - 17abc}{a^2x^4} + \frac{-5b^4 + 22ab^2c - 12a^2c^2}{a^3x^3} + \frac{5b^5 - 27ab^3c + 29a^2c^2}{a^4x^2} \right) dx}{a(b^2 - 4ac)} \\
 &= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 29a^2c^2)}{a^5(b^2 - 4ac)x} \\
 &= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 29a^2c^2)}{a^5(b^2 - 4ac)x} \\
 &= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 29a^2c^2)}{a^5(b^2 - 4ac)x} \\
 &= -\frac{5b^2 - 12ac}{4a^2(b^2 - 4ac)x^4} + \frac{b(5b^2 - 17ac)}{3a^3(b^2 - 4ac)x^3} - \frac{5b^4 - 22ab^2c + 12a^2c^2}{2a^4(b^2 - 4ac)x^2} + \frac{b(5b^4 - 27ab^2c + 29a^2c^2)}{a^5(b^2 - 4ac)x}
 \end{aligned}$$

Mathematica [A] time = 0.413161, size = 272, normalized size = 0.86

$$\frac{-\frac{12a(-9a^2b^2c^2 - 5a^2bc^3x + 2a^3c^3 + 5ab^3c^2x + 6ab^4c - b^5cx - b^6)}{(b^2 - 4ac)(a + x(b + cx))} + 12 \log(x)(3a^2c^2 - 12ab^2c + 5b^4) - 6(3a^2c^2 - 12ab^2c + 5b^4) \log(a + x(b + cx))}{12a^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]

[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2*b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b^5*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/(b^2 - 4*a*c)*(a + x*(b + c*x))) + (12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x

)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)]/(12*a^6)

Maple [B] time = 0.018, size = 619, normalized size = 2.

$$-\frac{1}{4a^2x^4} + \frac{c}{x^2a^3} - \frac{3b^2}{2a^4x^2} + 3\frac{\ln(x)c^2}{a^4} - 12\frac{\ln(x)b^2c}{a^5} + 5\frac{\ln(x)b^4}{a^6} + \frac{2b}{3a^3x^3} - 6\frac{bc}{a^4x} + 4\frac{b^3}{a^5x} - 5\frac{bc^3x}{a^3(cx^2 + bx + a)(4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)

[Out]
$$-1/4/a^2/x^4 + 1/a^3/x^2*c - 3/2/a^4/x^2*b^2 + 3/a^4*\ln(x)*c^2 - 12/a^5*\ln(x)*b^2*c + 5/a^6*\ln(x)*b^4 + 2/3/a^3*b/x^3 - 6*b/a^4/x*c + 4*b^3/a^5/x - 5/a^3/(c*x^2+b*x+a)*b*c^3/(4*a*c-b^2)*x + 5/a^4/(c*x^2+b*x+a)*b^3*c^2/(4*a*c-b^2)*x - 1/a^5/(c*x^2+b*x+a)*b^5*c/(4*a*c-b^2)*x + 2/a^2/(c*x^2+b*x+a)/(4*a*c-b^2)*c^3 - 9/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c^2 + 6/a^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c - 1/a^5/(c*x^2+b*x+a)/(4*a*c-b^2)*b^6 - 6/a^3/(4*a*c-b^2)*c^3*\ln(c*x^2+b*x+a) + 51/2/a^4/(4*a*c-b^2)*c^2*\ln(c*x^2+b*x+a)*b^2 - 16/a^5/(4*a*c-b^2)*c*\ln(c*x^2+b*x+a)*b^4 + 5/2/a^6/(4*a*c-b^2)*\ln(c*x^2+b*x+a)*b^6 - 70/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3 + 105/a^4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c^2 - 42/a^5/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*c + 5/a^6/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 5.72475, size = 3563, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + \sqrt{b^2 - 4*a*c})*(2*c*x + b))/(c*x^2 + b*x + a) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*\log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*\log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*\sqrt{-b^2 + 4*a*c}*\arctan(-\sqrt{-b^2 + 4*a*c}*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*\log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*\log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4)] \end{aligned}$$

Sympy [B] time = 47.4723, size = 6181, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)

[Out]
$$\begin{aligned} & (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) \\ & - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6) \cdot \log(x + (4608a^{19}c^{*7}(-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} - 26432a^{18}b^2c^6 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} + 38640a^{17}b^4c^5 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} - 26124a^{16}b^6c^4 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} + 9603a^{15}b^8c^3 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} - 6912a^{15}c^9 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} - 1989a^{14}b^{10}c^2 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} + 37616a^{14}b^2c^8 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6)) + 219a^{13}b^{12}c \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6))^{*2} - 96472a^{13}b^4c^7 \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6)) - 10a^{12}b^{14} \cdot (-b\sqrt{-(4ac - b^2)}^{*3}) \cdot (70a^3c^3 - 105a^2b^2c^2 + 42ab^4c - 5b^6) / (2a^6(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) - (3a^2c^2 - 12ab^2c + 5b^4) / (2a^6)) \end{aligned}$$

$$\begin{aligned}
& a^{**4}c - b^{**6}) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6})^{**2} + 112 \\
& 063a^{**12}b^{**6}c^{**6}(-b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} \\
& + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6}) - 6 \\
& 9023a^{**11}b^{**8}c^{**5}(-b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} \\
& + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6}) - \\
& 20736a^{**11}c^{**11} + 24355a^{**10}b^{**10}c^{**4}(-b\sqrt{-(4ac - b^2)^{**3}})(70 \\
& a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} \\
& - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6}) + 373872a^{**10}b^{**2}c^{**10} - 4964a^{**9}b^{**12}c^{**3}(-b\sqrt{-(4ac - b^2)^{**3}}) \\
& (70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} \\
& + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6}) - 2277288a^{**9}b^{**4}c^{**9} + 54 \\
& 5a^{**8}b^{**14}c^{**2}(-b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6}) + 648 \\
& 7391a^{**8}b^{**6}c^{**8} - 25a^{**7}b^{**16}c^{**}(-b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6}) - 9943570a^{**7}b^{**8}c^{**7} + 9090837a^{**6}b^{**10}c^{**6} - 5264714 \\
& a^{**5}b^{**12}c^{**5} + 1984426a^{**4}b^{**14}c^{**4} - 486146a^{**3}b^{**16}c^{**3} + 74720 \\
& a^{**2}b^{**18}c^{**2} - 6550ab^{**20}c + 250b^{**22})/(90720a^{**10}b^{**11}c^{**11} - 84413 \\
& 0a^{**9}b^{**3}c^{**10} + 3174507a^{**8}b^{**5}c^{**9} - 5885010a^{**7}b^{**7}c^{**8} + 61682 \\
& 25a^{**6}b^{**9}c^{**7} - 3960180a^{**5}b^{**11}c^{**6} + 1618470a^{**4}b^{**13}c^{**5} - 423 \\
& 276a^{**3}b^{**15}c^{**4} + 68670a^{**2}b^{**17}c^{**3} - 6300ab^{**19}c^{**2} + 250b^{**21} \\
& c)) + (b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} + 42 \\
& ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6}) * \log(x + (4608a^{**19}c^{**7}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} + 42 \\
& ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6}))^{**2} - 26432a^{**18} \\
& b^{**2}c^{**6}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} + \\
& 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6})^{**2} + 38640a^{**17}b^{**4}c^{**5}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c \\
& + 5b^{**4})/(2a^{**6})^{**2} - 26124 \\
& a^{**16}b^{**6}c^{**4}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12 \\
& ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6})^{**2} + 96 \\
& 03a^{**15}b^{**8}c^{**3}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2} \\
& + 42ab^{**4}c - 5b^{**6})/(2a^{**6}(64a^{**3}c^{**3} - 48a^{**2}b^{**2}c^{**2} + 12 \\
& ab^{**4}c - b^{**6})) - (3a^{**2}c^{**2} - 12ab^{**2}c + 5b^{**4})/(2a^{**6})^{**2} - \\
& 6912a^{**15}c^{**9}(b\sqrt{-(4ac - b^2)^{**3}})(70a^{**3}c^{**3} - 105a^{**2}b^{**2}c^{**2}
\end{aligned}$$

$$\begin{aligned}
& **2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12* \\
& a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 1989*a \\
& **14*b**10*c**2*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c \\
& **2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12* \\
& a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 376 \\
& 16*a**14*b**2*c**8*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b** \\
& 2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + \\
& 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 219 \\
& *a**13*b**12*c*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c* \\
& **2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a \\
& *b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 9647 \\
& 2*a**13*b**4*c**7*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2 \\
& *c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1 \\
& 2*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 10*a \\
& **12*b**14*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c**2 + \\
& 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b** \\
& 4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 112063*a \\
& **12*b**6*c**6*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c* \\
& **2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a \\
& *b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 69023*a \\
& **11*b**8*c**5*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c* \\
& **2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a \\
& *b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 20736*a \\
& **11*c**11 + 24355*a**10*b**10*c**4*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c* \\
& **3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48* \\
& a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4 \\
&)/(2*a**6)) + 373872*a**10*b**2*c**10 - 4964*a**9*b**12*c**3*(b*\sqrt{-(4*a* \\
& c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2 \\
& *a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c* \\
& **2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 2277288*a**9*b**4*c**9 + 545*a**8*b* \\
& *14*c**2*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 4 \\
& 2*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4* \\
& c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 6487391*a**8* \\
& b**6*c**8 - 25*a**7*b**16*c*(b*\sqrt{-(4*a*c - b**2)**3}*(70*a**3*c**3 - 105 \\
& *a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b** \\
& 2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a** \\
& 6)) - 9943570*a**7*b**8*c**7 + 9090837*a**6*b**10*c**6 - 5264714*a**5*b**12 \\
& *c**5 + 1984426*a**4*b**14*c**4 - 486146*a**3*b**16*c**3 + 74720*a**2*b**18 \\
& *c**2 - 6550*a*b**20*c + 250*b**22)/(90720*a**10*b*c**11 - 844130*a**9*b**3 \\
& *c**10 + 3174507*a**8*b**5*c**9 - 5885010*a**7*b**7*c**8 + 6168225*a**6*b** \\
& 9*c**7 - 3960180*a**5*b**11*c**6 + 1618470*a**4*b**13*c**5 - 423276*a**3*b* \\
& *15*c**4 + 68670*a**2*b**17*c**3 - 6300*a*b**19*c**2 + 250*b**21*c)) - (12* \\
& a**5*c - 3*a**4*b**2 + x**5*(348*a**2*b*c**3 - 324*a*b**3*c**2 + 60*b**5*c) \\
& + x**4*(-72*a**3*c**3 + 480*a**2*b**2*c**2 - 354*a*b**4*c + 60*b**6) + x** \\
& 3*(208*a**3*b*c**2 - 172*a**2*b**3*c + 30*a*b**5) + x**2*(-36*a**4*c**2 + 4
\end{aligned}$$

```

9*a**3*b**2*c - 10*a**2*b**4) + x*(-20*a**4*b*c + 5*a**3*b**3))/(x**6*(48*a
**6*c**2 - 12*a**5*b**2*c) + x**5*(48*a**6*b*c - 12*a**5*b**3) + x**4*(48*a
**7*c - 12*a**6*b**2)) + (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)*log(x + (-207
36*a**11*c**11 + 373872*a**10*b**2*c**10 - 2277288*a**9*b**4*c**9 - 6912*a*
**9*c**9*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 6487391*a**8*b**6*c**8 + 376
16*a**8*b**2*c**8*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 9943570*a**7*b**8*
c**7 - 96472*a**7*b**4*c**7*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 4608*a**
7*c**7*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 9090837*a**6*b**10*c**6 +
112063*a**6*b**6*c**6*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 26432*a**6*b**
2*c**6*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 5264714*a**5*b**12*c**5 -
69023*a**5*b**8*c**5*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 38640*a**5*b**4
*c**5*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 1984426*a**4*b**14*c**4 + 2
4355*a**4*b**10*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 26124*a**4*b**6
*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 486146*a**3*b**16*c**3 - 49
64*a**3*b**12*c**3*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 9603*a**3*b**8*c*
**3*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 74720*a**2*b**18*c**2 + 545*a*
**2*b**14*c**2*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 1989*a**2*b**10*c**2*(
3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 6550*a*b**20*c - 25*a*b**16*c*(3*a
**2*c**2 - 12*a*b**2*c + 5*b**4) + 219*a*b**12*c*(3*a**2*c**2 - 12*a*b**2*c
+ 5*b**4)**2 + 250*b**22 - 10*b**14*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**
2)/(90720*a**10*b*c**11 - 844130*a**9*b**3*c**10 + 3174507*a**8*b**5*c**9 -
5885010*a**7*b**7*c**8 + 6168225*a**6*b**9*c**7 - 3960180*a**5*b**11*c**6
+ 1618470*a**4*b**13*c**5 - 423276*a**3*b**15*c**4 + 68670*a**2*b**17*c**3
- 6300*a*b**19*c**2 + 250*b**21*c))/a**6

```

Giac [A] time = 1.12268, size = 468, normalized size = 1.47

$$\frac{(5b^7 - 42ab^5c + 105a^2b^3c^2 - 70a^3bc^3) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right) - (5b^4 - 12ab^2c + 3a^2c^2) \log(cx^2 + bx + a)}{(a^6b^2 - 4a^7c)\sqrt{-b^2+4ac}} + \frac{(5b^4 - 12ab^2c + 3a^2c^2) \log(cx^2 + bx + a)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] $-(5b^7 - 42a^2b^5c + 105a^2b^3c^2 - 70a^3b^2c^3) \arctan((2cx + b)/\sqrt{-b^2 + 4ac}) / ((a^6b^2 - 4a^7c) \sqrt{-b^2 + 4ac}) - 1/2(5b^4 - 12a^2b^2c + 3a^2c^2) \log(cx^2 + bx + a) / a^6 + (5b^4 - 12a^2b^2c + 3a^2c^2) \log(\text{abs}(x)) / a^6 - 1/12(3a^5b^2 - 12a^6c - 12(5a^2b^5c - 27a^2b^3c^2 + 29a^3b^2c^3) x^5 - 6(10a^2b^6 - 59a^2b^4c + 80a^3b^2c^2 - 12a^4c^3) x^4 - 2(15a^2b^5 - 86a^3b^3c + 104a^4b^2c^2) x^3 + (10a^3b^4 - 49a^4b^2c + 36a^5c^2) x^2 - 5(a^4b^3 - 4a^5b^2c) x) /$

$$(c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4$$

3.29 $\int x^2 \sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=257

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{x(7b^2 - 16ac) \sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{b(35b^2 - 116ac) \sqrt{ax^2 + bx^3 + cx^4}}{960c^3}$$

[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.588441, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{x(7b^2 - 16ac) \sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{b(35b^2 - 116ac) \sqrt{ax^2 + bx^3 + cx^4}}{960c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1919

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q


```
)*)Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
.)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x^2 \sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} + \frac{\int \frac{x^3(-3ab - \frac{1}{2}(7b^2 - 16ac)x)}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{40c} \\
 &= -\frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} - \frac{\int \frac{x^2(-a(7b^2 - 16ac) - \frac{1}{4}b(35b^2 - 116ac))\sqrt{ax^2 + bx^3 + cx^4}}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{120c^2} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} + \frac{x^2(b + 8cx)\sqrt{ax^2 + bx^3 + cx^4}}{40c} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2} \\
 &= \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{960c^3} - \frac{(105b^4 - 460ab^2c + 256a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{1920c^4x} - \frac{(7b^2 - 16ac)x\sqrt{ax^2 + bx^3 + cx^4}}{240c^2}
 \end{aligned}$$

Mathematica [A] time = 0.241455, size = 180, normalized size = 0.7

$$\frac{2\sqrt{cx}(a + x(b + cx))(128c^2(-2a^2 + acx^2 + 3c^2x^4) + 4b^2c(115a - 14cx^2) + 8bc^2x(6cx^2 - 29a) + 70b^3cx - 105b^4) + 15x^2}{3840c^{9/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x^4)) + 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b

$+ 2*c*x + 2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)]])/(3840*c^{(9/2)*\text{Sqrt}[x^2*(a + x*(b + c*x))])}$

Maple [A] time = 0.007, size = 310, normalized size = 1.2

$$\frac{1}{3840x} \sqrt{cx^4 + bx^3 + ax^2} \left(768x^2 (cx^2 + bx + a)^{3/2} c^{9/2} - 672c^{7/2} (cx^2 + bx + a)^{3/2} xb - 512c^{7/2} (cx^2 + bx + a)^{3/2} a + 560 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] $\frac{1}{3840} (cx^4 + bx^3 + ax^2)^{1/2} \left(768x^2 (cx^2 + bx + a)^{3/2} c^{9/2} - 672c^{7/2} (cx^2 + bx + a)^{3/2} xb - 512c^{7/2} (cx^2 + bx + a)^{3/2} a + 560c^{5/2} (cx^2 + bx + a)^{3/2} b^2 + 720c^{7/2} (cx^2 + bx + a)^{1/2} xab - 420c^{5/2} (cx^2 + bx + a)^{1/2} x^2 b^3 + 360c^{5/2} (cx^2 + bx + a)^{1/2} ab^2 - 210c^{3/2} (cx^2 + bx + a)^{1/2} b^4 + 720 \ln\left(\frac{1}{2} (2(cx^2 + bx + a)^{1/2} c^{1/2} + 2cx + b)\right) / c^{1/2} a^2 b^3 c^3 - 600 \ln\left(\frac{1}{2} (2(cx^2 + bx + a)^{1/2} c^{1/2} + 2cx + b)\right) / c^{1/2} a^2 b^3 c^2 + 105 \ln\left(\frac{1}{2} (2(cx^2 + bx + a)^{1/2} c^{1/2} + 2cx + b)\right) / c^{1/2} b^5 c \right) / x / (cx^2 + bx + a)^{1/2} / c^{11/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^3 + ax^2} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)

Fricas [A] time = 1.73634, size = 898, normalized size = 3.49

$$\frac{15(7b^5 - 40ab^3c + 48a^2bc^2) \sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c + (b^2 + 4ac)x}}{x}\right) + 4(384c^5x^4 + 48bc^4x^3 - 105b^4c^3)}{7680c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x^2 (a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)

Giac [A] time = 1.18191, size = 382, normalized size = 1.49

$$\frac{1}{1920} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(6 \left(8x \operatorname{sgn}(x) + \frac{b \operatorname{sgn}(x)}{c} \right) x - \frac{7b^2c^2 \operatorname{sgn}(x) - 16ac^3 \operatorname{sgn}(x)}{c^4} \right) x + \frac{35b^3c \operatorname{sgn}(x) - 116abc^2 \operatorname{sgn}(x)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4 - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(-

$$2*(\sqrt{c}*x - \sqrt{c*x^2 + b*x + a})*\sqrt{c} - b)/c^{(9/2)} + 1/3840*(105*b^5*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) - 600*a*b^3*c*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) + 720*a^2*b*c^2*\log(\text{abs}(-b + 2*\sqrt{a}*\sqrt{c})) + 210*\sqrt{a}*b^4*\sqrt{c} - 920*a^{(3/2)}*b^2*c^{(3/2)} + 512*a^{(5/2)}*c^{(5/2)})*\text{sgn}(x)/c^{(9/2)}$$

3.30 $\int x\sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=205

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} - \frac{x(b^2 - 4ac)(5b^2 - 4ac)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{x}{2\sqrt{ax^2 + bx^3 + cx^4}}\right)}{128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

[Out] $-\left(\left(5b^2 - 12ac\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(96c^2\right) + \left(b\left(15b^2 - 52ac\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(192c^3x\right) + \left(x\left(b + 6cx\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(24c\right) - \left(\left(b^2 - 4ac\right)\left(5b^2 - 4ac\right)xx\sqrt{a + bx + cx^2}\right)\text{ArcTanh}\left[\left(b + 2cx\right)/\left(2\sqrt{c}\sqrt{a + bx + cx^2}\right)\right]/\left(128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}\right)$

Rubi [A] time = 0.370274, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1919, 1949, 12, 1914, 621, 206}

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} - \frac{x(b^2 - 4ac)(5b^2 - 4ac)\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{x}{2\sqrt{ax^2 + bx^3 + cx^4}}\right)}{128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] $-\left(\left(5b^2 - 12ac\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(96c^2\right) + \left(b\left(15b^2 - 52ac\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(192c^3x\right) + \left(x\left(b + 6cx\right)\sqrt{ax^2 + bx^3 + cx^4}\right)/\left(24c\right) - \left(\left(b^2 - 4ac\right)\left(5b^2 - 4ac\right)xx\sqrt{a + bx + cx^2}\right)\text{ArcTanh}\left[\left(b + 2cx\right)/\left(2\sqrt{c}\sqrt{a + bx + cx^2}\right)\right]/\left(128c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}\right)$

Rule 1919

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p]/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1) + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +

$c*x^{(2*n - q)}^{(p - 1)}$, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]

Rule 1949

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_) * ((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)] , x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} + \frac{\int \frac{x^2(-2ab - \frac{1}{2}(5b^2 - 12ac)x)}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} - \frac{\int \frac{x(-\frac{1}{2}a(5b^2 - 12ac) - \frac{1}{4}b(15b^2 - 52ac))}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{48c^2} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c} \\
&= -\frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96c^2} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192c^3x} + \frac{x(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{24c}
\end{aligned}$$

Mathematica [A] time = 0.192789, size = 150, normalized size = 0.73

$$\frac{2\sqrt{cx}(a + x(b + cx))(b(8c^2x^2 - 52ac) + 24c^2x(a + 2cx^2) - 10b^2cx + 15b^3) - 3x(16a^2c^2 - 24ab^2c + 5b^4)\sqrt{a + x(b + cx)}}{384c^{7/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])/(384*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.008, size = 265, normalized size = 1.3

$$\frac{1}{384x} \sqrt{cx^4 + bx^3 + ax^2} \left(96x (cx^2 + bx + a)^{3/2} c^{7/2} - 80c^{5/2} (cx^2 + bx + a)^{3/2} b - 48c^{7/2} \sqrt{cx^2 + bx + ax} + 60c^{5/2} \sqrt{cx^2 + bx + ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] $\frac{1}{384} (cx^4 + bx^3 + ax^2)^{1/2} (96x (cx^2 + bx + a)^{3/2} c^{7/2} - 80c^{5/2} (cx^2 + bx + a)^{3/2} b - 48c^{7/2} \sqrt{cx^2 + bx + ax} + 60c^{5/2} \sqrt{cx^2 + bx + ax})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^3 + ax^2} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)

Fricas [A] time = 1.64592, size = 737, normalized size = 3.6

$$\frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c + (b^2 + 4ac)x}}{x}\right) + 4(48c^4x^3 + 8bc^3x^2 + 15b^3c - 52c^2x)}{768c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

3.31 $\int \sqrt{ax^2 + bx^3 + cx^4} dx$

Optimal. Leaf size=163

$$\frac{b(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}} - \frac{b(b+2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx}$$

[Out] $-(b*(b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2) * \text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(5/2)*x*\text{Sqrt}[a + b*x + c*x^2])$

Rubi [A] time = 0.0576031, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1903, 640, 612, 621, 206}

$$\frac{b(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}} - \frac{b(b+2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a*x^2 + b*x^3 + c*x^4], x]$

[Out] $-(b*(b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*x + c*x^2) * \text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(5/2)*x*\text{Sqrt}[a + b*x + c*x^2])$

Rule 1903

$\text{Int}[\text{Sqrt}[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]$
 $\rightarrow \text{Dist}[\text{Sqrt}[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*\text{Sqrt}[a + b*x^(n - q) + c*x^(2*(n - q))]), \text{Int}[x^(q/2)*\text{Sqrt}[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /;$ FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]

Rule 640

$\text{Int}[(d_. + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]$
 $\rightarrow \text{Simp}[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + \text{Dist}[(2*c*d - b$

*e)/(2*c), Int[(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt{ax^2 + bx^3 + cx^4} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4} \int x\sqrt{a + bx + cx^2} dx}{x\sqrt{a + bx + cx^2}} \\
 &= \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} - \frac{(b\sqrt{ax^2 + bx^3 + cx^4}) \int \sqrt{a + bx + cx^2} dx}{2cx\sqrt{a + bx + cx^2}} \\
 &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{(b(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}) \int \sqrt{a + bx + cx^2} dx}{16c^2x\sqrt{a + bx + cx^2}} \\
 &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{(b(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}) \int \sqrt{a + bx + cx^2} dx}{8c^2x\sqrt{a + bx + cx^2}} \\
 &= -\frac{b(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{8c^2x} + \frac{(a + bx + cx^2)\sqrt{ax^2 + bx^3 + cx^4}}{3cx} + \frac{b(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4} \int \sqrt{a + bx + cx^2} dx}{16c^{5/2}x\sqrt{a + bx + cx^2}}
 \end{aligned}$$

Mathematica [A] time = 0.219505, size = 119, normalized size = 0.73

$$\frac{2\sqrt{cx}(a+x(b+cx))(8c(a+cx^2)-3b^2+2bcx)+3bx(b^2-4ac)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (2*Sqrt[c]*x*(a+x*(b+c*x))*(-3*b^2+2*b*c*x+8*c*(a+c*x^2))+3*b*(b^2-4*a*c)*x*Sqrt[a+x*(b+c*x)]*ArcTanh[(b+2*c*x)/(2*Sqrt[c]*Sqrt[a+x*(b+c*x)])]/(48*c^(5/2)*Sqrt[x^2*(a+x*(b+c*x))])

Maple [A] time = 0.006, size = 167, normalized size = 1.

$$\frac{1}{48x}\sqrt{cx^4+bx^3+ax^2}\left(16(cx^2+bx+a)^{3/2}c^{5/2}-12c^{5/2}\sqrt{cx^2+bx+axb}-6c^{3/2}\sqrt{cx^2+bx+ab^2}-12\ln\left(\frac{1}{2}\frac{2\sqrt{cx^2+bx+axb}}{c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2), x)

[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^2-12*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x/(c*x^2+b*x+a)^(1/2)/c^(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^3 + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)

Fricas [A] time = 1.61506, size = 591, normalized size = 3.63

$$\left[\frac{3(b^3 - 4abc)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c + (b^2 + 4ac)x}}{x}\right) - 4(8c^3x^2 + 2bc^2x - 3b^2c + 8ac^2)\sqrt{cx^4 + bx^3 + ax^2}}{96c^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2)/(c^3*x), - 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^2 + bx^3 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)

Giac [A] time = 1.14418, size = 224, normalized size = 1.37

$$\frac{1}{24} \sqrt{cx^2 + bx + a} \left(2 \left(4x \operatorname{sgn}(x) + \frac{b \operatorname{sgn}(x)}{c} \right) x - \frac{3b^2 \operatorname{sgn}(x) - 8ac \operatorname{sgn}(x)}{c^2} \right) - \frac{(b^3 \operatorname{sgn}(x) - 4abc \operatorname{sgn}(x)) \log\left(\left| -2 \left(\sqrt{cx} - \frac{b}{2c} \right) \right|\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x) - 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/48*(3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

$$3.32 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x} dx$$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.0777735, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1918, 1914, 621, 206}

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]

[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1918

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^p_], x_Symbol] :> Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]

Rule 1914


```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{(b^2 - 4ac) \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{\left((b^2 - 4ac) x \sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{\left((b^2 - 4ac) x \sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{4c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4cx} - \frac{(b^2 - 4ac) x \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.14577, size = 100, normalized size = 0.84

$$\frac{x(2\sqrt{c}(b + 2cx)(a + x(b + cx)) - (b^2 - 4ac)\sqrt{a + x(b + cx)} \log(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx))}{8c^{3/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

[Out] $(x*(2*\sqrt{c}*(b + 2*c*x)*(a + x*(b + c*x)) - (b^2 - 4*a*c)*\sqrt{a + x*(b + c*x)})*\log[b + 2*c*x + 2*\sqrt{c}*\sqrt{a + x*(b + c*x)}])/(8*c^{(3/2)}*\sqrt{x}^{2*(a + x*(b + c*x))})$

Maple [A] time = 0.004, size = 146, normalized size = 1.2

$$\frac{1}{8x} \sqrt{cx^4 + bx^3 + ax^2} \left(4 \sqrt{cx^2 + bx + ac^{5/2}x} + 2 \sqrt{cx^2 + bx + ac^{3/2}b} + 4 \ln \left(\frac{1}{2} \frac{2 \sqrt{cx^2 + bx + a\sqrt{c}} + 2cx + b}{\sqrt{c}} \right) \right) ac^2 - \ln \left(\frac{1}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c*x^4+b*x^3+a*x^2)^{(1/2)}/x,x)$

[Out] $\frac{1}{8}*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(4*(c*x^2+b*x+a)^{(1/2)}*c^{(5/2)}*x+2*(c*x^2+b*x+a)^{(1/2)}*c^{(3/2)}*b+4*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)}))*a*c^2-\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*b^2*c)/(c*x^2+b*x+a)^{(1/2)}/c^{(5/2)}/x$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x^4+b*x^3+a*x^2)^{(1/2)}/x,x, \text{algorithm}=\text{"maxima"})$

[Out] $\text{integrate}(\sqrt{c*x^4 + b*x^3 + a*x^2}/x, x)$

Fricas [A] time = 1.66874, size = 502, normalized size = 4.22

$$\left[\frac{(b^2 - 4ac)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(2c^2x + bc) (b^2 - 4ac)\sqrt{-cx}}{16c^2x}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")

[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)

Giac [A] time = 1.16641, size = 169, normalized size = 1.42

$$\frac{1}{8} \left(2 \sqrt{cx^2 + bx + a} \left(2x + \frac{b}{c} \right) + \frac{(b^2 - 4ac) \log \left(\left| -2 \left(\sqrt{cx} - \sqrt{cx^2 + bx + a} \right) \sqrt{c} - b \right| \right)}{c^{\frac{3}{2}}} \right) \operatorname{sgn}(x) - \frac{(b^2 \log(|-b + 2\sqrt{a}\sqrt{c}|))}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/c^(3/2)

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} dx$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{ax}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] + (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/((2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4]))

Rubi [A] time = 0.126323, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1921, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{ax}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] + (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/((2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4]))

Rule 1921

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

Rule 1933

Int[((A_) + (B_)*(x_)^(j_))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]

Rule 843

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{1}{2} \int \frac{2a + bx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{2a+bx}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} + \frac{\left(ax\sqrt{a + bx + cx^2}\right) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(bx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} - \frac{\left(2ax\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(bx\sqrt{a + bx + cx^2}\right)}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} - \frac{\sqrt{ax}\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{bx}{\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.110016, size = 134, normalized size = 0.77

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}-2\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] - 2*Sqrt[a]*Sqrt[c]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.004, size = 126, normalized size = 0.7

$$-\frac{1}{2x}\sqrt{cx^4 + bx^3 + ax^2}\left(2\sqrt{a}\ln\left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x}\right)\sqrt{c} - 2\sqrt{cx^2 + bx + a}\sqrt{c} - b\ln\left(\frac{1}{2}\left(2\sqrt{cx^2 + bx + a}\sqrt{c} + \dots\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)

[Out] $-1/2*(c*x^4+b*x^3+a*x^2)^{(1/2)}*(2*a^{(1/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*c^{(1/2)}-2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}-b*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)}))/x/(c*x^2+b*x+a)^{(1/2)}/c^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)`

Fricas [A] time = 1.84208, size = 1457, normalized size = 8.42

$$\left[\frac{b\sqrt{cx} \log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right) + 2\sqrt{acx} \log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)}{4cx} \right] +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")`

[Out] $[1/4*(b*\sqrt{c})*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) + 2*\sqrt{a}*c*x*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) + 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*c/(c*x), -1/2*(b*\sqrt{-c})*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) - \sqrt{a}*c*x*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) - 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*c/(c*x), 1/4*(4*\sqrt{-a}*c*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*\sqrt{c})*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) + 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*c/(c*x), 1/2*(2*\sqrt{-a}*c*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) - b*\sqrt{-c})*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*\sqrt{c*x^4 + b*x$

$$^3 + a*x^2)*c)/(c*x)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.34 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} dx$$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{cx}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (\text{Sqrt}[c]*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]$

Rubi [A] time = 0.124074, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1920, 1933, 843, 621, 206, 724}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{cx}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]

[Out] $-(\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (\text{Sqrt}[c]*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]$

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1933

```
Int[((A_) + (B_)*(x_)^(j_.))/Sqrt[(b_)*(x_)^(n_.) + (a_)*(x_)^(q_.) + (c_)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{1}{2} \int \frac{b + 2cx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{b+2cx}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} + \frac{\left(bx\sqrt{a + bx + cx^2}\right) \int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(cx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} - \frac{\left(bx\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\left(2cx\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2} - \frac{bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\sqrt{cx}\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.127059, size = 131, normalized size = 0.76

$$\frac{\sqrt{a + x(b + cx)} \left(bx \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) + 2\sqrt{a} \left(\sqrt{a + x(b + cx)} - \sqrt{cx} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right) \right)}{2\sqrt{a}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]

[Out] -(Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)] - Sqrt[c]*x*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(2*Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 174, normalized size = 1.

$$\frac{1}{2ax^2} \sqrt{cx^4 + bx^3 + ax^2} \left(2c^{5/2} \sqrt{cx^2 + bx + ax^2} - c^{3/2} \sqrt{a} \ln \left(\frac{1}{x} \left(2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a} \right) \right) \right) xb - 2 \left(cx^2 + bx + a \right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)

```
[Out] 1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^2-c^(3/2)*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x*b-2*(c*x^2+b*x+a)^(3/2)*c^(3/2)+2*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x*b+2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a*c^2)/x^2/(c*x^2+b*x+a)^(1/2)/a/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)
```

Fricas [A] time = 1.96342, size = 1489, normalized size = 8.61

$$\frac{2a\sqrt{cx^2} \log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right) + \sqrt{ab}x^2 \log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)}{4ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*a*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2*arctan(1/2*s
```

```

qrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a/(a*x^2)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.35 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} dx$$

Optimal. Leaf size=114

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(3/2))

Rubi [A] time = 0.147701, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(3/2))

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1951

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*(A_) + (B_.)*(x_)^(r_.), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b

```
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_)*(x_)^2 + (b_)*(x_)^(n_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} + \frac{1}{4} \int \frac{b + 2cx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} - \frac{\int \frac{b^2 - 4ac}{2\sqrt{ax^2 + bx^3 + cx^4}} dx}{4a} \\
 &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} - \frac{(b^2 - 4ac) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8a} \\
 &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} + \frac{(b^2 - 4ac) \operatorname{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a + bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{4a} \\
 &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2x^3} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{4ax^2} + \frac{(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a + bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{8a^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.100333, size = 112, normalized size = 0.98

$$\frac{\sqrt{x^2(a+x(b+cx))} \left(x^2 (b^2 - 4ac) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) - 2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)} \right)}{8a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(8*a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])

Maple [B] time = 0.006, size = 207, normalized size = 1.8

$$-\frac{1}{8a^2x^3}\sqrt{cx^4+bx^3+ax^2}\left(4ca^{3/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^2+2c\sqrt{cx^2+bx+ax^3}b-4c\sqrt{cx^2+bx+ax^2}a-\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)

[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*c*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^2+2*c*(c*x^2+b*x+a)^(1/2)*x^3*b-4*c*(c*x^2+b*x+a)^(1/2)*x^2*a-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^2*b^2-2*(c*x^2+b*x+a)^(3/2)*x*b+2*(c*x^2+b*x+a)^(1/2)*x^2*b^2+4*(c*x^2+b*x+a)^(3/2)*a)/x^3/(c*x^2+b*x+a)^(1/2)/a^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)

Fricas [A] time = 1.69192, size = 517, normalized size = 4.54

$$\left[\frac{(b^2 - 4ac)\sqrt{a}x^3 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x - 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) + 4\sqrt{cx^4 + bx^3 + ax^2}(abx + 2a^2) (b^2 - 4ac)\sqrt{a}}{16a^2x^3}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")

[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")

[Out] Timed out

$$3.36 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5} dx$$

Optimal. Leaf size=155

$$\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4}$$

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(5/2))

Rubi [A] time = 0.255838, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1920, 1951, 12, 1904, 206}

$$\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{b(b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} - \frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]

[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(5/2))

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
.*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

```

Rule 1904

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} + \frac{1}{6} \int \frac{b + 2cx}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} - \frac{\int \frac{\frac{1}{2}(3b^2 - 8ac) + bcx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx}{12a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} + \frac{\int \frac{3b(b^2 - 4ac)}{4\sqrt{ax^2 + bx^3 + cx^4}} dx}{12a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} + \frac{(b(b^2 - 4ac)) \int}{16a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{(b(b^2 - 4ac)) \operatorname{Su}}{16a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{3x^4} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{12ax^3} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{24a^2x^2} - \frac{b(b^2 - 4ac) \operatorname{tanh}}{16a}
\end{aligned}$$

Mathematica [A] time = 0.143892, size = 131, normalized size = 0.85

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(-2\sqrt{a}\sqrt{a + x(b + cx)} (8a^2 + 2ax(b + 4cx) - 3b^2x^2) - 3bx^3 (b^2 - 4ac) \operatorname{tanh}^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) \right)}{48a^{5/2}x^4\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5, x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b^2*x^2 + 2*a*x*(b + 4*c*x)) - 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))/(48*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])

Maple [A] time = 0.006, size = 234, normalized size = 1.5

$$\frac{1}{48x^4a^3} \sqrt{cx^4 + bx^3 + ax^2} \left(12ca^{3/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) x^3b + 6c\sqrt{cx^2 + bx + a}x^4b^2 - 12c\sqrt{cx^2 + bx + a}x^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)`

[Out] $\frac{1}{48}(c x^4+b x^3+a x^2)^{1/2} \left(12 c a^{3/2} \ln\left(\frac{(2 a+b x+2 a^{1/2})(c x^2+b x+a)^{1/2}}{x}\right) x^3 b+6 c(c x^2+b x+a)^{1/2} x^4 b^2-12 c(c x^2+b x+a)^{1/2} x^3 a b-3 a^{1/2} \ln\left(\frac{(2 a+b x+2 a^{1/2})(c x^2+b x+a)^{1/2}}{x}\right) x^3 b^3-6(c x^2+b x+a)^{3/2} x^2 b^2+6(c x^2+b x+a)^{1/2} x^3 b^3+12(c x^2+b x+a)^{3/2} x a b-16(c x^2+b x+a)^{3/2} a^2\right) / x^4 / (c x^2+b x+a)^{1/2} / a^3$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^4+b x^3+a x^2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)`

Fricas [A] time = 1.84133, size = 609, normalized size = 3.93

$$\left[\frac{3(b^3 - 4abc)\sqrt{a}x^4 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) + 4\sqrt{cx^4 + bx^3 + ax^2}(2a^2bx + 8a^3 - (3ab^2 - 3a^2b))}{96a^3x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")`

[Out] $[-1/96(3(b^3 - 4a*b*c)*\sqrt{a})x^4*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) + 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2)/(a^3*x^4), 1/48(3(b^3 - 4a*b*c)*\sqrt{-a})x^4*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2)/(a^3*x^4)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)

[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")

[Out] Timed out

$$3.37 \quad \int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6} dx$$

Optimal. Leaf size=205

$$-\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192a^3x^2} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} + \frac{(b^2 - 4ac)(5b^2 - 4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(24*a*x^4) + ((5*b^2 - 12*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(15*b^2 - 52*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/(128*a^{(7/2)})$

Rubi [A] time = 0.385848, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1920, 1951, 12, 1904, 206}

$$-\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{192a^3x^2} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} + \frac{(b^2 - 4ac)(5b^2 - 4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6, x]

[Out] $-\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(24*a*x^4) + ((5*b^2 - 12*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(15*b^2 - 52*a*c)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/(128*a^{(7/2)})$

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1904

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^6} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} + \frac{1}{8} \int \frac{b + 2cx}{x^3 \sqrt{ax^2 + bx^3 + cx^4}} dx \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} - \frac{\int \frac{\frac{1}{2}(5b^2 - 12ac) + 2bcx}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx}{24a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} + \frac{\int \frac{\frac{1}{4}b(15b^2 - 52ac)}{x \sqrt{ax^2 + bx^3 + cx^4}} dx}{4} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15b^2 - 52ac)}{192a^2x^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15b^2 - 52ac)}{192a^2x^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15b^2 - 52ac)}{192a^2x^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{4x^5} - \frac{b\sqrt{ax^2 + bx^3 + cx^4}}{24ax^4} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{96a^2x^3} - \frac{b(15b^2 - 52ac)}{192a^2x^2}
\end{aligned}$$

Mathematica [A] time = 0.225157, size = 160, normalized size = 0.78

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(3x^4 (16a^2c^2 - 24ab^2c + 5b^4) \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) - 2\sqrt{a}\sqrt{a+x(b+cx)} (8a^2x(b+3cx) + 48a^3 - 384a^{7/2}x^5\sqrt{a+x(b+cx)}) \right)}{384a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3 + 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(384*a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])

Maple [B] time = 0.006, size = 387, normalized size = 1.9

$$\frac{1}{384x^5a^4} \sqrt{cx^4 + bx^3 + ax^2} \left(48c^2a^{5/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) x^4 + 24c^2\sqrt{cx^2 + bx + ax^5}ab - 72ca^{3/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)`

[Out] $\frac{1}{384}(c^2x^4+b^2x^3+ax^2)^{1/2}(48c^2a^{5/2}\ln((2a+bx+2a^{1/2})(cx^2+bx+a)^{1/2})/x)x^4+24c^2(c^2x^2+b^2x+a)^{1/2}x^5ab-72ca^{3/2}\ln((2a+bx+2a^{1/2})(cx^2+bx+a)^{1/2})/x)x^4b^2-48c^2(c^2x^2+b^2x+a)^{1/2}x^4a^2-30c(c^2x^2+b^2x+a)^{1/2}x^5b^3-24c(c^2x^2+b^2x+a)^{3/2}x^3ab+84c(c^2x^2+b^2x+a)^{1/2}x^4ab^2+15a^{1/2}\ln((2a+bx+2a^{1/2})(cx^2+bx+a)^{1/2})/x)x^4b^4+48c(c^2x^2+b^2x+a)^{3/2}x^2a^2+30(c^2x^2+b^2x+a)^{3/2}x^3b^3-30(c^2x^2+b^2x+a)^{1/2}x^4b^4-60(c^2x^2+b^2x+a)^{3/2}x^2ab^2+80(c^2x^2+b^2x+a)^{3/2}xa^2b-96(c^2x^2+b^2x+a)^{3/2}a^3/x^5/(cx^2+bx+a)^{1/2}/a^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)`

Fricas [A] time = 2.12879, size = 757, normalized size = 3.69

$$\frac{3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{ax^5} \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4(8a^3bx + 48a^4 + (15ab^3 - 52a^2b^2c))\sqrt{ax^5}}{768a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")`

[Out] $\frac{1}{768}(3(5b^4 - 24ab^2c + 16a^2c^2)\sqrt{a}x^5\log(-(8a^3bx + 48a^4 + (15ab^3 - 52a^2b^2c))\sqrt{a})/x^3) - 4(8a^3bx + 48a^4 + (15ab^3 - 52a^2b^2c))\sqrt{a}x^5$

```
b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b*x + 4
8*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*
x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2(a + bx + cx^2)}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.38 $\int x(ax^2 + bx^3 + cx^4)^{3/2} dx$

Optimal. Leaf size=422

$$\frac{x^2(560a^2c^2 - 568ab^2c + 99b^4)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} - \frac{bx(2416a^2c^2 - 1560ab^2c + 231b^4)\sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} + \frac{(18896a^2b^2c^2}{$$

[Out] $((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(71680*c^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(32768*c^(13/2)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rubi [A] time = 1.20339, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1919, 1945, 1949, 12, 1914, 621, 206}

$$\frac{x^2(560a^2c^2 - 568ab^2c + 99b^4)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} - \frac{bx(2416a^2c^2 - 1560ab^2c + 231b^4)\sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} + \frac{(18896a^2b^2c^2}{$$

Antiderivative was successfully verified.

[In] $\text{Int}[x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x]$

[Out] $((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(71680*c^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(32768*c^(13/2)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1) + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x(ax^2 + bx^3 + cx^4)^{3/2} dx &= \frac{x(3b + 14cx)(ax^2 + bx^3 + cx^4)^{3/2}}{112c} + \frac{3 \int x^2 \left(-4ab - \frac{1}{2}(11b^2 - 28ac)x\right) \sqrt{ax^2 + bx^3 + cx^4}}{112c} \\
&= -\frac{x^3(b(11b^2 + 68ac) + 10c(11b^2 - 28ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{4480c^2} + \frac{x(3b + 14cx)(ax^2 + bx^3 + cx^4)^{3/2}}{112c} \\
&= \frac{(99b^4 - 568ab^2c + 560a^2c^2)x^2 \sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} - \frac{x^3(b(11b^2 + 68ac) + 10c(11b^2 - 28ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{4480c^2} \\
&= -\frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)x \sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} + \frac{(99b^4 - 568ab^2c + 560a^2c^2)x^2 \sqrt{ax^2 + bx^3 + cx^4}}{35840c^3} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(231b^4 - 1560ab^2c + 2416a^2c^2)x \sqrt{ax^2 + bx^3 + cx^4}}{71680c^4} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} \\
&= \frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 18896a^2b^2c^2 - 6720a^3c^3) \sqrt{ax^2 + bx^3 + cx^4}}{286720c^5}
\end{aligned}$$

Mathematica [A] time = 0.426287, size = 236, normalized size = 0.56

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{(16a^2c^2 - 72ab^2c + 33b^4) \left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}(4c(5a+2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{4096c^{11/2}x^3(a+x(b+cx))^{3/2}} \right) + (372abc - 288a^2c)}{8c}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(a + b*x + c*x^2 - (11*b*(a + x*(b + c*x)))/
(14*c*x) + ((-231*b^3 + 372*a*b*c + 330*b^2*c*x - 280*a*c^2*x)*(a + x*(b +
c*x)))/(560*c^3*x^3) + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(
b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))]/(40
96*c^(11/2)*x^3*(a + x*(b + c*x))^(3/2)))/(8*c)
```

Maple [A] time = 0.008, size = 649, normalized size = 1.5

$$\frac{1}{1146880x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(26880 \ln \left(\frac{1}{2} \frac{2\sqrt{cx^2 + bx + a}\sqrt{c} + 2cx + b}{\sqrt{c}} \right) a^4 c^5 + 3465 \ln \left(\frac{1}{2} \frac{2\sqrt{cx^2 + bx + a}\sqrt{c} + 2cx + b}{\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/1146880*(c*x^4+b*x^3+a*x^2)^(3/2)*(26880*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^4*c^5+3465*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^8*c+143360*x^3*(c*x^2+b*x+a)^(5/2)*c^(13/2)-59136*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b^3+18480*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^5-6930*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^7-112640*(c*x^2+b*x+a)^(5/2)*c^(11/2)*x^2*b-71680*(c*x^2+b*x+a)^(5/2)*c^(11/2)*x*a+84480*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b^2+95232*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a*b+17920*(c*x^2+b*x+a)^(3/2)*c^(11/2)*x*a^2-134400*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b^2*c^4+117600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^4*c^3-35280*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^6*c^2-40320*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^3+26880*(c*x^2+b*x+a)^(1/2)*c^(11/2)*x*a^3-13860*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^6+13440*(c*x^2+b*x+a)^(1/2)*c^(9/2)*a^3*b-63840*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^3+42840*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^5+36960*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^4+8960*(c*x^2+b*x+a)^(3/2)*c^(9/2)*a^2*b-80640*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b^2-127680*(c*x^2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b^2+85680*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b^4)/x^3/(c*x^2+b*x+a)^(3/2)/c^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)

Fricas [A] time = 2.03112, size = 1600, normalized size = 3.79

$$\left[\frac{105 (33 b^8 - 336 a b^6 c + 1120 a^2 b^4 c^2 - 1280 a^3 b^2 c^3 + 256 a^4 c^4) \sqrt{c x} \log \left(-\frac{8 c^2 x^3 + 8 b c x^2 + 4 \sqrt{c x^4 + b x^3 + a x^2} (2 c x + b) \sqrt{c + (b^2 + 4 a c) x}}{x} \right) + \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 87040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \left(x^2 (a + b x + c x^2) \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] $\text{Integral}(x*(x**2*(a + b*x + c*x**2))**(3/2), x)$

Giac [A] time = 1.29441, size = 703, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")`

[Out]
$$\begin{aligned} & 1/573440*\sqrt{c*x^2 + b*x + a}*(2*(4*(2*(8*(10*(4*(14*c*x*\text{sgn}(x) + 17*b*\text{sgn}(x)))*x + (b^2*c^6*\text{sgn}(x) + 84*a*c^7*\text{sgn}(x))/c^7)*x - (11*b^3*c^5*\text{sgn}(x) - 5 \\ & 2*a*b*c^6*\text{sgn}(x))/c^7)*x + (99*b^4*c^4*\text{sgn}(x) - 568*a*b^2*c^5*\text{sgn}(x) + 560* \\ & a^2*c^6*\text{sgn}(x))/c^7)*x - (231*b^5*c^3*\text{sgn}(x) - 1560*a*b^3*c^4*\text{sgn}(x) + 2416 \\ & *a^2*b*c^5*\text{sgn}(x))/c^7)*x + (1155*b^6*c^2*\text{sgn}(x) - 8988*a*b^4*c^3*\text{sgn}(x) + \\ & 18896*a^2*b^2*c^4*\text{sgn}(x) - 6720*a^3*c^5*\text{sgn}(x))/c^7)*x - (3465*b^7*c*\text{sgn}(x) \\ & - 30660*a*b^5*c^2*\text{sgn}(x) + 81648*a^2*b^3*c^3*\text{sgn}(x) - 58816*a^3*b*c^4*\text{sgn}(\\ & x))/c^7) - 3/32768*(33*b^8*\text{sgn}(x) - 336*a*b^6*c*\text{sgn}(x) + 1120*a^2*b^4*c^2*s \\ & \text{gn}(x) - 1280*a^3*b^2*c^3*\text{sgn}(x) + 256*a^4*c^4*\text{sgn}(x))*\log(\text{abs}(-2*(\sqrt{c})*x \\ & - \sqrt{c*x^2 + b*x + a})*\sqrt{c} - b))/c^{(13/2)} + 1/1146880*(3465*b^8*\log(\\ & \text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) - 35280*a*b^6*c*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c} \\ &)) + 117600*a^2*b^4*c^2*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) - 134400*a^3*b^2*c \\ & ^3*\log(\text{abs}(-b + 2*\sqrt{a})*\sqrt{c})) + 26880*a^4*c^4*\log(\text{abs}(-b + 2*\sqrt{a})* \\ & \sqrt{c})) + 6930*\sqrt{a}*b^7*\sqrt{c} - 61320*a^{(3/2)}*b^5*c^{(3/2)} + 163296*a \\ & ^{(5/2)}*b^3*c^{(5/2)} - 117632*a^{(7/2)}*b*c^{(7/2))*\text{sgn}(x)/c^{(13/2)} \end{aligned}$$

$$3.39 \quad \int (ax^2 + bx^3 + cx^4)^{3/2} dx$$

Optimal. Leaf size=364

$$\frac{b(1168a^2c^2 - 728ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(5488a^2b^2c^2 - 2048a^3c^3 - 2520ab^4c + 315b^6)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$

[Out] $-(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(17920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/7 - (3*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2048*c^{(11/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rubi [A] time = 1.03943, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1906, 1945, 1949, 12, 1914, 621, 206}

$$\frac{b(1168a^2c^2 - 728ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(5488a^2b^2c^2 - 2048a^3c^3 - 2520ab^4c + 315b^6)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $-(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(17920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/7 - (3*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2048*c^{(11/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rule 1906

```

Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] := Simp[(x*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(p*(2*n - q) + 1), x] + Dis
t[[(n - q)*p]/(p*(2*n - q) + 1), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]

```

Rule 1945

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[[(n - q)*p]/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1)]*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]

```

Rule 1949

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1)]*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1914

```

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

```

```
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (ax^2 + bx^3 + cx^4)^{3/2} dx &= \frac{1}{7}x(ax^2 + bx^3 + cx^4)^{3/2} + \frac{3}{14} \int x^2(2a + bx)\sqrt{ax^2 + bx^3 + cx^4} dx \\
&= \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x(ax^2 + bx^3 + cx^4)^{3/2} + \frac{\int \frac{x^4(-4a(b^2-6ac)-\frac{1}{2}b(9b^2-4ac))}{\sqrt{ax^2+bx^3+cx^4}} dx}{280c} \\
&= -\frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} + \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} + \frac{1}{7}x(ax^2 + bx^3 + cx^4)^{3/2} \\
&= \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} - \frac{b(9b^2 - 44ac)x^2\sqrt{ax^2 + bx^3 + cx^4}}{2240c^2} + \frac{x^3(b^2 + 24ac + 10bcx)\sqrt{ax^2 + bx^3 + cx^4}}{280c} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(7b^2 - 32ac)(3b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}}{4480c^3} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x} \\
&= -\frac{b(105b^4 - 728ab^2c + 1168a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{17920c^4} + \frac{(315b^6 - 2520ab^4c + 5488a^2b^2c^2 - 2048a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{35840c^5x}
\end{aligned}$$

Mathematica [A] time = 0.267093, size = 197, normalized size = 0.54

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{7(4abc-3b^3) \left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}(4c(5a+2cx^2)-3b^2+8bcx)+3(b^2-4ac)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2048c^{9/2}(a+x(b+cx))^{3/2}} + \frac{(-16ac+21b^2-30bcx)(a+bx+cx)}{40c^2} \right)}{7cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

```
[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*(x^2*(a + x*(b + c*x)) + ((21*b^2 - 16*a*c - 30*b*c*x)*(a + x*(b + c*x)))/(40*c^2) + (7*(-3*b^3 + 4*a*b*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))]/(2048*c^(9/2)*(a + x*(b + c*x))^(3/2)))/(7*c*x^3)
```

Maple [A] time = 0.009, size = 479, normalized size = 1.3

$$\frac{1}{71680x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(10240x^2 (cx^2 + bx + a)^{\frac{5}{2}} c^{11/2} - 7680 (cx^2 + bx + a)^{\frac{5}{2}} c^{9/2}xb - 4096 (cx^2 + bx + a)^{\frac{5}{2}} c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/71680*(c*x^4+b*x^3+a*x^2)^(3/2)*(10240*x^2*(c*x^2+b*x+a)^(5/2)*c^(11/2)-7680*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b-4096*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a+5376*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b^2+4480*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b-3360*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^3+2240*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^2-1680*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^4+6720*(c*x^2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b-6720*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b^3+1260*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^5+3360*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^2-3360*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^4+630*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^6+6720*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b*c^4-8400*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^3*c^3+2940*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^5*c^2-315*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^7*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 1.98469, size = 1299, normalized size = 3.57

$$\left[\frac{105(3b^7 - 28ab^5c + 80a^2b^3c^2 - 64a^3bc^3)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c + (b^2 + 4ac)x}}{x}\right) - 4(5120c^7x^6 + 6400b^6c^6x^5 + 315b^6c^5x^4 - 16(9b^3c^4 - 44ab^2c^3 - 2048a^3c^4 + 128(b^2c^5 + 64a^2c^6))x^3 + 8(21b^4c^3 - 124ab^2c^4 + 128a^2c^5)x^2 - 2(105b^5c^2 - 728ab^3c^3 + 1168a^2b^2c^4)x)\sqrt{c}}{c^6x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6*c^5*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6*c^5*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax^2 + bx^3 + cx^4)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)

Giac [A] time = 1.30208, size = 579, normalized size = 1.59

$$\frac{1}{35840} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2 \left(8 \left(10(4cx \operatorname{sgn}(x) + 5b \operatorname{sgn}(x))x + \frac{b^2 c^5 \operatorname{sgn}(x) + 64ac^6 \operatorname{sgn}(x)}{c^6} \right) x - \frac{9b^3 c^4 \operatorname{sgn}(x) - 44ab^2 c^5 \operatorname{sgn}(x)}{c^6} \right) x + \frac{21b^4 c^3 \operatorname{sgn}(x) - 124a^2 b^2 c^4 \operatorname{sgn}(x) + 128a^2 c^5 \operatorname{sgn}(x)}{c^6} \right) x - \frac{105b^5 c^2 \operatorname{sgn}(x) - 728a^2 b^3 c^3 \operatorname{sgn}(x) + 1168a^2 b^2 c^4 \operatorname{sgn}(x)}{c^6} \right) x + \frac{315b^6 c \operatorname{sgn}(x) - 2520a^2 b^4 c^2 \operatorname{sgn}(x) + 5488a^2 b^2 c^3 \operatorname{sgn}(x) - 2048a^3 c^4 \operatorname{sgn}(x)}{c^6} \right) + \frac{3}{2048} (3b^7 \operatorname{sgn}(x) - 28a^2 b^5 c \operatorname{sgn}(x) + 80a^2 b^3 c^2 \operatorname{sgn}(x) - 64a^3 b^2 c^3 \operatorname{sgn}(x)) \log(\operatorname{abs}(-2(\sqrt{c}x - \sqrt{cx^2 + bx + a})\sqrt{c} - b)) / c^{11/2} - \frac{1}{71680} (315b^7 \log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) - 2940a^2 b^5 c \log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) + 8400a^2 b^3 c^2 \log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) - 6720a^3 b^2 c^3 \log(\operatorname{abs}(-b + 2\sqrt{a}\sqrt{c})) + 630\sqrt{a}b^6 \sqrt{c} - 5040a^{3/2}b^4 c^{3/2} + 10976a^{5/2}b^2 c^{5/2} - 4096a^{7/2}c^{7/2}) \operatorname{sgn}(x) / c^{11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*x + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b*c^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*c^5*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2*b*c^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a^2*b^2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*a*b^5*c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b^2*c^3*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2) - 1/71680*(315*b^7*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 8400*a^2*b^3*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 6720*a^3*b^2*c^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(11/2)

$$3.40 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x} dx$$

Optimal. Leaf size=288

$$\frac{(240a^2c^2 - 216ab^2c + 35b^4) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(1296a^2c^2 - 760ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} - \frac{x(6cx(7b^2 - 20ac^2) - (3b^2 - 4ac^2) \sqrt{ax^2 + bx^3 + cx^4})}{60c^2x}$$

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.519235, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1919, 1934, 1949, 12, 1914, 621, 206}

$$\frac{(240a^2c^2 - 216ab^2c + 35b^4) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(1296a^2c^2 - 760ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} - \frac{x(6cx(7b^2 - 20ac^2) - (3b^2 - 4ac^2) \sqrt{ax^2 + bx^3 + cx^4})}{60c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
```

```

*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]

```

Rule 1934

```

Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^p]*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*
(2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
- q))^p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((
n - q)*p)/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] &&
NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]

```

Rule 1949

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^p_
)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1914

```

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]

```

```
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x} dx &= \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx} + \frac{\int (-2ab + \frac{1}{2}(-7b^2 + 20ac)x) \sqrt{ax^2 + bx^3 + cx^4} dx}{20c} \\
&= -\frac{x(b(7b^2 + 12ac) + 6c(7b^2 - 20ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{960c^2} + \frac{(3b + 10cx)(ax^2 + bx^3 + cx^4)^{3/2}}{60cx} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{x(b(7b^2 + 12ac) + 6c(7b^2 - 20ac)x) \sqrt{ax^2 + bx^3 + cx^4}}{960c^2} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x} \\
&= \frac{(35b^4 - 216ab^2c + 240a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{3840c^3} - \frac{b(105b^4 - 760ab^2c + 1296a^2c^2) \sqrt{ax^2 + bx^3 + cx^4}}{7680c^4x}
\end{aligned}$$

Mathematica [A] time = 0.242008, size = 180, normalized size = 0.62

$$\frac{(x^2(a + x(b + cx)))^{3/2} \left(\frac{(7b^2 - 4ac) \left(2\sqrt{c(b+2cx)} \sqrt{a+x(b+cx)} (4c(5a+2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right)}{512c^{7/2}(a+x(b+cx))^{3/2}} + x(a + x(b + cx)) \right)}{6cx^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]

[Out] ((x^2*(a + x*(b + c*x)))^(3/2)*((-7*b*(a + x*(b + c*x)))/(10*c) + x*(a + x*(b + c*x)) + ((7*b^2 - 4*a*c)*(2*sqrt[c]*(b + 2*c*x)*sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])))/(512*c^(7/2)*(a + x*(b + c*x))^(3/2)))/(6*c*x^3)

Maple [A] time = 0.009, size = 431, normalized size = 1.5

$$\frac{1}{15360x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(2560x (cx^2 + bx + a)^{5/2} c^{9/2} - 1792c^{7/2} (cx^2 + bx + a)^{5/2} b - 640c^{9/2} (cx^2 + bx + a)^{3/2} xa + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)

[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(9/2)-1792*c^(7/2)*(c*x^2+b*x+a)^(5/2)*b-640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*x*a+1120*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x*b^2-320*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b+560*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^3-960*c^(9/2)*(c*x^2+b*x+a)^(1/2)*x*a^2+1920*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a*b^2-420*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^4-480*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b+960*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^3-210*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^5-960*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^4*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^6*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(11/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)

Fricas [A] time = 1.83513, size = 1099, normalized size = 3.82

$$\left[\frac{15(7b^6 - 60ab^4c + 144a^2b^2c^2 - 64a^3c^3)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right) - 4(1280c^6x^5 + 1664 \dots}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*\sqrt{c})*x \\ & * \log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{c} \\ & + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c \\ & + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 \\ & - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*\sqrt{c*x^4 + b*x^3 + a*x^2} \\ & / (c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*\sqrt{-c})*x* \\ & \arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{-c} / (c^2*x^3 + b*c*x^2 + a*c*x) \\ & - 2*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + \\ & 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x) \\ & *\sqrt{c*x^4 + b*x^3 + a*x^2} / (c^5*x)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)

[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)

Giac [A] time = 1.32493, size = 493, normalized size = 1.71

$$\frac{1}{7680} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2 \left(8(10cx\operatorname{sgn}(x) + 13b\operatorname{sgn}(x))x + \frac{3b^2c^4\operatorname{sgn}(x) + 140ac^5\operatorname{sgn}(x)}{c^5} \right) x - \frac{7b^3c^3\operatorname{sgn}(x) - 36ab}{c^5} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/7680*\sqrt{c*x^2 + b*x + a}*(2*(4*(2*(8*(10*c*x*\operatorname{sgn}(x) + 13*b*\operatorname{sgn}(x)))*x + \\ & (3*b^2*c^4*\operatorname{sgn}(x) + 140*a*c^5*\operatorname{sgn}(x))/c^5)*x - (7*b^3*c^3*\operatorname{sgn}(x) - 36*a*b*c \end{aligned}$$

$$\begin{aligned}
& ^4*\text{sgn}(x))/c^5)*x + (35*b^4*c^2*\text{sgn}(x) - 216*a*b^2*c^3*\text{sgn}(x) + 240*a^2*c^4 \\
& *\text{sgn}(x))/c^5)*x - (105*b^5*c*\text{sgn}(x) - 760*a*b^3*c^2*\text{sgn}(x) + 1296*a^2*b*c^3 \\
& *\text{sgn}(x))/c^5) - 1/1024*(7*b^6*\text{sgn}(x) - 60*a*b^4*c*\text{sgn}(x) + 144*a^2*b^2*c^2* \\
& \text{sgn}(x) - 64*a^3*c^3*\text{sgn}(x))*\log(\text{abs}(-2*(\text{sqrt}(c))*x - \text{sqrt}(c*x^2 + b*x + a))* \\
& \text{sqrt}(c) - b))/c^{(9/2)} + 1/15360*(105*b^6*\log(\text{abs}(-b + 2*\text{sqrt}(a))*\text{sqrt}(c))) - \\
& 900*a*b^4*c*\log(\text{abs}(-b + 2*\text{sqrt}(a))*\text{sqrt}(c))) + 2160*a^2*b^2*c^2*\log(\text{abs}(-b \\
& + 2*\text{sqrt}(a))*\text{sqrt}(c))) - 960*a^3*c^3*\log(\text{abs}(-b + 2*\text{sqrt}(a))*\text{sqrt}(c))) + 210 \\
& *\text{sqrt}(a)*b^5*\text{sqrt}(c) - 1520*a^{(3/2)}*b^3*c^{(3/2)} + 2592*a^{(5/2)}*b*c^{(5/2))*s \\
& \text{gn}(x)/c^{(9/2)}
\end{aligned}$$

$$3.41 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$$

Optimal. Leaf size=198

$$\frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{3bx(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}$$

[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.178102, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1917, 1918, 1914, 621, 206}

$$\frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{3bx(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]

[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1917

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1)))^(p + 1)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && EqQ[m + p*(n - 1) - 1, 0]

Rule 1918

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2
*c*(2*p + 1)), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^2} dx &= \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} - \frac{b \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3} dx}{2c} \\
&= -\frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} + \frac{(3b(b^2 - 4ac)) \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx}{32c^2} \\
&= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
&= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
&= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5} \\
&= \frac{3b(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{128c^3x} - \frac{b(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2 + bx^3 + cx^4)^{5/2}}{5cx^5}
\end{aligned}$$

Mathematica [A] time = 0.182453, size = 163, normalized size = 0.82

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(4b^2c(2cx^2-25a)+8bc^2x(7a+22cx^2)+128c^2(a+cx^2)^2-10b^3cx+15b^4\right)-15\sqrt{c}\sqrt{a+x(b+cx)}\right)}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a + 22*c*x^2)) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 289, normalized size = 1.5

$$\frac{1}{1280x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(256 (cx^2 + bx + a)^{5/2} c^{7/2} - 160 c^{7/2} (cx^2 + bx + a)^{3/2} xb - 80 c^{5/2} (cx^2 + bx + a)^{3/2} b^2 - 240 \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)`

[Out] $\frac{1}{1280}(c^2x^4+bx^3+ax^2)^{3/2} \left(256(c^2x^2+bx+a)^{5/2}c^{7/2} - 160c^{7/2}(c^2x^2+bx+a)^{3/2}xb - 80c^{5/2}(c^2x^2+bx+a)^{3/2}b^2 - 240c^{7/2}(c^2x^2+bx+a)^{1/2}x^2ab + 60c^{5/2}(c^2x^2+bx+a)^{1/2}x^2b^3 - 120c^{5/2}(c^2x^2+bx+a)^{1/2}a^2b^2 + 30c^{3/2}(c^2x^2+bx+a)^{1/2}b^4 - 240\ln\left(\frac{1}{2}(2(c^2x^2+bx+a)^{1/2}c^{1/2}+2cx+b)/c^{1/2}\right)a^2b^3c^3 + 120\ln\left(\frac{1}{2}(2(c^2x^2+bx+a)^{1/2}c^{1/2}+2cx+b)/c^{1/2}\right)a^2b^3c^2 - 15\ln\left(\frac{1}{2}(2(c^2x^2+bx+a)^{1/2}c^{1/2}+2cx+b)/c^{1/2}\right)b^5c \right) / x^3 / (c^2x^2+bx+a)^{3/2} / c^{9/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)`

Fricas [A] time = 1.69484, size = 878, normalized size = 4.43

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right) + 4(128c^5x^4 + 176bc^4x^3 + 15b^4c - 100a^2b^2c^2 + 128a^2c^3 + 8(b^2c^3 + 32a^2c^4)x^2 - 2(5b^3c^2 - 28a^2b^2c^3)x)\sqrt{c^2x^4 + bx^3 + ax^2}}{2560c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")`

[Out] $\frac{1}{2560}(15(b^5 - 8a^2b^3c + 16a^2b^2c^2)\sqrt{c}x \log(-8c^2x^3 + 8b^2c^2x^2 - 4\sqrt{c^2x^4 + bx^3 + ax^2}(2cx + b)\sqrt{c} + (b^2 + 4a^2c)x^2)/x) + 4(128c^5x^4 + 176b^2c^4x^3 + 15b^4c - 100a^2b^2c^2 + 128a^2c^3 + 8(b^2c^3 + 32a^2c^4)x^2 - 2(5b^3c^2 - 28a^2b^2c^3)x)\sqrt{c^2x^4 + bx^3 + ax^2}) / (c^4x), \frac{1}{1280}(15(b^5 - 8a^2b^3c + 16a^2b^2c^2)\sqrt{c}x \arctan(1/2\sqrt{c^2x^4 + bx^3 + ax^2}(2cx + b)\sqrt{-c}) / (c^2x^3 + b^2c^2x^2 + a^2c^2x) + 2(128c^5x^4 + 176b^2c^4x^3 + 15b^4c - 100a^2b^2c^2 + 128a^2c^3 + 8(b^2c^3 + 32a^2c^4)x^2 - 2(5b^3c^2 - 28a^2b^2c^3)x)\sqrt{-c}) / (c^2x^3 + b^2c^2x^2 + a^2c^2x)$

$*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*\text{sqrt}(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)

[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)

Giac [A] time = 1.27319, size = 383, normalized size = 1.93

$$\frac{1}{640} \sqrt{cx^2 + bx + a} \left(2 \left(4 \left(2(8cx\text{sgn}(x) + 11b\text{sgn}(x))x + \frac{b^2c^3\text{sgn}(x) + 32ac^4\text{sgn}(x)}{c^4} \right) x - \frac{5b^3c^2\text{sgn}(x) - 28abc^3\text{sgn}(x)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")

[Out] $\frac{1}{640} \sqrt{c*x^2 + b*x + a} * (2 * (4 * (2 * (8*c*x*\text{sgn}(x) + 11*b*\text{sgn}(x)) * x + (b^2*c^3*\text{sgn}(x) + 32*a*c^4*\text{sgn}(x))/c^4) * x - (5*b^3*c^2*\text{sgn}(x) - 28*a*b*c^3*\text{sgn}(x))/c^4) * x + (15*b^4*c*\text{sgn}(x) - 100*a*b^2*c^2*\text{sgn}(x) + 128*a^2*c^3*\text{sgn}(x))/c^4 + 3/256 * (b^5*\text{sgn}(x) - 8*a*b^3*c*\text{sgn}(x) + 16*a^2*b*c^2*\text{sgn}(x)) * \log(\text{abs}(-2*(\text{sqrt}(c)*x - \text{sqrt}(c*x^2 + b*x + a))*\text{sqrt}(c) - b))/c^{7/2} - 1/1280 * (15*b^5 * \log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) - 120*a*b^3*c * \log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) + 240*a^2*b*c^2 * \log(\text{abs}(-b + 2*\text{sqrt}(a)*\text{sqrt}(c))) + 30*\text{sqrt}(a)*b^4*\text{sqrt}(c) - 200*a^{3/2}*b^2*c^{3/2} + 256*a^{5/2}*c^{5/2})*\text{sgn}(x)/c^{7/2}$

$$3.42 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3} dx$$

Optimal. Leaf size=165

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

[Out] $(-3*(b^2 - 4*a*c)*(b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(128*c^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rubi [A] time = 0.127758, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1918, 1914, 621, 206}

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3 + c*x^4)^{(3/2)}/x^3, x]$

[Out] $(-3*(b^2 - 4*a*c)*(b + 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(128*c^{(5/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rule 1918

$\text{Int}[(x_)^{(m_.)}*((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m-n+q+1)}*(b+2*c*x^{(n-q)})*(a*x^q + b*x^n + c*x^{(2*n-q)})^p)/(2*c*(n-q)*(2*p+1)), x] - \text{Dist}[(p*(b^2-4*a*c))/(2*c*(2*p+1)), \text{Int}[x^{(m+q)}*(a*x^q + b*x^n + c*x^{(2*n-q)})^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^3} dx &= \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} - \frac{(3(b^2 - 4ac)) \int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x} dx}{16c} \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{(3(b^2 - 4ac))}{16c} \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{(3(b^2 - 4ac))}{16c} \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{(3(b^2 - 4ac))}{16c} \\ &= -\frac{3(b^2 - 4ac)(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{64c^2x} + \frac{(b + 2cx)(ax^2 + bx^3 + cx^4)^{3/2}}{8cx^3} + \frac{3(b^2 - 4ac)^2}{16c} \end{aligned}$$

Mathematica [A] time = 0.0644026, size = 132, normalized size = 0.8

$$\frac{x\sqrt{a + x(b + cx)} \left(2\sqrt{c}(b + 2cx)\sqrt{a + x(b + cx)} (4c(5a + 2cx^2) - 3b^2 + 8bcx) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}} \right) \right)}{128c^{5/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(128*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 265, normalized size = 1.6

$$\frac{1}{128x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(32x (cx^2 + bx + a)^{\frac{3}{2}} c^{\frac{7}{2}} + 16c^{\frac{5}{2}} (cx^2 + bx + a)^{\frac{3}{2}} b + 48c^{\frac{7}{2}} \sqrt{cx^2 + bx + axa} - 12c^{\frac{5}{2}} \sqrt{cx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)

[Out] 1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)+16*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b+48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x*a-12*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x*b^2+24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^3+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*c^3-24*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(7/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)

Fricas [A] time = 1.64564, size = 725, normalized size = 4.39

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)\sqrt{c} + (b^2 + 4ac)x}{x}\right) + 4(16c^4x^3 + 24bc^3x^2 - 3b^3c + 20abc)}{256c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")

[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)

[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)

Giac [A] time = 1.28001, size = 313, normalized size = 1.9

$$\frac{1}{64} \sqrt{cx^2 + bx + a} \left(2 \left(4(2cx \operatorname{sgn}(x) + 3b \operatorname{sgn}(x))x + \frac{b^2c^2 \operatorname{sgn}(x) + 20ac^3 \operatorname{sgn}(x)}{c^3} \right) x - \frac{3b^3c \operatorname{sgn}(x) - 20abc^2 \operatorname{sgn}(x)}{c^3} \right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")

[Out] $\frac{1}{64}\sqrt{c x^2 + b x + a} (2 (4 (2 c x \operatorname{sgn}(x) + 3 b \operatorname{sgn}(x)) x + (b^2 c^2 \operatorname{sgn}(x) + 20 a c^3 \operatorname{sgn}(x)) / c^3) x - (3 b^3 c \operatorname{sgn}(x) - 20 a b c^2 \operatorname{sgn}(x)) / c^3 - 3 / 128 (b^4 \operatorname{sgn}(x) - 8 a b^2 c \operatorname{sgn}(x) + 16 a^2 c^2 \operatorname{sgn}(x)) \log(\operatorname{abs}(-2 (\operatorname{sqrt}(c) x - \operatorname{sqrt}(c x^2 + b x + a)) \operatorname{sqrt}(c) - b)) / c^{5/2} + 1 / 128 (3 b^4 \log(\operatorname{abs}(-b + 2 \operatorname{sqrt}(a) \operatorname{sqrt}(c))) - 24 a b^2 c \log(\operatorname{abs}(-b + 2 \operatorname{sqrt}(a) \operatorname{sqrt}(c))) + 48 a^2 c^2 \log(\operatorname{abs}(-b + 2 \operatorname{sqrt}(a) \operatorname{sqrt}(c))) + 6 \operatorname{sqrt}(a) b^3 \operatorname{sqrt}(c) - 40 a^{3/2} b c^{3/2}) \operatorname{sgn}(x) / c^{5/2}$

$$3.43 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} dx$$

Optimal. Leaf size=227

$$\frac{a^{3/2}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(8ac+b^2+2bcx)}{8}$$

[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.255295, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1921, 1945, 1933, 843, 621, 206, 724}

$$\frac{a^{3/2}x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(b^2-12ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(8ac+b^2+2bcx)}{8}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]

[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1921

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^p]/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q

+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1)]*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
&& EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_.))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^(2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} dx &= \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{1}{2} \int \frac{(2a + bx)\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx \\
 &= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{\int \frac{8a^2c - \frac{1}{2}b(b^2 - 12ac)x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\
 &= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{(x\sqrt{a + bx + cx^2}) \int \frac{8a^2c - \frac{1}{2}b}{x\sqrt{a + bx + cx^2}} dx}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} + \frac{(a^2x\sqrt{a + bx + cx^2}) \int \frac{8a^2c - \frac{1}{2}b}{x\sqrt{a + bx + cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} - \frac{(2a^2x\sqrt{a + bx + cx^2}) \operatorname{Subst}\left[\int \frac{8a^2c - \frac{1}{2}b}{x\sqrt{a + bx + cx^2}} dx\right]}{\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{(b^2 + 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8cx} + \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + bx + cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}}
 \end{aligned}$$

Mathematica [A] time = 0.23753, size = 166, normalized size = 0.73

$$\frac{x\sqrt{a + x(b + cx)} \left(-48a^{3/2}c^{3/2} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right) + 2\sqrt{c}\sqrt{a + x(b + cx)} (8c(4a + cx^2) + 3b^2 + 14bcx) - 3b(b^2 - 12ac) \right)}{48c^{3/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x]

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*c*x + 8*c*(4*a + c*x^2)) - 48*a^(3/2)*c^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S

$\text{qrt}[a + x*(b + c*x)] - 3*b*(b^2 - 12*a*c)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)])]/(48*c^{3/2}*\text{Sqrt}[x^2*(a + x*(b + c*x))])$

Maple [A] time = 0.005, size = 222, normalized size = 1.

$$-\frac{1}{48x^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(48c^{5/2}a^{3/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) - 16(cx^2 + bx + a)^{3/2}c^{5/2} - 12c^{5/2}\sqrt{cx^2 + bx + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)

[Out] $-1/48*(c*x^4+b*x^3+a*x^2)^{(3/2)}*(48*c^{(5/2)}*a^{(3/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)-16*(c*x^2+b*x+a)^{(3/2)}*c^{(5/2)}-12*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x*b-48*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*a-6*c^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}*b^2-36*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*a*b*c^2+3*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*b^3*c)/x^3/(c*x^2+b*x+a)^{(3/2)}/c^{(5/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)

Fricas [A] time = 2.56221, size = 1825, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{3/2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**3/2/x**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.44 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5} dx$$

Optimal. Leaf size=219

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{3\sqrt{abx}\sqrt{a+bx+cx^2}}{4x}$$

[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.242014, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1920, 1945, 1933, 843, 621, 206, 724}

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} + \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{3\sqrt{abx}\sqrt{a+bx+cx^2}}{4x}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]

[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1920

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +

1] && NeQ[m + p*q + 1, 0]

Rule 1945

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p]/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1933

Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{3}{2} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^2} dx \\
 &= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{3 \int \frac{4abc + c(b^2 + 4ac)x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c} \\
 &= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{(3x\sqrt{a + bx + cx^2}) \int \frac{4abc + c(b^2 + 4ac)}{x\sqrt{a + bx + cx^2}}}{8c\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} + \frac{(3abx\sqrt{a + bx + cx^2}) \int \frac{1}{x\sqrt{a + bx + cx^2}}}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} - \frac{(3abx\sqrt{a + bx + cx^2}) \text{Subst}\left(\int \frac{1}{4a-x}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= \frac{3(3b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} - \frac{3\sqrt{abx}\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{\dots}}\right)}{2\sqrt{ax^2 + bx^3 + cx^4}}
 \end{aligned}$$

Mathematica [A] time = 0.184463, size = 158, normalized size = 0.72

$$\frac{\sqrt{a + x(b + cx)} \left(3x(4ac + b^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) + 2\sqrt{c}\sqrt{a + x(b + cx)}(x(5b + 2cx) - 4a) - 12\sqrt{ab}\sqrt{cx} \tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{\dots}}\right) \right)}{8\sqrt{c}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]

[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*c*x)) - 12*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b

+ c*x)))] + 3*(b^2 + 4*a*c)*x*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])]/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 254, normalized size = 1.2

$$\frac{1}{8ax^4} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(8c^{5/2} (cx^2 + bx + a)^{3/2} x^2 + 12c^{5/2} \sqrt{cx^2 + bx + ax^2} a - 12c^{3/2} a^{3/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + ax^2}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)

[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(8*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^2+12*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^2*a-12*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x*b-8*(c*x^2+b*x+a)^(5/2)*c^(3/2)+8*c^(3/2)*(c*x^2+b*x+a)^(3/2)*x*b+18*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x*a*b+12*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a^2*c^2+3*c*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a*b^2)/x^4/(c*x^2+b*x+a)^(3/2)/a/c^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)

Fricas [A] time = 2.21845, size = 1735, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.45 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6} dx$$

Optimal. Leaf size=219

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} + \frac{3b\sqrt{cx}\sqrt{a+bx+cx^2}}{4x^2}$$

[Out] $(-3*(b - 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x^4)^{(3/2)}/(2*x^5) - (3*(b^2 + 4*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (3*b*\text{Sqrt}[c]*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rubi [A] time = 0.237785, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1920, 1941, 1933, 843, 621, 206, 724}

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} + \frac{3b\sqrt{cx}\sqrt{a+bx+cx^2}}{4x^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x^2 + b*x^3 + c*x^4)^{(3/2)}/x^6, x]$

[Out] $(-3*(b - 2*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x^4)^{(3/2)}/(2*x^5) - (3*(b^2 + 4*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (3*b*\text{Sqrt}[c]*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])$

Rule 1920

$\text{Int}[(x_)^{(m_.)}*((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(p_.)}, x_Symbol] :> \text{Simp}[(x^{(m+1)}*(a*x^q + b*x^n + c*x^{(2*n-q)})^p)/(m+p*q+1), x] - \text{Dist}[(n-q)*p/(m+p*q+1), \text{Int}[x^{(m+n)}*(b+2*c*x^{(n-q)})*(a*x^q + b*x^n + c*x^{(2*n-q)})^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +

1] && NeQ[m + p*q + 1, 0]

Rule 1941

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p]/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Simp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m + p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1933

Int[((A_) + (B_)*(x_)^(j_))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3] && EqQ[q, 2]

Rule 843

Int[((d_) + (e_)*(x_)^(m_))*((f_) + (g_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{3}{4} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
 &= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{3}{8} \int \frac{-b^2 - 4ac - 4bcx}{\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{(3x\sqrt{a + bx + cx^2}) \int \frac{-b^2 - 4ac - 4bcx}{x\sqrt{a + bx + cx^2}} dx}{8\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{(3bcx\sqrt{a + bx + cx^2}) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{2\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} + \frac{(3bcx\sqrt{a + bx + cx^2}) \text{Subst}\left(\int \frac{1}{4c\sqrt{a + bx + cx^2}} dx\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\
 &= -\frac{3(b - 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{4x^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{2x^5} - \frac{3(b^2 + 4ac)x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + bx + cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}
 \end{aligned}$$

Mathematica [A] time = 0.199347, size = 162, normalized size = 0.74

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(3x^2 (4ac + b^2) \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) + 2\sqrt{a} \left((2a + x(5b - 4cx))\sqrt{a + x(b + cx)} - 6b\sqrt{cx^2} \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) \right) \right)}{8\sqrt{ax^3}\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x]

[Out] -(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x*(b + c*x)] - 6*b*Sqrt[c]*x^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(8*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])

Maple [A] time = 0.006, size = 338, normalized size = 1.5

$$-\frac{1}{8a^2x^5} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(12c^{5/2}a^{5/2} \ln\left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x}\right) x^2 - 2c^{5/2} (cx^2 + bx + a)^{3/2} x^3 b - 4c^{5/2} (cx^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)`

[Out] $-1/8*(c*x^4+b*x^3+a*x^2)^{(3/2)}*(12*c^{(5/2)}*a^{(5/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^2-2*c^{(5/2)}*(c*x^2+b*x+a)^{(3/2)}*x^3*b-4*c^{(5/2)}*(c*x^2+b*x+a)^{(3/2)}*x^2*a-6*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x^3*a*b+3*c^{(3/2)}*a^{(3/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^2*b^2-12*c^{(5/2)}*(c*x^2+b*x+a)^{(1/2)}*x^2*a^2+2*c^{(3/2)}*(c*x^2+b*x+a)^{(5/2)}*x*b-2*c^{(3/2)}*(c*x^2+b*x+a)^{(3/2)}*x^2*b^2+4*(c*x^2+b*x+a)^{(5/2)}*a*c^{(3/2)}-6*c^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}*x^2*a*b^2-12*c^2*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})*x^2*a^2*b)/x^5/(c*x^2+b*x+a)^{(3/2)}/a^2/c^{(3/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)`

Fricas [A] time = 2.24241, size = 1737, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")`


```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{3/2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.46 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7} dx$$

Optimal. Leaf size=257

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

[Out] $((b^2 - 8*a*c + 2*b*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 + b*x^3 + c*x^4)^{(3/2)}/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/(4*a*x^5) + (b*(b^2 - 12*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(16*a^{(3/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (c^{(3/2)})*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]$

Rubi [A] time = 0.350564, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1920, 1951, 1941, 1933, 843, 621, 206, 724}

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}} + \frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} + \frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x]

[Out] $((b^2 - 8*a*c + 2*b*c*x)*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 + b*x^3 + c*x^4)^{(3/2)}/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^{(3/2)})/(4*a*x^5) + (b*(b^2 - 12*a*c)*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2])])/(16*a^{(3/2)}*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]) + (c^{(3/2)})*x*\text{Sqrt}[a + b*x + c*x^2]*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]$

Rule 1920

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*q + 1), x] - Dist[((n - q)*p)/(m + p*q + 1), Int[x^(m + n)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &

```
& EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
  IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
  1] && NeQ[m + p*q + 1, 0]
```

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
  *((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
  *x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
  1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
  + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
  x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
  ] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
  && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)
  ]*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
  *((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n
  - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n
  - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n
  - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Si
  mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
  p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
  b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
  r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
  Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
  + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1933

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
  _.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(
  n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(A + B*x^(n - q))/(x^(q/
  2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
  , n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
  && EqQ[q, 2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
  _.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
  c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
  x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

$\text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ !\text{IGtQ}[m, 0]$

Rule 621

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] \ :> \ \text{Dist}[2, \text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \ /; \ \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 724

$\text{Int}[1/(((d_) + (e_)*(x_))*\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2]), x_Symbol] \ :> \ \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] \ /; \ \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} + \frac{1}{2} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^4} dx \\
&= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} - \int \frac{\left(\frac{1}{2}(b^2 - 8ac) - bcx\right)\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} + \int \frac{bcx\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} + \int \frac{bcx\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} + \int \frac{bcx\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} + \int \frac{bcx\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx \\
&= \frac{(b^2 - 8ac + 2bcx)\sqrt{ax^2 + bx^3 + cx^4}}{8ax^2} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{3x^6} - \frac{b(ax^2 + bx^3 + cx^4)^{3/2}}{4ax^5} + \int \frac{bcx\sqrt{ax^2 + bx^3 + cx^4}}{x^3} dx
\end{aligned}$$

Mathematica [A] time = 0.306707, size = 175, normalized size = 0.68

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(3bx^3 (b^2 - 12ac) \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) - 2\sqrt{a} \left(\sqrt{a + x(b + cx)} (8a^2 + 2ax(7b + 16cx) + 3b^2x^2) - 2bx^3 \right) \right)}{48a^{3/2}x^4\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^2 + 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) - 24*a*c^(3/2)*x^3*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(48*a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])

Maple [A] time = 0.007, size = 435, normalized size = 1.7

$$\frac{1}{48 x^6 a^3} (cx^4 + bx^3 + ax^2)^{\frac{3}{2}} \left(32 c^{7/2} (cx^2 + bx + a)^{3/2} x^4 a - 36 c^{5/2} a^{5/2} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) x^3 b + 48 c^{7/2} \sqrt{cx^2 + bx + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)

[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*c^(7/2)*(c*x^2+b*x+a)^(3/2)*x^4*a-36*c^(5/2)*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*b+48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*x^4*a^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^4*b^2-32*c^(5/2)*(c*x^2+b*x+a)^(5/2)*x^2*a+28*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^3*a*b-6*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^2+3*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*b^3+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b+2*c^(3/2)*(c*x^2+b*x+a)^(5/2)*x^2*b^2-2*c^(3/2)*(c*x^2+b*x+a)^(3/2)*x^3*b^3+4*c^(3/2)*(c*x^2+b*x+a)^(5/2)*x*a*b-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*x^3*a*b^3-16*(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2)+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x^3*a^3*c^3)/x^6/(c*x^2+b*x+a)^(3/2)/a^3/c^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)

Fricas [A] time = 2.49978, size = 1871, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/48*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**3/2/x**7, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.47 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8} dx$$

Optimal. Leaf size=197

$$\frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}} - \frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7}$$

[Out] $-\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{(32ax^3)} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{(64a^2x^2)} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{(8x^4)} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{(4x^7)} - \frac{3(b^2 - 4ac)^2 \operatorname{ArcTanh}\left[\frac{x(2a + bx)}{(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4})}\right]}{(128a^{5/2})}$

Rubi [A] time = 0.362848, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$\frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{3(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}} - \frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(ax^2 + bx^3 + cx^4)^{3/2}/x^8, x]$

[Out] $-\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{(32ax^3)} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{(64a^2x^2)} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{(8x^4)} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{(4x^7)} - \frac{3(b^2 - 4ac)^2 \operatorname{ArcTanh}\left[\frac{x(2a + bx)}{(2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4})}\right]}{(128a^{5/2})}$

Rule 1920

$\operatorname{Int}[(x_)^{(m_.)} * ((b_.) * (x_)^{(n_.)} + (a_.) * (x_)^{(q_.)} + (c_.) * (x_)^{(r_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)} * (a*x^q + b*x^n + c*x^{(2*n-q)})^p) / (m + p*q + 1), x] - \operatorname{Dist}[(n - q) * p / (m + p*q + 1), \operatorname{Int}[x^{(m+n)} * (b + 2*c*x^{(n-q)}) * (a*x^q + b*x^n + c*x^{(2*n-q)})^{(p-1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, x\} \& \& \operatorname{EqQ}[r, 2*n - q] \& \& \operatorname{PosQ}[n - q] \& \& \operatorname{IntegerQ}[p] \& \& \operatorname{NeQ}[b^2 - 4*a*c, 0] \& \& \operatorname{IGtQ}[n, 0] \& \& \operatorname{GtQ}[p, 0] \& \& \operatorname{RationalQ}[m, q] \& \& \operatorname{LeQ}[m + p*q + 1, -(n - q) +$

1] && NeQ[m + p*q + 1, 0]

Rule 1941

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(x^(m + 1)*(A*(m + p*q + (n - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Simp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m + p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1951

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} + \frac{3}{8} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^5} dx \\
 &= -\frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} + \frac{1}{16} \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{4x^7} - \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{32ax^3} + \frac{b(3b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{64a^2x^2} - \frac{(b + 6cx)\sqrt{ax^2 + bx^3 + cx^4}}{8x^4} - \int \frac{b^2 - 12ac - 4bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx
 \end{aligned}$$

Mathematica [A] time = 0.116674, size = 141, normalized size = 0.72

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(2\sqrt{a}(2a + bx)\sqrt{a + x(b + cx)} (8a^2 + 4ax(2b + 5cx) - 3b^2x^2) + 3x^4 (b^2 - 4ac)^2 \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) \right)}{128a^{5/2}x^5\sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]

[Out] -(Sqrt[x^2*(a + x*(b + c*x))]*(2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x)) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))]/(128*a^(5/2)*x^5*Sqrt[a + x*(b + c*x)])

Maple [B] time = 0.009, size = 501, normalized size = 2.5

$$-\frac{1}{128x^7a^4}(cx^4+bx^3+ax^2)^{\frac{3}{2}}\left(48c^2a^{7/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^4+24c^2(cx^2+bx+a)^{3/2}x^5ab-24ca^{5/2}\ln\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)

[Out]
$$-1/128*(c*x^4+b*x^3+a*x^2)^{(3/2)}*(48*c^2*a^{(7/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^4+24*c^2*(c*x^2+b*x+a)^{(3/2)}*x^5*a*b-24*c*a^{(5/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^4*b^2-16*c^2*(c*x^2+b*x+a)^{(3/2)}*x^4*a^2+24*c^2*(c*x^2+b*x+a)^{(1/2)}*x^5*a^2*b-2*c*(c*x^2+b*x+a)^{(3/2)}*x^5*b^3-48*c^2*(c*x^2+b*x+a)^{(1/2)}*x^4*a^3-24*c*(c*x^2+b*x+a)^{(5/2)}*x^3*a*b+20*c*(c*x^2+b*x+a)^{(3/2)}*x^4*a*b^2-6*c*(c*x^2+b*x+a)^{(1/2)}*x^5*a*b^3+3*a^{(3/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^4*b^4+16*c*(c*x^2+b*x+a)^{(5/2)}*x^2*a^2+36*c*(c*x^2+b*x+a)^{(1/2)}*x^4*a^2*b^2+2*(c*x^2+b*x+a)^{(5/2)}*x^3*b^3-2*(c*x^2+b*x+a)^{(3/2)}*x^4*b^4+4*(c*x^2+b*x+a)^{(5/2)}*x^2*a*b^2-6*(c*x^2+b*x+a)^{(1/2)}*x^4*a*b^4-16*(c*x^2+b*x+a)^{(5/2)}*x*a^2*b+32*(c*x^2+b*x+a)^{(5/2)}*a^3)/x^7/(c*x^2+b*x+a)^{(3/2)}/a^4$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)

Fricas [A] time = 2.3415, size = 743, normalized size = 3.77

$$\left[\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{a}x^5 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x - 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4(24a^3bx + 16a^4 - (3ab^3 - 20a^2bc))}{256a^3x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^2
+ 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a)
)/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 +
20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 -
(3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3
+ a*x^2))/(a^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.48 \quad \int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9} dx$$

Optimal. Leaf size=249

$$-\frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} + \frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3}}\right)}{256a^{7/2}}$$

[Out] $-\left(\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3b(b^2 - 4ac)^2 \operatorname{ArcTanh}\left[\frac{x(2a + bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3}}\right]}{256a^{7/2}}\right)$

Rubi [A] time = 0.503853, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1920, 1941, 1951, 12, 1904, 206}

$$-\frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} + \frac{3b(b^2 - 4ac)^2 \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3}}\right)}{256a^{7/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(ax^2 + bx^3 + cx^4)^{3/2}/x^9, x]$

[Out] $-\left(\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3b(b^2 - 4ac)^2 \operatorname{ArcTanh}\left[\frac{x(2a + bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3}}\right]}{256a^{7/2}}\right)$

Rule 1920

$\operatorname{Int}[(x_)^{(m_.)}*((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m+1)}*(ax^q + bx^n + cx^{(2n-q)})^p)/(m+p*q+1), x] - \operatorname{Dist}[\frac{(n-q)*p}{m+p*q+1}, \operatorname{Int}[x^{(m+n)}*(b+2cx^{(n-q)})*(ax^q + bx^n + cx^{(2n-q)})^{(p-1)}, x], x] /;$ FreeQ[{a, b, c}, x] & & EqQ[r, 2n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4ac, 0] &&

IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) + 1] && NeQ[m + p*q + 1, 0]

Rule 1941

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(x^(m + 1)*(A*(m + p*q + (n - q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p]/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(n + m)*Simp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m + p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1951

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9} dx &= -\frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3}{10} \int \frac{(b + 2cx)\sqrt{ax^2 + bx^3 + cx^4}}{x^6} dx \\
 &= -\frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} + \frac{3}{160} \int \frac{2(b^2 - 8ac) - 4bcx}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} - \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{5x^8} - \int \frac{b}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{3(b + 4cx)\sqrt{ax^2 + bx^3 + cx^4}}{40x^5} \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2} \\
 &= -\frac{(b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{80ax^4} + \frac{b(5b^2 - 28ac)\sqrt{ax^2 + bx^3 + cx^4}}{320a^2x^3} - \frac{(15b^4 - 100ab^2c + 128a^2b^2c^2)\sqrt{ax^2 + bx^3 + cx^4}}{640a^3x^2}
 \end{aligned}$$

Mathematica [A] time = 0.176466, size = 177, normalized size = 0.71

$$\frac{\sqrt{x^2(a + x(b + cx))} \left(15bx^5 (b^2 - 4ac)^2 \tanh^{-1} \left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}} \right) - 2\sqrt{a}\sqrt{a+x(b+cx)} (8a^2x^2 (b^2 + 7bcx + 16c^2x^2) + 16a^3x) \right)}{1280a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]

[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])))/(1280*a^(7/2)*x^6*Sqrt[a + x*(b +

c*x)])

Maple [B] time = 0.008, size = 534, normalized size = 2.1

$$-\frac{1}{1280x^8a^5}(cx^4+bx^3+ax^2)^{\frac{3}{2}}\left(-240c^2a^{7/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^5b-120(cx^2+bx+a)^{3/2}c^2x^6ab^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)

[Out]
$$-1/1280*(c*x^4+b*x^3+a*x^2)^{(3/2)}*(-240*c^2*a^{(7/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^5*b-120*(c*x^2+b*x+a)^{(3/2)}*c^2*x^6*a*b^2+120*c*a^{(5/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^5*b^3+80*(c*x^2+b*x+a)^{(3/2)}*c^2*x^5*a^2*b+10*(c*x^2+b*x+a)^{(3/2)}*c*x^6*b^4-120*(c*x^2+b*x+a)^{(1/2)}*c^2*x^6*a^2*b^2+120*(c*x^2+b*x+a)^{(5/2)}*c*x^4*a*b^2-100*(c*x^2+b*x+a)^{(3/2)}*c*x^5*a*b^3+240*(c*x^2+b*x+a)^{(1/2)}*c^2*x^5*a^3*b+30*(c*x^2+b*x+a)^{(1/2)}*c*x^6*a*b^4-15*a^{(3/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^5*b^5-80*(c*x^2+b*x+a)^{(5/2)}*c*x^3*a^2*b-10*(c*x^2+b*x+a)^{(5/2)}*x^4*b^4+10*(c*x^2+b*x+a)^{(3/2)}*x^5*b^5-180*(c*x^2+b*x+a)^{(1/2)}*c*x^5*a^2*b^3-20*(c*x^2+b*x+a)^{(5/2)}*x^3*a*b^3+30*(c*x^2+b*x+a)^{(1/2)}*x^5*a*b^5+80*(c*x^2+b*x+a)^{(5/2)}*x^2*a^2*b^2-160*(c*x^2+b*x+a)^{(5/2)}*x*a^3*b+256*(c*x^2+b*x+a)^{(5/2)}*a^4)/x^8/(c*x^2+b*x+a)^{(3/2)}/a^5$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^9} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)

Fricas [A] time = 2.71046, size = 898, normalized size = 3.61

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{ax^6} \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4(176a^4bx + 128a^5 + (15ab^4 - 100a^2b^2c + 128a^3c^2)x^4 - 2(5a^2b^3 - 28a^3bc)x^3 + 8(a^3b^2 + 32a^4c)x^2)\sqrt{c^2x^4 + b^2x^3 + a^2x^2}}{2560a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")

[Out] Timed out

$$3.49 \quad \int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=143

$$\frac{x(3b^2 - 4ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.174007, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1928, 1949, 12, 1914, 621, 206}

$$\frac{x(3b^2 - 4ac)\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1928

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - 2*n + q + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^(p + 1)/(c*(m + p*q + 2*(n - q)*p + 1)), x] - Dist[1/(c*(m + p*q + 2*(n - q)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q + (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]

Rule 1949

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*(A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1914

```

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

```

Rule 621

```

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{\int \frac{x\left(a + \frac{3bx}{2}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\int \frac{(3b^2 - 4ac)x}{4\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c^2} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{(3b^2 - 4ac) \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8c^2} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\left((3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{8c^2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{\left((3b^2 - 4ac)x\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4c - x^2} dx\right)}{4c^2\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{2c} - \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4c^2x} + \frac{(3b^2 - 4ac)x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.136673, size = 105, normalized size = 0.73

$$\frac{x \left((3b^2 - 4ac) \sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + x(b + cx)}} \right) + 2\sqrt{c}(2cx - 3b)(a + x(b + cx)) \right)}{8c^{5/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 144, normalized size = 1.

$$\frac{x}{8} \sqrt{cx^2 + bx + a} \left(4 \sqrt{cx^2 + bx + ac^{5/2}} x - 6 \sqrt{cx^2 + bx + ac^{3/2}} b - 4 \ln \left(1/2 \frac{2 \sqrt{cx^2 + bx + a} \sqrt{c} + 2cx + b}{\sqrt{c}} \right) ac^2 + 3 \ln \left(1/2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)`

[Out] $\frac{1}{8}x(c^2x^2+bx+a)^{1/2}(4(c^2x^2+bx+a)^{1/2}c^{5/2}x-6(c^2x^2+bx+a)^{1/2}c^{3/2}b-4\ln(1/2(2(c^2x^2+bx+a)^{1/2}c^{1/2}+2cx+b)/c^{1/2}))a^2c^2+3\ln(1/2(2(c^2x^2+bx+a)^{1/2}c^{1/2}+2cx+b)/c^{1/2}))b^2c)/(c^2x^4+b^3x^3+a^2x^2)^{1/2}/c^{7/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)`

Fricas [A] time = 1.56927, size = 514, normalized size = 3.59

$$\left[\frac{(3b^2 - 4ac)\sqrt{cx} \log\left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(2c^2x - 3bc)}{16c^3x}, \frac{(3b^2 - 4ac)\sqrt{cx} \arctan\left(\frac{2cx+b}{\sqrt{cx^4 + bx^3 + ax^2}}\right) - 2\sqrt{cx^4 + bx^3 + ax^2}(2c^2x - 3bc)}{16c^3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $[-1/16*((3*b^2 - 4*a*c)*\sqrt{c})*x*\log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) - 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*\sqrt{-c})*x*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c^2*x - 3*b*c))/(c^3*x)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2), x)

[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2), x, algorithm="giac")

[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)

$$3.50 \quad \int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.0776804, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1917, 1914, 621, 206}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{bx\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1917

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n)*(a*x^(n - 1) + b*x^n + c*x^(n + 1)))^(p + 1)/(2*c*(p + 1)), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && EqQ[m + p*(n - 1) - 1, 0]

Rule 1914

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[

$a*x^q + b*x^n + c*x^{(2*n - q)}$, Int[$x^{(m - q/2)}/\text{Sqrt}[a + b*x^{(n - q)} + c*x^{(2*(n - q))}$], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[$1/\text{Sqrt}[(a_) + (b_)*(x_) + (c_)*(x_)^2]$, x_Symbol] := Dist[2, Subst[Int[$1/(4*c - x^2)$], x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[$((a_) + (b_)*(x_)^2)^{-1}$, x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{b \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2c} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{(bx\sqrt{a + bx + cx^2}) \int \frac{1}{\sqrt{a + bx + cx^2}} dx}{2c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{(bx\sqrt{a + bx + cx^2}) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{\sqrt{ax^2 + bx^3 + cx^4}}{cx} - \frac{bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.0540195, size = 89, normalized size = 0.86

$$\frac{x \left(2\sqrt{c}(a + x(b + cx)) - b\sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right)}{2c^{3/2}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

```
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 88, normalized size = 0.9

$$\frac{x}{2} \sqrt{cx^2 + bx + a} \left(2 \sqrt{cx^2 + bx + a} c^{3/2} - b \ln \left(\frac{1}{2} \left(2 \sqrt{cx^2 + bx + a} \sqrt{c} + 2cx + b \right) \frac{1}{\sqrt{c}} \right) c \right) \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} c^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*c^(3/2)-b*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2)))/((c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A] time = 1.59089, size = 429, normalized size = 4.17

$$\left[\frac{b\sqrt{cx} \log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right) + 4\sqrt{cx^4+bx^3+ax^2}c}{4c^2x}, \frac{b\sqrt{-cx} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)}{2c^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)

Giac [A] time = 1.15572, size = 146, normalized size = 1.42

$$\frac{b \arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-cc}} + \frac{b\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right) - 2\sqrt{ac}}{\left(\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}\right)^2 - c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c) + (b*(sqrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c)

$$3.51 \quad \int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]))/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.0366343, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1914, 621, 206}

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]))/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[
1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{ax^2 + bx^3 + cx^4}} dx &= \frac{\left(x\sqrt{a + bx + cx^2}\right) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{\left(2x\sqrt{a + bx + cx^2}\right) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2 + bx^3 + cx^4}} \\ &= \frac{x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2 + bx^3 + cx^4}} \end{aligned}$$

Mathematica [A] time = 0.0412742, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a + bx + cx^2} \log\left(2\sqrt{c}\sqrt{a + bx + cx^2} + b + 2cx\right)}{\sqrt{c}\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] (x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]])/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 65, normalized size = 0.9

$$x\sqrt{cx^2 + bx + a} \ln\left(\frac{1}{2}\left(2\sqrt{cx^2 + bx + a}\sqrt{c} + 2cx + b\right)\frac{1}{\sqrt{c}}\right) \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2), x)

[Out] $1/(c*x^4+b*x^3+a*x^2)^{(1/2)}*x*(c*x^2+b*x+a)^{(1/2)}*\ln(1/2*(2*(c*x^2+b*x+a)^{(1/2)}*c^{(1/2)}+2*c*x+b)/c^{(1/2)})/c^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)`

Fricas [A] time = 1.54597, size = 298, normalized size = 4.2

$$\left[\frac{\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c+(b^2+4ac)x}}{x}\right)}{2\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)}{c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)`

[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)

Giac [A] time = 1.15887, size = 50, normalized size = 0.7

$$-\frac{2 \arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")

[Out] -2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/sqrt(-c)

$$3.52 \quad \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])

Rubi [A] time = 0.0158112, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1904, 206}

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx = -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a + bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)\right) \\ = -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0211026, size = 70, normalized size = 1.56

$$\frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4], x]

[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))

Maple [A] time = 0.005, size = 66, normalized size = 1.5

$$-x\sqrt{cx^2 + bx + a} \ln\left(\frac{1}{x}\left(2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}\right)\right) \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2), x)

[Out] -1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)

Fricas [A] time = 1.61596, size = 300, normalized size = 6.67

$$\left[\frac{\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)

[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)

Giac [A] time = 1.15699, size = 80, normalized size = 1.78

$$-\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{cx-\sqrt{cx^2+bx+a}}}{\sqrt{-a}}\right)}{\sqrt{-a} \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c  
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

$$3.53 \quad \int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}} dx$$

Optimal. Leaf size=77

$$\frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))

Rubi [A] time = 0.0540936, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1927, 1904, 206}

$$\frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}} - \frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))

Rule 1927

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(2*a*(n - q)*(p + 1)), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p + 1)]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{ax^2 + bx^3 + cx^4}} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} - \frac{b \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} + \frac{b \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a} \\ &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{ax^2} + \frac{b \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{2a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0555978, size = 89, normalized size = 1.16

$$\frac{bx\sqrt{a+x(b+cx)} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}(a+x(b+cx))}{2a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]), x]

[Out] (-2*Sqrt[a]*(a + x*(b + c*x)) + b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 88, normalized size = 1.1

$$-\frac{1}{2}\sqrt{cx^2 + bx + a} \left(2\sqrt{cx^2 + bx + aa^{3/2}} - b \ln\left(\frac{1}{x} \left(2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a} \right) \right) ax \right) \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}} a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)`

[Out] $-1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a*x)/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(5/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)`

Fricas [A] time = 1.73191, size = 444, normalized size = 5.77

$$\left[\frac{\sqrt{ab}x^2 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}a}{4a^2x^2}, -\frac{\sqrt{-ab}x^2 \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{2(acx^3 + abx^2 + a^2x)}\right)}{2a^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] $[1/4*(\sqrt{a}*b*x^2*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*a)/(a^2*x^2), -1/2*(\sqrt{-a}*b*x^2*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*\sqrt{c*x^4 + b*x^3 + a*x^2})*a)/(a^2*x^2)]$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2(a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.54 \quad \int \frac{1}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx$$

Optimal. Leaf size=119

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/(8*a^{(5/2)})$

Rubi [A] time = 0.148656, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1929, 1951, 12, 1904, 206}

$$-\frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x^2*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]),x]$

[Out] $-\text{Sqrt}[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])])/(8*a^{(5/2)})$

Rule 1929

$\text{Int}[(x_)^{(m_.)}*((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(x^{(m - q + 1)}*(a*x^q + b*x^n + c*x^{(2*n - q)})^{(p + 1)})/(a*(m + p*q + 1)), x] - \text{Dist}[1/(a*(m + p*q + 1)), \text{Int}[x^{(m + n - q)}*(b*(m + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^{(n - q)})*(a*x^q + b*x^n + c*x^{(2*n - q)})^p, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{EqQ}[r, 2*n - q] \&\& \text{PosQ}[n - q] \&\& \text{IntegerQ}[p] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GeQ}[p, -1] \&\& \text{LtQ}[p, 0] \&\& \text{RationalQ}[m, q] \&\& \text{LtQ}[m + p*q + 1, 0]$

Rule 1951

$\text{Int}[(x_)^{(m_.)}*((c_.)*(x_)^{(j_.)} + (b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)})^{(p_.)}*((A_) + (B_.)*(x_)^{(r_.)}), x_Symbol] \rightarrow \text{Simp}[(A*x^{(m - q + 1)}*(a*x^q + b$


```
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_)*(x_)^2 + (b_)*(x_)^(n_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 \sqrt{ax^2 + bx^3 + cx^4}} dx &= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{\int \frac{-\frac{3b}{2} - cx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{\int \frac{-\frac{3b^2}{4} + ac}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} + \frac{(3b^2 - 4ac) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{8a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{(3b^2 - 4ac) \text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{4a^2} \\
&= -\frac{\sqrt{ax^2 + bx^3 + cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2 + bx^3 + cx^4}}{4a^2x^2} - \frac{(3b^2 - 4ac) \tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{8a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0955229, size = 112, normalized size = 0.94

$$\frac{x^2 \left(-(3b^2 - 4ac) \right) \sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}} \right) - 2\sqrt{a}(2a - 3bx)(a + x(b + cx))}{8a^{5/2}x\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] (-2*Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x)) - (3*b^2 - 4*a*c)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(8*a^(5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 152, normalized size = 1.3

$$-\frac{1}{8x} \sqrt{cx^2 + bx + a} \left(4 \sqrt{cx^2 + bx + a} a^{5/2} - 6 a^{3/2} \sqrt{cx^2 + bx + a} x b - 4 c \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) \right) a^2 x^2 + 3 \ln \left(\frac{2}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)

[Out] $-1/8*(c*x^2+b*x+a)^{(1/2)}*(4*(c*x^2+b*x+a)^{(1/2)}*a^{(5/2)}-6*a^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}*x*b-4*c*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*a^2*x^2+3*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)*x^2*a*b^2)/x/(c*x^4+b*x^3+a*x^2)^{(1/2)}/a^{(7/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)`

Fricas [A] time = 1.7908, size = 527, normalized size = 4.43

$$\left[\frac{(3b^2 - 4ac)\sqrt{a}x^3 \log\left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right) - 4\sqrt{cx^4 + bx^3 + ax^2}(3abx - 2a^2)(3b^2 - 4ac)}{16a^3x^3}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")`

[Out] `[-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{x^2 (a + bx + cx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.55 \quad \int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=262

$$\frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac) \sqrt{ax^2 + bx^3 + cx^4}}{4c^3x(b^2 - 4ac)} + \frac{3x(5b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/((2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.50583, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{(5b^2 - 12ac) \sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac) \sqrt{ax^2 + bx^3 + cx^4}}{4c^3x(b^2 - 4ac)} + \frac{3x(5b^2 - 4ac) \sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/((2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1923

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n -

```
q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c
*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,
-1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1949

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_.) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1914

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2bx\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{2 \int \frac{x^2(6ab+\frac{3}{2}(5b^2-12ac)x)}{\sqrt{ax^2+bx^3+cx^4}} dx}{3c(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{2bx\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3(b^2 - 4ac)} \\
&= \frac{2x^4(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2c^2(b^2 - 4ac)} - \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4c^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.258017, size = 183, normalized size = 0.7

$$\frac{x \left(2\sqrt{c} (4a^2c(6cx - 13b) + a(-62b^2cx + 15b^3 - 20bc^2x^2 + 8c^3x^3) + b^2x(15b^2 + 5bcx - 2c^2x^2)) - 3(16a^2c^2 - 24ab^2c + 8c^3) \sqrt{x^2(a + x(b + cx))} \right)}{8c^{7/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (x*(2*sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) - 3*(5*b^4 - 24*a*

$$b^2c + 16a^2c^2) \sqrt{a + x(b + cx)} \operatorname{ArcTanh}\left[\frac{(b + 2cx)}{(2\sqrt{c}\sqrt{a + x(b + cx)})}\right] \Big/ (8c^{7/2}(-b^2 + 4ac) \sqrt{x^2(a + x(b + cx))})$$

Maple [A] time = 0.009, size = 283, normalized size = 1.1

$$-\frac{x^3(cx^2 + bx + a)}{32ac - 8b^2} \left(-16c^{9/2}x^3a + 4c^{7/2}x^3b^2 + 40c^{7/2}x^2ab - 10c^{5/2}x^2b^3 - 48c^{7/2}xa^2 + 124c^{5/2}xab^2 - 30c^{3/2}xb^4 + 48\sqrt{cx^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2), x)

[Out]
$$-1/8x^3(c^2x^2+bx+a)/c^{9/2}(-16c^{9/2}x^3a+4c^{7/2}x^3b^2+40c^{7/2}x^2ab-10c^{5/2}x^2b^3-48c^{7/2}xa^2+124c^{5/2}xab^2-30c^{3/2}xb^4+48\sqrt{cx^2})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(cx^4 + bx^3 + ax^2)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="maxima")

[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

Fricas [A] time = 2.27583, size = 1316, normalized size = 5.02

$$\left[\frac{3((5b^4c - 24ab^2c^2 + 16a^2c^3)x^3 + (5b^5 - 24ab^3c + 16a^2bc^2)x^2 + (5ab^4 - 24a^2b^2c + 16a^3c^2)x)\sqrt{c} \log\left(-\frac{8c^2x^3 + 8bcx^2 - \dots}{16((b^2c^5 - 4ac \dots)}\right)}{16((b^2c^5 - 4ac \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*\sqrt{c}*\log(- \\ & (8*c^2*x^3 + 8*b*c*x^2 - 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*\sqrt{-c}*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*\sqrt{c*x^4 + b*x^3 + a*x^2})/(b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

$$3.56 \quad \int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=201

$$\frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2x(b^2 - 4ac)} + \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} - \frac{3bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b}{2\sqrt{c}}\right)}{2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.304659, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2x(b^2 - 4ac)} + \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} - \frac{3bx\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b}{2\sqrt{c}}\right)}{2c^{5/2}\sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1923

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] := -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&

PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]

Rule 1949

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*(A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 1914

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{\int \frac{x(2ab+(3b^2-8ac)x)}{\sqrt{ax^2+bx^3+cx^4}} dx}{c(b^2 - 4ac)} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)x} - \int \frac{3bx^2}{\sqrt{ax^2+bx^3+cx^4}} dx \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)x} - \frac{3bx^2}{\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)x} - \frac{3bx^2}{\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)x} - \frac{3bx^2}{\sqrt{ax^2+bx^3+cx^4}} \\
&= \frac{2x^3(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)} + \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{c^2(b^2 - 4ac)x} - \frac{3bx^2}{\sqrt{ax^2+bx^3+cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.15881, size = 141, normalized size = 0.7

$$\frac{x \left(2\sqrt{c} (8a^2c + a(-3b^2 + 10bcx + 4c^2x^2) - b^2x(3b + cx)) + 3b(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1} \left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}} \right) \right)}{2c^{5/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (x*(2*sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x^2)) + 3*b*(b^2 - 4*a*c)*sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*sqrt[c]*sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 199, normalized size = 1.

$$\frac{x^3 (cx^2 + bx + a)}{8ac - 2b^2} \left(8c^{7/2}x^2a - 2c^{5/2}x^2b^2 + 20c^{5/2}xab - 6c^{3/2}xb^3 + 16c^{5/2}a^2 - 6c^{3/2}ab^2 - 12 \ln \left(\frac{1}{2} \frac{2\sqrt{cx^2 + bx + a}\sqrt{c} + \sqrt{c}}{\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] $\frac{1}{2}x^3(c^2x^2+bx+a)/c^{7/2}(8c^{7/2}x^2a-2c^{5/2}x^2b^2+20c^{5/2}xab-6c^{3/2}xb^3+16c^{5/2}a^2-6c^{3/2}ab^2-12\ln(1/2*(2*(c^2x^2+bx+a)^{1/2}*c^{1/2}+2*c*x+b)/c^{1/2})*(c^2x^2+bx+a)^{1/2}*a*b*c^2+3*\ln(1/2*(2*(c^2x^2+bx+a)^{1/2}*c^{1/2}+2*c*x+b)/c^{1/2})*(c^2x^2+bx+a)^{1/2}*b^3*c)/(c^4x^4+b^3x^3+a^2x^2)^{3/2}/(4*a*c-b^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

Fricas [A] time = 2.09024, size = 1026, normalized size = 5.1

$$\frac{3 \left((b^3c - 4abc^2)x^3 + (b^4 - 4ab^2c)x^2 + (ab^3 - 4a^2bc)x \right) \sqrt{c} \log \left(-\frac{8c^2x^3 + 8bcx^2 - 4\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{c} + (b^2 + 4ac)x}{x} \right) + 4\sqrt{cx^4 + bx^3 + ax^2}}{4 \left((b^2c^4 - 4ac^5)x^3 + (b^3c^3 - 4abc^4)x^2 + (ab^2c^3 - 4a^2c^4)x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2), x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A] time = 1.16984, size = 263, normalized size = 1.31

$$\frac{2 \left(\frac{b^3 c^2 - 3 a b c^3}{b^2 c^4 - 4 a c^5} + \frac{a b^2 c^2 - 2 a^2 c^3}{(b^2 c^4 - 4 a c^5) x} \right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}} + \frac{3 b \arctan \left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}} \right)}{\sqrt{-c} c^2} + \frac{b \left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x} \right) - 2 \sqrt{a} c}{\left(\left(\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x} \right)^2 - c \right) c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="giac")
```

```
[Out] 2*((b^3*c^2 - 3*a*b*c^3)/(b^2*c^4 - 4*a*c^5) + (a*b^2*c^2 - 2*a^2*c^3)/((b^2*c^4 - 4*a*c^5)*x))/sqrt(c + b/x + a/x^2) + 3*b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c^2) + (b*(sqrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqrt(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c^2)
```

$$3.57 \quad \int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx(b^2-4ac)} + \frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.175293, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1923, 1949, 12, 1914, 621, 206}

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx(b^2-4ac)} + \frac{x\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1923

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - 2*n + q + 1)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/((n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p,

-1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]

Rule 1949

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] - Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1914

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}} dx}{b^2 - 4ac} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{2 \int \frac{(b^2-4ac)x}{2\sqrt{ax^2+bx^3+cx^4}} dx}{c(b^2 - 4ac)} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}} dx}{c} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{(x\sqrt{a + bx + cx^2}) \int \frac{1}{\sqrt{a+bx+cx^2}} dx}{c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{(2x\sqrt{a + bx + cx^2}) \text{Subst}\left(\int \frac{1}{4c-x^2} dx\right)}{c\sqrt{ax^2 + bx^3 + cx^4}} \\
&= \frac{2x^2(2a + bx)}{(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2b\sqrt{ax^2 + bx^3 + cx^4}}{c(b^2 - 4ac)x} + \frac{x\sqrt{a + bx + cx^2} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2 + bx^3 + cx^4}}
\end{aligned}$$

Mathematica [A] time = 0.122982, size = 112, normalized size = 0.73

$$\frac{x \left(2\sqrt{c}(-ab + 2acx + b^2(-x)) + (b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) \right)}{c^{3/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] -((x*(2*Sqrt[c]*(-a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 166, normalized size = 1.1

$$-\frac{x^3(cx^2 + bx + a)}{4ac - b^2} \left(4c^{5/2}xa - 2c^{3/2}xb^2 - 2c^{3/2}ab - 4 \ln \left(\frac{1}{2} \frac{2\sqrt{cx^2 + bx + a}\sqrt{c} + 2cx + b}{\sqrt{c}} \right) \sqrt{cx^2 + bx + a}c^2 + \ln \left(\frac{1}{2} \left(2\sqrt{cx^2 + bx + a} + \sqrt{c} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] $-x^3*(c*x^2+b*x+a)/c^{5/2}*(4*c^{5/2}*x*a-2*c^{3/2}*x*b^2-2*c^{3/2}*a*b-4*\ln(1/2*(2*(c*x^2+b*x+a)^{1/2}*c^{1/2}+2*c*x+b)/c^{1/2})*(c*x^2+b*x+a)^{1/2}*a*c^2+\ln(1/2*(2*(c*x^2+b*x+a)^{1/2}*c^{1/2}+2*c*x+b)/c^{1/2})*(c*x^2+b*x+a)^{1/2}*b^2*c)/(c*x^4+b*x^3+a*x^2)^{3/2}/(4*a*c-b^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

Fricas [A] time = 2.15883, size = 873, normalized size = 5.71

$$\frac{\left((b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x \right) \sqrt{c} \log \left(-\frac{8c^2x^3 + 8bcx^2 + 4\sqrt{cx^4 + bx^3 + ax^2}(2cx+b)\sqrt{c} + (b^2 + 4ac)x}{x} \right) - 4\sqrt{cx^4 + bx^3 + ax^2}}{2 \left((b^2c^3 - 4ac^4)x^3 + (b^3c^2 - 4abc^3)x^2 + (ab^2c^2 - 4a^2c^3)x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out] $[1/2*((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*\sqrt{c}*\log(-\frac{8*c^2*x^3 + 8*b*c*x^2 + 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{c} + (b^2 + 4*a*c)*x}{x}) - 4*\sqrt{c*x^4 + b*x^3 + a*x^2} + 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(a*b*c + (b^2*c - 2*a*c^2)*x)]/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x), -((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*\sqrt{-c}*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(2*c*x + b)*\sqrt{-c}/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*\sqrt{c*x^4 + b*x^3 + a*x^2}$

$$x^2)(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)

Giac [A] time = 1.19216, size = 149, normalized size = 0.97

$$-\frac{2\left(\frac{abc}{(b^2c^2-4ac^3)x} + \frac{b^2c-2ac^2}{b^2c^2-4ac^3}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}} - \frac{2 \arctan\left(\frac{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}} - \frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] -2*(a*b*c/((b^2*c^2 - 4*a*c^3)*x) + (b^2*c - 2*a*c^2)/(b^2*c^2 - 4*a*c^3))/sqrt(c + b/x + a/x^2) - 2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c)

$$3.58 \quad \int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=40

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rubi [A] time = 0.0399566, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1916}

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])

Rule 1916

Int[(x_)^(m_)/((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(3/2), x_Symbol] :> Simp[(x^((n-1)/2)*(4*a + 2*b*x))/((b^2 - 4*a*c)*Sqrt[a*x^(n-1) + b*x^n + c*x^(n+1)]), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3*n-1)/2] && EqQ[q, n-1] && EqQ[r, n+1] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}} dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0741147, size = 37, normalized size = 0.92

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.004, size = 53, normalized size = 1.3

$$-2 \frac{(cx^2 + bx + a)(bx + 2a)x^3}{(4ac - b^2)(cx^4 + bx^3 + ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] -2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

Fricas [A] time = 1.98544, size = 150, normalized size = 3.75

$$\frac{2\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)}{(b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)

Giac [A] time = 1.19208, size = 61, normalized size = 1.52

$$\frac{2 \left(\frac{b}{b^2 - 4ac} + \frac{2a}{(b^2 - 4ac)x} \right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] 2*(b/(b^2 - 4*a*c) + 2*a/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)

$$3.59 \quad \int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

[Out] $(-2*x*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^2+b*x^3+c*x^4])$

Rubi [A] time = 0.0402122, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1915}

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3/(a*x^2+b*x^3+c*x^4)^(3/2),x]$

[Out] $(-2*x*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^2+b*x^3+c*x^4])$

Rule 1915

$\text{Int}[(x_)^(m_)/((b_)*(x_)^(n_)+(a_)*(x_)^(q_)+(c_)*(x_)^(r_))^(3/2), x_Symbol] \rightarrow \text{Simp}[(-2*x^((n-1)/2)*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^(n-1)+b*x^n+c*x^(n+1)]), x] /; \text{FreeQ}\{a, b, c, n\}, x \ \&\& \ \text{EqQ}[m, (3*(n-1))/2] \ \&\& \ \text{EqQ}[q, n-1] \ \&\& \ \text{EqQ}[r, n+1] \ \&\& \ \text{NeQ}[b^2-4*a*c, 0]$

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A] time = 0.0244011, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] $(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*\text{Sqrt}[x^2*(a + x*(b + c*x))])$

Maple [A] time = 0.003, size = 52, normalized size = 1.3

$$2 \frac{(cx^2 + bx + a)(2cx + b)x^3}{(4ac - b^2)(cx^4 + bx^3 + ax^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2), x)

[Out] $2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="maxima")

[Out] integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

Fricas [A] time = 1.94596, size = 151, normalized size = 3.87

$$\frac{2\sqrt{cx^4 + bx^3 + ax^2}(2cx + b)}{(b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] $-2\sqrt{c x^4 + b x^3 + a x^2} (2 c x + b) / ((b^2 c - 4 a c^2) x^3 + (b^3 - 4 a b c) x^2 + (a b^2 - 4 a^2 c) x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2(a + bx + cx^2)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)

Giac [A] time = 1.16545, size = 61, normalized size = 1.56

$$-\frac{2\left(\frac{2c}{b^2-4ac} + \frac{b}{(b^2-4ac)x}\right)}{\sqrt{c + \frac{b}{x} + \frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] $-2*(2*c/(b^2 - 4*a*c) + b/((b^2 - 4*a*c)*x))/\sqrt{c + b/x + a/x^2}$

$$3.60 \quad \int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=94

$$\frac{2x(-2ac + b^2 + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)

Rubi [A] time = 0.0676333, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1922, 1904, 206}

$$\frac{2x(-2ac + b^2 + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)

Rule 1922

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -(n - q)*(2*p + 3)]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} + \frac{\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{a} \\ &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{x(2a + bx)}{\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a} \\ &= \frac{2x(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{\tanh^{-1}\left(\frac{x(2a + bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}}\right)}{a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.123031, size = 109, normalized size = 1.16

$$\frac{x(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a + bx}{2\sqrt{a}\sqrt{a + x(b + cx)}}\right) - 2\sqrt{a}x(-2ac + b^2 + bcx)}{a^{3/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]
```

```
[Out] (-2*Sqrt[a]*x*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(a^(3/2)*(-b^2 + 4
*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.006, size = 164, normalized size = 1.7

$$\frac{x^3 (cx^2 + bx + a)}{4ac - b^2} \left(4a^{5/2}c - 2a^{3/2}xbc - 2a^{3/2}b^2 - 4 \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) \sqrt{cx^2 + bx + a} a^2c + \ln \left(\frac{1}{x} (2a + \dots) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] x^3*(c*x^2+b*x+a)*(4*a^(5/2)*c-2*a^(3/2)*x*b*c-2*a^(3/2)*b^2-4*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^2*c+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(5/2)/(4*a*c-b^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)

Fricas [B] time = 2.06535, size = 869, normalized size = 9.24

$$\left[\frac{((b^2c - 4ac^2)x^3 + (b^3 - 4abc)x^2 + (ab^2 - 4a^2c)x)\sqrt{a} \log \left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x - 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3} \right) + 4\sqrt{cx^4 + bx^3 + ax^2}}{2((a^2b^2c - 4a^3c^2)x^3 + (a^2b^3 - 4a^3bc)x^2 + (a^3b^2 - 4a^4c)x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(a))/(2*((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)))]

$$+ a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2(a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**3/2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.61 \quad \int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=144

$$-\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{a^2 x^2 (b^2 - 4ac)} + \frac{3b \tanh^{-1} \left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} \right)}{2a^{5/2}} + \frac{2(-2ac + b^2 + bcx)}{a(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/2))

Rubi [A] time = 0.163171, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1924, 1951, 12, 1904, 206}

$$-\frac{(3b^2 - 8ac) \sqrt{ax^2 + bx^3 + cx^4}}{a^2 x^2 (b^2 - 4ac)} + \frac{3b \tanh^{-1} \left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} \right)}{2a^{5/2}} + \frac{2(-2ac + b^2 + bcx)}{a(b^2 - 4ac) \sqrt{ax^2 + bx^3 + cx^4}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/2))

Rule 1924

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]

&& LtQ[m + p*q + 1, n - q]

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{3b^2}{2} + 4ac - bcx}{x\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{2 \int -\frac{3b(b^2 - 4ac)}{4\sqrt{ax^2 + bx^3 + cx^4}} dx}{a^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} - \frac{(3b) \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{(3b) \text{Subst} \left(\int \frac{1}{4a - x^2} dx, \right)}{a^2} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(3b^2 - 8ac)\sqrt{ax^2 + bx^3 + cx^4}}{a^2(b^2 - 4ac)x^2} + \frac{3b \tanh^{-1} \left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2 + bx^3 + cx^4}} \right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.105634, size = 138, normalized size = 0.96

$$\frac{2\sqrt{a}(-4a^2c + a(b^2 - 10bcx - 8c^2x^2) + 3b^2x(b + cx)) - 3bx(b^2 - 4ac)\sqrt{a + x(b + cx)} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{5/2}(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x]

[Out] (2*sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) - 3*b*(b^2 - 4*a*c)*x*sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*sqrt[a]*sqrt[a + x*(b + c*x)])])/(2*a^(5/2)*(-b^2 + 4*a*c)*sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.006, size = 201, normalized size = 1.4

$$-\frac{x^2(cx^2 + bx + a)}{8ac - 2b^2} \left(16a^{5/2}x^2c^2 - 6a^{3/2}x^2b^2c + 20a^{5/2}xbc - 6a^{3/2}xb^3 - 12 \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) \right) \sqrt{cx^2 + bx + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out]
$$-1/2*x^2*(c*x^2+b*x+a)*(16*a^(5/2)*x^2*c^2-6*a^(3/2)*x^2*b^2*c+20*a^(5/2)*x*b*c-6*a^(3/2)*x*b^3-12*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x*a^2*b*c+3*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x*a*b^3+8*a^(7/2)*c-2*a^(5/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(7/2)/(4*a*c-b^2)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)`

Fricas [A] time = 1.87612, size = 1041, normalized size = 7.23

$$\frac{3 \left((b^3c - 4abc^2)x^4 + (b^4 - 4ab^2c)x^3 + (ab^3 - 4a^2bc)x^2 \right) \sqrt{a} \log \left(-\frac{8abx^2 + (b^2 + 4ac)x^3 + 8a^2x + 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3} \right) - 4\sqrt{cx^4 + bx^3 + ax^2}}{4 \left((a^3b^2c - 4a^4c^2)x^4 + (a^3b^3 - 4a^4bc)x^3 + (a^4b^2 - 4a^5c)x^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{a}*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3) - 4*\sqrt{c*x^4 + b*x^3 + a*x^2}*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2), \\ & -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-a}*\arctan(1/2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(b*x + 2*a)*\sqrt{-a}/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*\sqrt{c*x^4 + b*x^3 + a*x^2}*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b \end{aligned}$$

```
*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2(a + bx + cx^2)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.62 \quad \int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=209

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3x^2(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2x^3(b^2 - 4ac)} - \frac{3(5b^2 - 4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}} + \frac{2}{ax(b^2 - 4ac)}$$

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(7/2))

Rubi [A] time = 0.286989, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1907, 1951, 12, 1904, 206}

$$\frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3x^2(b^2 - 4ac)} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2x^3(b^2 - 4ac)} - \frac{3(5b^2 - 4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}} + \frac{2}{ax(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2), x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(7/2))

Rule 1907

Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol] := -Simp[(x^(-q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r

, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
.*(A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q]
&& EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)
*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1904

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-2(b^2 - 2ac) + \frac{1}{2}(-b^2 + 4ac) - 2bcx}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{\int \frac{-\frac{1}{4}b(15b^2 - 52ac) - \frac{1}{2}c(5b^2 - 12ac)}{x\sqrt{ax^2 + bx^3 + cx^4}}}{a^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(5b^2 - 12ac)\sqrt{ax^2 + bx^3 + cx^4}}{2a^2(b^2 - 4ac)x^3} + \frac{b(15b^2 - 52ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.166795, size = 181, normalized size = 0.87

$$\frac{2\sqrt{a}(2a^2(b^2 + 10bcx - 12c^2x^2) - 8a^3c + abx(-5b^2 + 62bcx + 52c^2x^2) - 15b^3x^2(b + cx)) + 3x^2(16a^2c^2 - 24ab^2c + 5b^4)}{8a^{7/2}x(b^2 - 4ac)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2), x]

[Out] $-(2*\text{Sqrt}[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 12*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^2*\text{Sqrt}[a + x*(b + c*x)]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])])/(8*a^{7/2}*(b^2 - 4*a*c)*x*\text{Sqrt}[x^2*(a + x*(b + c*x))])$

Maple [A] time = 0.007, size = 292, normalized size = 1.4

$$-\frac{x(cx^2 + bx + a)}{32ac - 8b^2} \left(48a^{7/2}x^2c^2 - 104a^{5/2}x^3bc^2 + 16a^{9/2}c - 40a^{7/2}xbc - 124a^{5/2}x^2b^2c + 30a^{3/2}x^3b^3c - 4a^{7/2}b^2 + 10a^{5/2}xb \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out]
$$-1/8*x*(c*x^2+b*x+a)*(48*a^(7/2)*x^2*c^2-104*a^(5/2)*x^3*b*c^2+16*a^(9/2)*c-40*a^(7/2)*x*b*c-124*a^(5/2)*x^2*b^2*c+30*a^(3/2)*x^3*b^3*c-4*a^(7/2)*b^2+10*a^(5/2)*x*b^3+30*a^(3/2)*x^2*b^4-48*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^3*c^2+72*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a^2*b^2*c-15*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a*b^4)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(9/2)/(4*a*c-b^2)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)`

Fricas [A] time = 2.20791, size = 1328, normalized size = 6.35

$$\left[\frac{3 \left((5b^4c - 24ab^2c^2 + 16a^2c^3)x^5 + (5b^5 - 24ab^3c + 16a^2bc^2)x^4 + (5ab^4 - 24a^2b^2c + 16a^3c^2)x^3 \right) \sqrt{a} \log \left(-\frac{8abx^2 + (b^2 - 4ac)x + 2a}{16(a^4b^2c - 4a^3b^2c^2 + 16a^2b^2c^3 - 4a^2b^2c^2 + 16a^2b^2c^2)} \right)}{16(a^4b^2c - 4a^3b^2c^2 + 16a^2b^2c^3 - 4a^2b^2c^2 + 16a^2b^2c^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

[Out]
$$[-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*\sqrt{a}*\log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*\sqrt{c*x^4 + b*x^3 + a*x^2})*(b*x + 2*a)*\sqrt{a})/x^3 + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^2$$

```

3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*
b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c - 24*a*b^
2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2))*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2*a^3*b^2 -
8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*
a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^
4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*
x^3)]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage₀x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```


$$3.63 \quad \int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=271

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{24a^4x^2(b^2 - 4ac)} + \frac{b(35b^2 - 116ac) \sqrt{ax^2 + bx^3 + cx^4}}{12a^3x^3(b^2 - 4ac)} - \frac{(7b^2 - 16ac) \sqrt{ax^2 + bx^3 + cx^4}}{3a^2x^4(b^2 - 4ac)}$$

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4) + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))

Rubi [A] time = 0.452449, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1904, 206}

$$\frac{(256a^2c^2 - 460ab^2c + 105b^4) \sqrt{ax^2 + bx^3 + cx^4}}{24a^4x^2(b^2 - 4ac)} + \frac{b(35b^2 - 116ac) \sqrt{ax^2 + bx^3 + cx^4}}{12a^3x^3(b^2 - 4ac)} - \frac{(7b^2 - 16ac) \sqrt{ax^2 + bx^3 + cx^4}}{3a^2x^4(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4) + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))

Rule 1924

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -

```

q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
&& LtQ[m + p*q + 1, n - q]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_.) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1904

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{7b^2}{2} + 8ac - 3bcx}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{2 \int \frac{-\frac{1}{4}b(35b^2 - 116ac) -}{x^2\sqrt{ax^2 + bx^3 + cx^4}} dx}{3a^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{12a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^2\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(7b^2 - 16ac)\sqrt{ax^2 + bx^3 + cx^4}}{3a^2(b^2 - 4ac)x^4} + \frac{b(35b^2 - 116ac)\sqrt{ax^2 + bx^3 + cx^4}}{12a^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.265435, size = 225, normalized size = 0.83

$$\frac{2\sqrt{a}(8a^3(b^2 + 7bcx + 16c^2x^2) + 2a^2x(-86b^2cx - 7b^3 + 244bc^2x^2 + 128c^3x^3) - 32a^4c + 5ab^2x^2(7b^2 - 106bcx - 92c^2x^2) - 48a^{9/2}x^2(4ac - b^2)\sqrt{x^2(a + x(b + cx))})}{48a^{9/2}x^2(4ac - b^2)\sqrt{x^2(a + x(b + cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 - 86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(48*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A] time = 0.009, size = 340, normalized size = 1.3

$$-\frac{cx^2 + bx + a}{192ac - 48b^2} \left(-512a^{7/2}x^4c^3 + 920a^{5/2}x^4b^2c^2 - 210a^{3/2}x^4b^4c + 720\sqrt{cx^2 + bx + a} \ln \left(\frac{2a + bx + 2\sqrt{a}\sqrt{cx^2 + bx + a}}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)`

[Out] `-1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*x^4*c^3+920*a^(5/2)*x^4*b^2*c^2-210*a^(3/2)*x^4*b^4*c+720*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*a^3*b*c^2-600*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*a^2*b^3*c+105*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^3*a*b^5-976*a^(7/2)*x^3*b*c^2+1060*a^(5/2)*x^3*b^3*c-210*a^(3/2)*x^3*b^5-256*a^(9/2)*x^2*c^2+344*a^(7/2)*x^2*b^2*c-70*a^(5/2)*x^2*b^4-112*a^(9/2)*x*b*c+28*a^(7/2)*x*b^3+64*a^(11/2)*c-16*a^(9/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)`

Fricas [A] time = 2.91328, size = 1550, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")`

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(x^2(a+bx+cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

$$3.64 \quad \int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}} dx$$

Optimal. Leaf size=343

$$\frac{b(1808a^2c^2 - 1680ab^2c + 315b^4)\sqrt{ax^2 + bx^3 + cx^4}}{64a^5x^2(b^2 - 4ac)} - \frac{(240a^2c^2 - 448ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{32a^4x^3(b^2 - 4ac)} - \frac{15(16a^2c^2 - 56ab^2c + 16a^2c^2 - 56ab^2c + 16a^2c^2 - 56ab^2c)}{128a^{11/2}}$$

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5) + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(11/2))

Rubi [A] time = 0.620551, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1904, 206}

$$\frac{b(1808a^2c^2 - 1680ab^2c + 315b^4)\sqrt{ax^2 + bx^3 + cx^4}}{64a^5x^2(b^2 - 4ac)} - \frac{(240a^2c^2 - 448ab^2c + 105b^4)\sqrt{ax^2 + bx^3 + cx^4}}{32a^4x^3(b^2 - 4ac)} - \frac{15(16a^2c^2 - 56ab^2c + 16a^2c^2 - 56ab^2c + 16a^2c^2 - 56ab^2c)}{128a^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4]) - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5) + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(11/2))

Rule 1924

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q +
b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Di
st[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n -
q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q +
(n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1),
x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]
&& LtQ[m + p*q + 1, n - q]

```

Rule 1951

```

Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1904

```

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/
Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
- 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax^2 + bx^3 + cx^4)^{3/2}} dx &= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{2 \int \frac{-\frac{9b^2}{2} + 10ac - 4bcx}{x^4\sqrt{ax^2 + bx^3 + cx^4}} dx}{a(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{\int \frac{-\frac{3}{4}b(21b^2 - 68ac) - \frac{3}{2}c}{x^3\sqrt{ax^2 + bx^3 + cx^4}} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)} \\
&= \frac{2(b^2 - 2ac + bcx)}{a(b^2 - 4ac)x^3\sqrt{ax^2 + bx^3 + cx^4}} - \frac{(9b^2 - 20ac)\sqrt{ax^2 + bx^3 + cx^4}}{4a^2(b^2 - 4ac)x^5} + \frac{b(21b^2 - 68ac)\sqrt{ax^2 + bx^3 + cx^4}}{8a^3(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.286878, size = 272, normalized size = 0.79

$$15x^4(240a^2b^2c^2 - 64a^3c^3 - 140ab^4c + 21b^6)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}(-16a^4(b^2 + 6bcx + 10c^2x^2) +$$

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Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]

[Out] (-2*Sqrt[a]*(64*a^5*c + 315*b^5*x^4*(b + c*x) + 105*a*b^3*x^3*(b^2 - 18*b*c*x - 16*c^2*x^2) - 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 8*a^3*x*(3*b^3 + 2

$$6*b^2*c*x + 98*b*c^2*x^2 - 60*c^3*x^3) + 2*a^2*b*x^2*(-21*b^3 - 308*b^2*c*x + 1352*b*c^2*x^2 + 904*c^3*x^3)) + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*x^4*\text{Sqrt}[a + x*(b + c*x)]*\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + x*(b + c*x)])]/(128*a^(11/2)*(-b^2 + 4*a*c)*x^3*\text{Sqrt}[x^2*(a + x*(b + c*x))])$$

Maple [A] time = 0.008, size = 446, normalized size = 1.3

$$-\frac{cx^2 + bx + a}{128x(4ac - b^2)} \left(3616 a^{7/2} x^5 b c^3 - 3360 a^{5/2} x^5 b^3 c^2 + 630 a^{3/2} x^5 b^5 c - 960 a^{9/2} x^4 c^3 + 5408 a^{7/2} x^4 b^2 c^2 - 3780 a^{5/2} x^4 b^4 c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2), x)

[Out] $-1/128/x*(c*x^2+b*x+a)*(3616*a^(7/2)*x^5*b*c^3-3360*a^(5/2)*x^5*b^3*c^2+630*a^(3/2)*x^5*b^5*c-960*a^(9/2)*x^4*c^3+5408*a^(7/2)*x^4*b^2*c^2-3780*a^(5/2)*x^4*b^4*c+630*a^(3/2)*x^4*b^6+960*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^4*c^3-3600*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^3*b^2*c^2+2100*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^2*b^4*c-315*\ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^6+1568*a^(9/2)*x^3*b*c^2-1232*a^(7/2)*x^3*b^3*c+210*a^(5/2)*x^3*b^5-320*a^(11/2)*x^2*c^2+416*a^(9/2)*x^2*b^2*c-84*a^(7/2)*x^2*b^4-192*a^(11/2)*x*b*c+48*a^(9/2)*x*b^3+128*a^(13/2)*c-32*a^(11/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(13/2)/(4*a*c-b^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2), x, algorithm="maxima")

[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)

Fricas [A] time = 4.08513, size = 1901, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")

[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2))*(b*x + 2*a)*sqrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (x^2 (a + bx + cx^2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)

3.65 $\int x^m (ax + bx^3 + cx^5) dx$

Optimal. Leaf size=37

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

[Out] $(a*x^{(2+m)})/(2+m) + (b*x^{(4+m)})/(4+m) + (c*x^{(6+m)})/(6+m)$

Rubi [A] time = 0.0129212, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^{m+2}}{m+2} + \frac{bx^{m+4}}{m+4} + \frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^m*(a*x + b*x^3 + c*x^5), x]$

[Out] $(a*x^{(2+m)})/(2+m) + (b*x^{(4+m)})/(4+m) + (c*x^{(6+m)})/(6+m)$

Rule 14

$\text{Int}[(u_*)*((c_*)*(x_*)^{(m_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}[\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*)*(v_*)] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rubi steps

$$\begin{aligned} \int x^m (ax + bx^3 + cx^5) dx &= \int (ax^{1+m} + bx^{3+m} + cx^{5+m}) dx \\ &= \frac{ax^{2+m}}{2+m} + \frac{bx^{4+m}}{4+m} + \frac{cx^{6+m}}{6+m} \end{aligned}$$

Mathematica [A] time = 0.0288071, size = 34, normalized size = 0.92

$$x^{m+2} \left(\frac{a}{m+2} + \frac{bx^2}{m+4} + \frac{cx^4}{m+6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a*x + b*x^3 + c*x^5),x]

[Out] $x^{(2+m)}*(a/(2+m) + (b*x^2)/(4+m) + (c*x^4)/(6+m))$

Maple [B] time = 0.003, size = 77, normalized size = 2.1

$$\frac{x^{2+m} (cm^2x^4 + 6cmx^4 + bm^2x^2 + 8cx^4 + 8bmx^2 + am^2 + 12bx^2 + 10am + 24a)}{(6+m)(4+m)(2+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(c*x^5+b*x^3+a*x),x)

[Out] $x^{(2+m)}*(c*m^2*x^4+6*c*m*x^4+b*m^2*x^2+8*c*x^4+8*b*m*x^2+a*m^2+12*b*x^2+10*a*m+24*a)/(6+m)/(4+m)/(2+m)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.31314, size = 161, normalized size = 4.35

$$\frac{((cm^2 + 6cm + 8c)x^6 + (bm^2 + 8bm + 12b)x^4 + (am^2 + 10am + 24a)x^2)x^m}{m^3 + 12m^2 + 44m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m + 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)$

Sympy [A] time = 1.09027, size = 280, normalized size = 7.57

$$\left\{ \begin{array}{l} -\frac{a}{4x^4} - \frac{b}{2x^2} + c \log(x) \\ -\frac{a}{2x^2} + b \log(x) + \frac{cx^2}{2} \\ a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48} + \frac{10amx^2x^m}{m^3+12m^2+44m+48} + \frac{24ax^2x^m}{m^3+12m^2+44m+48} + \frac{bm^2x^4x^m}{m^3+12m^2+44m+48} + \frac{8bmx^4x^m}{m^3+12m^2+44m+48} + \frac{12bx^4x^m}{m^3+12m^2+44m+48} + \frac{cm^2x^6x^m}{m^3+12m^2+44m+48} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)

[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) + b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m**2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))

Giac [B] time = 1.11225, size = 144, normalized size = 3.89

$$\frac{cm^2x^6x^m + 6cmx^6x^m + bm^2x^4x^m + 8cx^6x^m + 8bmx^4x^m + am^2x^2x^m + 12bx^4x^m + 10amx^2x^m + 24ax^2x^m}{m^3 + 12m^2 + 44m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $(c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12*m^2 + 44*m + 48)$

3.66 $\int x^2 (ax + bx^3 + cx^5) dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

Rubi [A] time = 0.0078675, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3 + c*x^5), x]

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x^2 (ax + bx^3 + cx^5) dx &= \int (ax^3 + bx^5 + cx^7) dx \\ &= \frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8} \end{aligned}$$

Mathematica [A] time = 0.0019507, size = 25, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8

Maple [A] time = 0.002, size = 20, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^5+b*x^3+a*x),x)

[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8

Maxima [A] time = 1.12666, size = 26, normalized size = 1.04

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4

Fricas [A] time = 1.0509, size = 47, normalized size = 1.88

$$\frac{1}{8}x^8c + \frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $1/8*x^8*c + 1/6*x^6*b + 1/4*x^4*a$

Sympy [A] time = 0.056171, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**5+b*x**3+a*x),x)`

[Out] $a*x**4/4 + b*x**6/6 + c*x**8/8$

Giac [A] time = 1.09845, size = 26, normalized size = 1.04

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")`

[Out] $1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4$

3.67 $\int x(ax + bx^3 + cx^5) dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

Rubi [A] time = 0.0072531, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {14}

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3 + c*x^5), x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x(ax + bx^3 + cx^5) dx &= \int (ax^2 + bx^4 + cx^6) dx \\ &= \frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7} \end{aligned}$$

Mathematica [A] time = 0.0018573, size = 25, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^5+b*x^3+a*x),x)

[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7

Maxima [A] time = 1.072, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3

Fricas [A] time = 1.08682, size = 47, normalized size = 1.88

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a$

Sympy [A] time = 0.056432, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x**5+b*x**3+a*x),x)`

[Out] $a*x**3/3 + b*x**5/5 + c*x**7/7$

Giac [A] time = 1.0907, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{5}bx^5 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")`

[Out] $1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3$

$$3.68 \quad \int (ax + bx^3 + cx^5) dx$$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6

Rubi [A] time = 0.0044085, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Int[a*x + b*x^3 + c*x^5,x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6

Rubi steps

$$\int (ax + bx^3 + cx^5) dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A] time = 0.0000472, size = 25, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

[In] Integrate[a*x + b*x^3 + c*x^5,x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(c*x^5+b*x^3+a*x,x)`

[Out] `1/2*a*x^2+1/4*b*x^4+1/6*c*x^6`

Maxima [A] time = 1.11375, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")`

[Out] `1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2`

Fricas [A] time = 1.04867, size = 47, normalized size = 1.88

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")`

[Out] `1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a`

Sympy [A] time = 0.055644, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Giac [A] time = 1.10021, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

$$3.69 \quad \int \frac{ax+bx^3+cx^5}{x} dx$$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

Rubi [A] time = 0.0056522, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x} dx &= \int (a+bx^2+cx^4) dx \\ &= ax + \frac{bx^3}{3} + \frac{cx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.0011684, size = 20, normalized size = 1.

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5

Maple [A] time = 0., size = 17, normalized size = 0.9

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x,x)

[Out] a*x+1/3*b*x^3+1/5*c*x^5

Maxima [A] time = 1.13096, size = 22, normalized size = 1.1

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x

Fricas [A] time = 1.22959, size = 39, normalized size = 1.95

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")

[Out] $1/5*c*x^5 + 1/3*b*x^3 + a*x$

Sympy [A] time = 0.055914, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)/x,x)`

[Out] $a*x + b*x**3/3 + c*x**5/5$

Giac [A] time = 1.09995, size = 22, normalized size = 1.1

$$\frac{1}{5}cx^5 + \frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")`

[Out] $1/5*c*x^5 + 1/3*b*x^3 + a*x$

$$3.70 \quad \int \frac{ax+bx^3+cx^5}{x^2} dx$$

Optimal. Leaf size=21

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Rubi [A] time = 0.0081022, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x^2} dx &= \int \left(\frac{a}{x} + bx + cx^3 \right) dx \\ &= \frac{bx^2}{2} + \frac{cx^4}{4} + a \log(x) \end{aligned}$$

Mathematica [A] time = 0.001807, size = 21, normalized size = 1.

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{bx^2}{2} + \frac{cx^4}{4} + a \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x^2,x)

[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)

Maxima [A] time = 1.14202, size = 23, normalized size = 1.1

$$\frac{1}{4} cx^4 + \frac{1}{2} bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)

Fricas [A] time = 1.22428, size = 46, normalized size = 2.19

$$\frac{1}{4} cx^4 + \frac{1}{2} bx^2 + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")

[Out] $1/4*c*x^4 + 1/2*b*x^2 + a*\log(x)$

Sympy [A] time = 0.089078, size = 17, normalized size = 0.81

$$a \log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)/x**2,x)`

[Out] $a*\log(x) + b*x**2/2 + c*x**4/4$

Giac [A] time = 1.1209, size = 27, normalized size = 1.29

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + \frac{1}{2}a\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")`

[Out] $1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*\log(x^2)$

$$3.71 \quad \int \frac{ax+bx^3+cx^5}{x^3} dx$$

Optimal. Leaf size=18

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

[Out] $-(a/x) + b*x + (c*x^3)/3$

Rubi [A] time = 0.0068819, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^3 + c*x^5)/x^3, x]$

[Out] $-(a/x) + b*x + (c*x^3)/3$

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned} \int \frac{ax + bx^3 + cx^5}{x^3} dx &= \int \left(b + \frac{a}{x^2} + cx^2 \right) dx \\ &= -\frac{a}{x} + bx + \frac{cx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0020158, size = 18, normalized size = 1.

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)/x^3,x)

[Out] -a/x+b*x+1/3*c*x^3

Maxima [A] time = 1.11163, size = 22, normalized size = 1.22

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")

[Out] 1/3*c*x^3 + b*x - a/x

Fricas [A] time = 1.19098, size = 42, normalized size = 2.33

$$\frac{cx^4 + 3bx^2 - 3a}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")

[Out] $1/3*(c*x^4 + 3*b*x^2 - 3*a)/x$

Sympy [A] time = 0.247622, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)

[Out] -a/x + b*x + c*x**3/3

Giac [A] time = 1.09081, size = 22, normalized size = 1.22

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")

[Out] 1/3*c*x^3 + b*x - a/x

3.72 $\int x^m (ax + bx^3 + cx^5)^2 dx$

Optimal. Leaf size=76

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7} (2ac + b^2)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

[Out] $(a^2 x^{3+m})/(3+m) + (2*a*b*x^{5+m})/(5+m) + ((b^2 + 2*a*c)*x^{7+m})/(7+m) + (2*b*c*x^{9+m})/(9+m) + (c^2*x^{11+m})/(11+m)$

Rubi [A] time = 0.0458617, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1585, 1108}

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7} (2ac + b^2)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]

[Out] $(a^2 x^{3+m})/(3+m) + (2*a*b*x^{5+m})/(5+m) + ((b^2 + 2*a*c)*x^{7+m})/(7+m) + (2*b*c*x^{9+m})/(9+m) + (c^2*x^{11+m})/(11+m)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1108

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d*x)^(m)*(a+b*x^2+c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m+1)/2]

Rubi steps

$$\begin{aligned}
\int x^m (ax + bx^3 + cx^5)^2 dx &= \int x^{2+m} (a + bx^2 + cx^4)^2 dx \\
&= \int (a^2 x^{2+m} + 2abx^{4+m} + (b^2 + 2ac)x^{6+m} + 2bcx^{8+m} + c^2 x^{10+m}) dx \\
&= \frac{a^2 x^{3+m}}{3+m} + \frac{2abx^{5+m}}{5+m} + \frac{(b^2 + 2ac)x^{7+m}}{7+m} + \frac{2bcx^{9+m}}{9+m} + \frac{c^2 x^{11+m}}{11+m}
\end{aligned}$$

Mathematica [A] time = 0.0787956, size = 69, normalized size = 0.91

$$x^{m+3} \left(\frac{a^2}{m+3} + \frac{x^4(2ac+b^2)}{m+7} + \frac{2abx^2}{m+5} + \frac{2bcx^6}{m+9} + \frac{c^2 x^8}{m+11} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]

[Out] x^(3+m)*(a^2/(3+m) + (2*a*b*x^2)/(5+m) + ((b^2 + 2*a*c)*x^4)/(7+m) + (2*b*c*x^6)/(9+m) + (c^2*x^8)/(11+m))

Maple [B] time = 0.006, size = 300, normalized size = 4.

$$x^{3+m} (c^2 m^4 x^8 + 24 c^2 m^3 x^8 + 2 b c m^4 x^6 + 206 c^2 m^2 x^8 + 52 b c m^3 x^6 + 744 c^2 m x^8 + 2 a c m^4 x^4 + b^2 m^4 x^4 + 472 b c m^2 x^6 + 94$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(c*x^5+b*x^3+a*x)^2,x)

[Out] x^(3+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^4*x^6+206*c^2*m^2*x^8+52*b*c*m^3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+472*b*c*m^2*x^6+945*c^2*x^8+56*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+2*a*b*m^4*x^2+548*a*c*m^2*x^4+274*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+374*a^2*m^2+4158*a*b*x^2+1888*a^2*m+3465*a^2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.4098, size = 602, normalized size = 7.92

$$\left((c^2m^4 + 24c^2m^3 + 206c^2m^2 + 744c^2m + 945c^2)x^{11} + 2(bcm^4 + 26bcm^3 + 236bcm^2 + 886bcm + 1155bc)x^9 + (b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")`

[Out]
$$\frac{((c^2m^4 + 24c^2m^3 + 206c^2m^2 + 744c^2m + 945c^2)x^{11} + 2(bcm^4 + 26bcm^3 + 236bcm^2 + 886bcm + 1155bc)x^9 + (b^2 + 2ac)m^4 + 28(b^2 + 2ac)m^3 + 274(b^2 + 2ac)m^2 + 1485b^2 + 2970ac + 1092(b^2 + 2ac)m)x^7 + 2(abm^4 + 30abm^3 + 320abm^2 + 1410abm + 2079ab)x^5 + (a^2m^4 + 32a^2m^3 + 374a^2m^2 + 1888a^2m + 3465a^2)x^3)x^m}{(m^5 + 35m^4 + 470m^3 + 3010m^2 + 9129m + 10395)}$$

Sympy [A] time = 3.99035, size = 1377, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)`

[Out]
$$\text{Piecewise}\left(\left(-a^2/(8x^{**8}) - ab/(3x^{**6}) - ac/(2x^{**4}) - b^2/(4x^{**4}) - bc/x^{**2} + c^2\log(x), \text{Eq}(m, -11)\right), \left(-a^2/(6x^{**6}) - ab/(2x^{**4}) - ac/x^{**2} - b^2/(2x^{**2}) + 2bc\log(x) + c^2x^{**2}/2, \text{Eq}(m, -9)\right), \left(-a^2/(4x^{**4}) - ab/x^{**2} + 2ac\log(x) + b^2\log(x) + bcx^{**2} + c^2x^{**4}/4, \text{Eq}(m, -7)\right), \left(-a^2/(2x^{**2}) + 2ab\log(x) + acx^{**2} + b^2x^{**2}/2 + bcx^{**4}/2 + c^2x^{**6}/6, \text{Eq}(m, -5)\right), \left(a^2\log(x) + abx^{**2} + acx^{**4}/2 + b^2x^{**4}/4 + bcx^{**6}/3 + c^2x^{**8}/8, \text{Eq}(m, -3)\right), \left(a^2m^{**4}x^{**3}x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 32a^2m^{**3}x^{**3}x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 374a^2m^{**2}x^{**3}x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 1155abx^{**2}x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 236bcm^2x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 26bcm^3x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 2bcm^4x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + (b^2 + 2ac)m^4x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 28(b^2 + 2ac)m^3x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 274(b^2 + 2ac)m^2x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 1485b^2x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 2970acx^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 1155bcx^9x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + 2(bcm^4 + 26bcm^3 + 236bcm^2 + 886bcm + 1155bc)x^9x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395) + (c^2m^4 + 24c^2m^3 + 206c^2m^2 + 744c^2m + 945c^2)x^{11}x^{**m}/(m^{**5} + 35m^{**4} + 470m^{**3} + 3010m^{**2} + 9129m + 10395)\right)$$

```

*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x
**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a*
*2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a
*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 30
10*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m
/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x
**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1
485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129
*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010
*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))

```

Giac [B] time = 1.1273, size = 539, normalized size = 7.09

$$c^2 m^4 x^{11} x^m + 24 c^2 m^3 x^{11} x^m + 2 b c m^4 x^9 x^m + 206 c^2 m^2 x^{11} x^m + 52 b c m^3 x^9 x^m + 744 c^2 m x^{11} x^m + b^2 m^4 x^7 x^m + 2 a c m^4 x^7 x^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x
^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c
*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x^m
```

$$\begin{aligned} &+ 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2*m^2 \\ &*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m + 10 \\ &92*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2*x^5*x \\ &^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820*a*b*m* \\ &x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x^m + 346 \\ &5*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395) \end{aligned}$$

3.73 $\int x^2 (ax + bx^3 + cx^5)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^2*x^13)/13

Rubi [A] time = 0.0360609, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1585, 1108}

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^2*x^13)/13

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1108

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}\int x^2 (ax + bx^3 + cx^5)^2 dx &= \int x^4 (a + bx^2 + cx^4)^2 dx \\ &= \int (a^2x^4 + 2abx^6 + (b^2 + 2ac)x^8 + 2bcx^{10} + c^2x^{12}) dx \\ &= \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}\end{aligned}$$

Mathematica [A] time = 0.0070494, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9(2ac + b^2) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^2*x^13)/13

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{(2ac + b^2)x^9}{9} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13

Maxima [A] time = 1.11908, size = 59, normalized size = 1.09

$$\frac{1}{13}c^2x^{13} + \frac{2}{11}bcx^{11} + \frac{1}{9}(b^2 + 2ac)x^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $1/13*c^2*x^{13} + 2/11*b*c*x^{11} + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5$

Fricas [A] time = 1.01494, size = 117, normalized size = 2.17

$$\frac{1}{13}x^{13}c^2 + \frac{2}{11}x^{11}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")`

[Out] $1/13*x^{13}*c^2 + 2/11*x^{11}*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 2/7*x^7*b*a + 1/5*x^5*a^2$

Sympy [A] time = 0.069509, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)`

[Out] $a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9 + b**2/9)$

Giac [A] time = 1.08607, size = 62, normalized size = 1.15

$$\frac{1}{13}c^2x^{13} + \frac{2}{11}bcx^{11} + \frac{1}{9}b^2x^9 + \frac{2}{9}acx^9 + \frac{2}{7}abx^7 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")`

[Out] $1/13*c^2*x^{13} + 2/11*b*c*x^{11} + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5$

3.74 $\int x(ax + bx^3 + cx^5)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4} + \frac{1}{8}x^8(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12}$$

[Out] $(a^2x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^{10})/5 + (c^2*x^{12})/12$

Rubi [A] time = 0.0544296, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1585, 1114, 631}

$$\frac{a^2x^4}{4} + \frac{1}{8}x^8(2ac + b^2) + \frac{1}{3}abx^6 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]

[Out] $(a^2x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^{10})/5 + (c^2*x^{12})/12$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 631

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq

$Q[a, 0])$

Rubi steps

$$\begin{aligned} \int x(ax + bx^3 + cx^5)^2 dx &= \int x^3(a + bx^2 + cx^4)^2 dx \\ &= \frac{1}{2} \text{Subst}\left(\int x(a + bx + cx^2)^2 dx, x, x^2\right) \\ &= \frac{1}{2} \text{Subst}\left(\int (a^2x + 2abx^2 + (b^2 + 2ac)x^3 + 2bcx^4 + c^2x^5) dx, x, x^2\right) \\ &= \frac{a^2x^4}{4} + \frac{1}{3}abx^6 + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{5}bcx^{10} + \frac{c^2x^{12}}{12} \end{aligned}$$

Mathematica [A] time = 0.0092113, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4(30a^2 + 15x^4(2ac + b^2) + 40abx^2 + 24bcx^6 + 10c^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)/120

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{(2ac + b^2)x^8}{8} + \frac{bcx^{10}}{5} + \frac{c^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12

Maxima [A] time = 1.0896, size = 59, normalized size = 1.09

$$\frac{1}{12}c^2x^{12} + \frac{1}{5}bcx^{10} + \frac{1}{8}(b^2 + 2ac)x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4

Fricas [A] time = 1.1233, size = 116, normalized size = 2.15

$$\frac{1}{12}x^{12}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{8}x^8b^2 + \frac{1}{4}x^8ca + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/12*x^12*c^2 + 1/5*x^10*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 1/3*x^6*b*a + 1/4*x^4*a^2

Sympy [A] time = 0.069945, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{bcx^{10}}{5} + \frac{c^2x^{12}}{12} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)

[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2/8)

Giac [A] time = 1.08953, size = 62, normalized size = 1.15

$$\frac{1}{12}c^2x^{12} + \frac{1}{5}bcx^{10} + \frac{1}{8}b^2x^8 + \frac{1}{4}acx^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.75 $\int (ax + bx^3 + cx^5)^2 dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

Rubi [A] time = 0.0263768, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1594, 1108}

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1108

Int[((d_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d*x)^(m*(a + b*x^2 + c*x^4))^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]

Rubi steps

$$\begin{aligned}
\int (ax + bx^3 + cx^5)^2 dx &= \int x^2 (a + bx^2 + cx^4)^2 dx \\
&= \int (a^2x^2 + 2abx^4 + (b^2 + 2ac)x^6 + 2bcx^8 + c^2x^{10}) dx \\
&= \frac{a^2x^3}{3} + \frac{2}{5}abx^5 + \frac{1}{7}(b^2 + 2ac)x^7 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}
\end{aligned}$$

Mathematica [A] time = 0.0075766, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7(2ac + b^2) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*x^11)/11

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{(2ac + b^2)x^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^2,x)

[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11

Maxima [A] time = 1.12043, size = 65, normalized size = 1.2

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{1}{3}a^2x^3 + \frac{2}{35}(5cx^7 + 7bx^5)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{11}c^2x^{11} + \frac{2}{9}b^2cx^9 + \frac{1}{7}b^2x^7 + \frac{1}{3}a^2x^3 + \frac{2}{35}(5cx^7 + 7b^2x^5)a$

Fricas [A] time = 1.11846, size = 115, normalized size = 2.13

$$\frac{1}{11}x^{11}c^2 + \frac{2}{9}x^9cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")`

[Out] $\frac{1}{11}x^{11}c^2 + \frac{2}{9}x^9c^2b + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7c^2a + \frac{2}{5}x^5b^2a + \frac{1}{3}x^3a^2$

Sympy [A] time = 0.069614, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**2,x)`

[Out] $a^2x^3/3 + 2*a*b*x^5/5 + 2*b*c*x^9/9 + c^2*x^{11}/11 + x^7*(2*a*c/7 + b^2/7)$

Giac [A] time = 1.09849, size = 62, normalized size = 1.15

$$\frac{1}{11}c^2x^{11} + \frac{2}{9}bcx^9 + \frac{1}{7}b^2x^7 + \frac{2}{7}acx^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")`

[Out] $\frac{1}{11}c^2x^{11} + \frac{2}{9}b^2cx^9 + \frac{1}{7}b^2x^7 + \frac{2}{7}a^2cx^7 + \frac{2}{5}a^2bx^5 + \frac{1}{3}a^2x^3$

$$3.76 \quad \int \frac{(ax+bx^3+cx^5)^2}{x} dx$$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10)/10

Rubi [A] time = 0.0456916, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1585, 1107, 611}

$$\frac{a^2x^2}{2} + \frac{1}{6}x^6(2ac + b^2) + \frac{1}{2}abx^4 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2/x, x]

[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10)/10

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 611

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,

0] && IGtQ[p, 0] && (EqQ[a, 0] || !PerfectSquareQ[b^2 - 4*a*c])

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3 + cx^5)^2}{x} dx &= \int x(a + bx^2 + cx^4)^2 dx \\
 &= \frac{1}{2} \text{Subst} \left(\int (a + bx + cx^2)^2 dx, x, x^2 \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(a^2 + 2abx + b^2 \left(1 + \frac{2ac}{b^2} \right) x^2 + 2bcx^3 + c^2x^4 \right) dx, x, x^2 \right) \\
 &= \frac{a^2x^2}{2} + \frac{1}{2}abx^4 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{4}bcx^8 + \frac{c^2x^{10}}{10}
 \end{aligned}$$

Mathematica [A] time = 0.0086665, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2(30a^2 + 10x^4(2ac + b^2) + 30abx^2 + 15bcx^6 + 6c^2x^8)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]

[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))/60

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{(2ac + b^2)x^6}{6} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^2/x,x)

[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10

Maxima [A] time = 1.02237, size = 59, normalized size = 1.09

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")

[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2

Fricas [A] time = 1.30918, size = 109, normalized size = 2.02

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}(b^2 + 2ac)x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")

[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2

Sympy [A] time = 0.069357, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)

[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/6)

Giac [A] time = 1.08928, size = 62, normalized size = 1.15

$$\frac{1}{10}c^2x^{10} + \frac{1}{4}bcx^8 + \frac{1}{6}b^2x^6 + \frac{1}{3}acx^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

$$3.77 \quad \int \frac{(ax+bx^3+cx^5)^2}{x^2} dx$$

Optimal. Leaf size=49

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

[Out] $a^2x + (2abx^3)/3 + ((b^2 + 2ac)x^5)/5 + (2bcx^7)/7 + (c^2x^9)/9$

Rubi [A] time = 0.0272706, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {1585, 1090}

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2, x]

[Out] $a^2x + (2abx^3)/3 + ((b^2 + 2ac)x^5)/5 + (2bcx^7)/7 + (c^2x^9)/9$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1090

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(ax + bx^3 + cx^5)^2}{x^2} dx &= \int (a + bx^2 + cx^4)^2 dx \\
 &= \int \left(a^2 + 2abx^2 + b^2 \left(1 + \frac{2ac}{b^2} \right) x^4 + 2bcx^6 + c^2x^8 \right) dx \\
 &= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}
 \end{aligned}$$

Mathematica [A] time = 0.0053894, size = 49, normalized size = 1.

$$a^2x + \frac{1}{5}x^5(2ac + b^2) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]

[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9

Maple [A] time = 0.001, size = 42, normalized size = 0.9

$$xa^2 + \frac{2abx^3}{3} + \frac{(2ac + b^2)x^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^2/x^2,x)

[Out] x*a^2+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9

Maxima [A] time = 1.12796, size = 55, normalized size = 1.12

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")

[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x

Fricas [A] time = 1.09929, size = 99, normalized size = 2.02

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}(b^2 + 2ac)x^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")

[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x

Sympy [A] time = 0.072256, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**2/x**2,x)

[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5)

Giac [A] time = 1.07832, size = 58, normalized size = 1.18

$$\frac{1}{9}c^2x^9 + \frac{2}{7}bcx^7 + \frac{1}{5}b^2x^5 + \frac{2}{5}acx^5 + \frac{2}{3}abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")

[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x

$$3.78 \quad \int \frac{x^8}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=100

$$\frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

[Out] $-(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)$

Rubi [A] time = 0.121738, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1585, 1114, 701, 634, 618, 206, 628}

$$\frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} + \frac{b(b^2 - 3ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x + b*x^3 + c*x^5),x]

[Out] $-(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 701

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
  ] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
  b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
  NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
  ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
  t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
  [2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
  t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
  x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
  Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
  imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
  e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{ax + bx^3 + cx^5} dx &= \int \frac{x^7}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{b}{c^2} + \frac{x}{c} + \frac{ab + (b^2 - ac)x}{c^2(a + bx + cx^2)} \right) dx, x, x^2 \right) \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{\text{Subst} \left(\int \frac{ab + (b^2 - ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2c^2} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} - \frac{(b(b^2 - 3ac)) \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} + \frac{(b^2 - ac) \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4c^3} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3} + \frac{(b(b^2 - 3ac)) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c^3} \\
&= -\frac{bx^2}{2c^2} + \frac{x^4}{4c} + \frac{b(b^2 - 3ac) \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2c^3 \sqrt{b^2 - 4ac}} + \frac{(b^2 - ac) \log(a + bx^2 + cx^4)}{4c^3}
\end{aligned}$$

Mathematica [A] time = 0.0937252, size = 93, normalized size = 0.93

$$\frac{(b^2 - ac) \log(a + bx^2 + cx^4) - \frac{2b(b^2 - 3ac) \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + cx^2 (cx^2 - 2b)}{4c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x + b*x^3 + c*x^5),x]

[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4]/(4*c^3)

Maple [A] time = 0.005, size = 142, normalized size = 1.4

$$\frac{x^4}{4c} - \frac{bx^2}{2c^2} - \frac{\ln(cx^4 + bx^2 + a)a}{4c^2} + \frac{\ln(cx^4 + bx^2 + a)b^2}{4c^3} + \frac{3ab}{2c^2} \arctan \left((2cx^2 + b) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}} - \frac{b^3}{2c^3} \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8/(c*x^5+b*x^3+a*x),x)`

[Out] $\frac{1}{4}x^4/c - \frac{1}{2}bx^2/c^2 - \frac{1}{4}c^2 \ln(c^2x^4 + b^2x^2 + a) + \frac{1}{4}c^3 \ln(c^2x^4 + b^2x^2 + a) * b^2 + \frac{3}{2}c^2 / (4ac - b^2)^{1/2} * \arctan((2cx^2 + b) / (4ac - b^2)^{1/2}) * ab - \frac{1}{2}c^3 / (4ac - b^2)^{1/2} * \arctan((2cx^2 + b) / (4ac - b^2)^{1/2}) * b^3$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^4 - 2bx^2}{4c^2} - \int \frac{(b^2-ac)x^3 + abx}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] $\frac{1}{4} * (c^2x^4 - 2b^2x^2) / c^2 - \text{integrate}(-((b^2 - a*c) * x^3 + a*b*x) / (c^2x^4 + b^2x^2 + a), x) / c^2$

Fricas [A] time = 1.329, size = 675, normalized size = 6.75

$$\left[\frac{(b^2c^2 - 4ac^3)x^4 - 2(b^3c - 4abc^2)x^2 - (b^3 - 3abc)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) + (b^4 - 5ab^2c + 4a^2c^2)}{4(b^2c^3 - 4ac^4)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")`

[Out] $\left[\frac{1}{4} * ((b^2 * c^2 - 4 * a * c^3) * x^4 - 2 * (b^3 * c - 4 * a * b * c^2) * x^2 - (b^3 - 3 * a * b * c) * \sqrt{b^2 - 4 * a * c} * \log((2 * c^2 * x^4 + 2 * b * c * x^2 + b^2 - 2 * a * c - (2 * c * x^2 + b) * \sqrt{b^2 - 4 * a * c})) / (c * x^4 + b * x^2 + a)) + (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2) * \log(c * x^4 + b * x^2 + a) / (b^2 * c^3 - 4 * a * c^4), \frac{1}{4} * ((b^2 * c^2 - 4 * a * c^3) * x^4 - 2 * (b^3 * c - 4 * a * b * c^2) * x^2 + 2 * (b^3 - 3 * a * b * c) * \sqrt{-b^2 + 4 * a * c} * \arctan(-(2 * c * x^2 + b) * \sqrt{-b^2 + 4 * a * c}) / (b^2 - 4 * a * c)) + (b^4 - 5 * a * b^2 * c + 4 * a^2 * c^2) * \log(c * x^4 + b * x^2 + a) / (b^2 * c^3 - 4 * a * c^4) \right]$

Sympy [B] time = 1.89168, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2} + \left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3} \right) \log \left(x^2 + \frac{2a^2c - ab^2 + 8ac^3 \left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3} \right) - 2b^2c^2 \left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3} \right)}{3abc - b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)

[Out] $-\frac{bx^2}{2c^2} + \frac{(-b\sqrt{-4ac+b^2})(3ac-b^2)}{4c^3(4ac-b^2)} \log(x^2 + \frac{2a^2c - ab^2 + 8ac^3(-b\sqrt{-4ac+b^2})(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{2b^2c^2(-b\sqrt{-4ac+b^2})(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3})}{3abc - b^3} + \frac{(b\sqrt{-4ac+b^2})(3ac-b^2)}{4c^3(4ac-b^2)} \log(x^2 + \frac{2a^2c - ab^2 + 8ac^3(b\sqrt{-4ac+b^2})(3ac-b^2)}{4c^3(4ac-b^2)} - \frac{ac-b^2}{4c^3})}{3abc - b^3} + \frac{x^4}{4c}$

Giac [A] time = 1.09998, size = 124, normalized size = 1.24

$$\frac{cx^4 - 2bx^2}{4c^2} + \frac{(b^2 - ac) \log(cx^4 + bx^2 + a)}{4c^3} - \frac{(b^3 - 3abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $\frac{1}{4} \frac{(cx^4 - 2bx^2)}{c^2} + \frac{1}{4} \frac{(b^2 - ac) \log(cx^4 + bx^2 + a)}{c^3} - \frac{1}{2} \frac{(b^3 - 3abc) \arctan((2cx^2 + b)/\sqrt{-b^2 + 4ac})}{(\sqrt{-b^2 + 4ac})c^3}$

$$3.79 \quad \int \frac{x^7}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

[Out] $-\left(\frac{b*x}{c^2}\right) + \frac{x^3}{3*c} + \left(\frac{b^2 - a*c - (b*(b^2 - 3*a*c))}{\text{Sqrt}[b^2 - 4*a*c]}\right) * \text{ArcTan}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*x}{\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]}\right] / \left(\text{Sqrt}[2]*c^{5/2}\right) * \text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]] + \left(\frac{b^2 - a*c + (b*(b^2 - 3*a*c))}{\text{Sqrt}[b^2 - 4*a*c]}\right) * \text{ArcTan}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*x}{\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]}\right] / \left(\text{Sqrt}[2]*c^{5/2}\right) * \text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]$

Rubi [A] time = 0.602493, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1122, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}} - \frac{bx}{c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^7/(a*x + b*x^3 + c*x^5), x]$

[Out] $-\left(\frac{b*x}{c^2}\right) + \frac{x^3}{3*c} + \left(\frac{b^2 - a*c - (b*(b^2 - 3*a*c))}{\text{Sqrt}[b^2 - 4*a*c]}\right) * \text{ArcTan}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*x}{\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]}\right] / \left(\text{Sqrt}[2]*c^{5/2}\right) * \text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]] + \left(\frac{b^2 - a*c + (b*(b^2 - 3*a*c))}{\text{Sqrt}[b^2 - 4*a*c]}\right) * \text{ArcTan}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*x}{\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]}\right] / \left(\text{Sqrt}[2]*c^{5/2}\right) * \text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]$

Rule 1585

$\text{Int}[(u_*)*(x_)^{(m_*)}*((a_*)*(x_)^{(p_*)} + (b_*)*(x_)^{(q_*)} + (c_*)*(x_)^{(r_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Int}[u*x^{(m+n*p)}*(a + b*x^{(q-p)} + c*x^{(r-p)})^n, x] /;$ FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{ax + bx^3 + cx^5} dx &= \int \frac{x^6}{a + bx^2 + cx^4} dx \\
&= \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^2)}{a+bx^2+cx^4} dx}{3c} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\int \frac{3ab+3(b^2-ac)x^2}{a+bx^2+cx^4} dx}{3c^2} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2 - ac - \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c^2} + \frac{\left(b^2 - ac + \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c^2} \\
&= -\frac{bx}{c^2} + \frac{x^3}{3c} + \frac{\left(b^2 - ac - \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2 - ac + \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.172064, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac} - ac\sqrt{b^2-4ac} + 3abc - b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^2\sqrt{b^2-4ac} - ac\sqrt{b^2-4ac} - 3abc + b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]

[Out] $-\left(\frac{bx}{c^2}\right) + \frac{x^3}{3c} + \frac{\left(-b^3 + 3ab^2c + b^2\sqrt{b^2 - 4ac} - ac\sqrt{b^2 - 4ac}\right) \text{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right]}{\left(\sqrt{2}\right)c^{5/2}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\left(b^3 - 3ab^2c + b^2\sqrt{b^2 - 4ac} - ac\sqrt{b^2 - 4ac}\right) \text{ArcTan}\left[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right]}{\left(\sqrt{2}\right)c^{5/2}\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}}$

Maple [B] time = 0.023, size = 467, normalized size = 2.3

$$\frac{x^3}{3c} - \frac{bx}{c^2} - \frac{\sqrt{2}a}{2c} \arctan\left(cx\sqrt{2}\frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2}b^2}{2c^2} \arctan\left(cx\sqrt{2}\frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right)$$


```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] time = 2.20684, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \text{RootSum}\left(t^4(256a^2c^7 - 128ab^2c^6 + 16b^4c^5) + t^2(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7) + a^5, \left(t \mapsto t \log\left(x + \sqrt{t^4(256a^2c^7 - 128ab^2c^6 + 16b^4c^5) + t^2(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7) + a^5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**5+b*x**3+a*x), x)


```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Giac [C] time = 2.48729, size = 5466, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 2*((a*c^3)^(1/4)*a*b*c^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))) - (a*
c^3)^(1/4)*a*b*c^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))) + 3*((a*c^3
)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) - ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))) + 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))) + 9*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)*a*c^3)*co
s(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2*c^2 - (a*c^3)^(3/4)
*a*c^3)*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
```


$$\begin{aligned}
& \left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|} \right)^3 \sinh\left(\frac{1}{2} \operatorname{imag_part}\left(\arcsin\left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|}\right)\right)\right) \\
& \left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|} \right)^3 + 3 \left((ac^3)^{3/4} b^2 c^2 - (ac^3)^{3/4} ac^3 \right) \cos\left(\frac{1}{4} \pi + \frac{1}{2} \operatorname{real_part}\left(\arcsin\left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|}\right)\right)\right) \\
& \sin\left(\frac{1}{4} \pi + \frac{1}{2} \operatorname{real_part}\left(\arcsin\left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|}\right)\right)\right) \sinh\left(\frac{1}{2} \operatorname{imag_part}\left(\arcsin\left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|}\right)\right)\right) \\
& \left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|} \right)^3 \log\left(-2x \left(\frac{a}{c}\right)^{1/4} \cos\left(\frac{1}{4} \pi + \frac{1}{2} \operatorname{arcsin}\left(\frac{1}{2} \sqrt{ac} \frac{b}{a|c|}\right)\right)\right) \\
& + x^2 + \sqrt{a/c} / \left(\sqrt{b^2 - 4ac} b c^5 \operatorname{abs}(c) - (b^2 - 4ac) c^6 \right) + \frac{1}{3} (c^2 x^3 - 3b c x) / c^3
\end{aligned}$$

$$3.80 \quad \int \frac{x^6}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=81

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} + \frac{x^2}{2c}$$

[Out] $x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2 * Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)$

Rubi [A] time = 0.0872205, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$, Rules used = {1585, 1114, 703, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} + \frac{x^2}{2c}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x + b*x^3 + c*x^5), x]

[Out] $x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2 * Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:= Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax + bx^3 + cx^5} dx &= \int \frac{x^5}{a + bx^2 + cx^4} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{a + bx + cx^2} dx, x, x^2 \right) \\
&= \frac{x^2}{2c} + \frac{\text{Subst} \left(\int \frac{-a-bx}{a+bx+cx^2} dx, x, x^2 \right)}{2c} \\
&= \frac{x^2}{2c} - \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} \\
&= \frac{x^2}{2c} - \frac{b \log(a + bx^2 + cx^4)}{4c^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c^2} \\
&= \frac{x^2}{2c} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2c^2 \sqrt{b^2 - 4ac}} - \frac{b \log(a + bx^2 + cx^4)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.0484484, size = 78, normalized size = 0.96

$$\frac{2(b^2-2ac) \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac-b^2}} - \frac{b \log(a + bx^2 + cx^4) + 2cx^2}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x + b*x^3 + c*x^5),x]

[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)

Maple [A] time = 0.003, size = 111, normalized size = 1.4

$$\frac{x^2}{2c} - \frac{b \ln(cx^4 + bx^2 + a)}{4c^2} - \frac{a}{c} \arctan \left((2cx^2 + b) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}} + \frac{b^2}{2c^2} \arctan \left((2cx^2 + b) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^5+b*x^3+a*x),x)

[Out] $\frac{1}{2}x^2/c - \frac{1}{4}b \ln(cx^4 + bx^2 + a)/c^2 - \frac{1}{c} \sqrt{4ac - b^2}^{(1/2)} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) + \frac{1}{2} \sqrt{c^2} \sqrt{4ac - b^2}^{(1/2)} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right) * b^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2c} - \frac{b \log(cx^4 + bx^2 + a)}{4c} - \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] $\frac{1}{2}x^2/c - \text{integrate}((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c$

Fricas [A] time = 1.27802, size = 556, normalized size = 6.86

$$\left[\frac{2(b^2c - 4ac^2)x^2 - (b^2 - 2ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^3 - 4abc) \log(cx^4 + bx^2 + a)}{4(b^2c^2 - 4ac^3)}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $\left[\frac{1}{4} * (2 * (b^2 * c - 4 * a * c^2) * x^2 - (b^2 - 2 * a * c) * \text{sqrt}(b^2 - 4 * a * c) * \log((2 * c^2 * x^4 + 2 * b * c * x^2 + b^2 - 2 * a * c + (2 * c * x^2 + b) * \text{sqrt}(b^2 - 4 * a * c)) / (c * x^4 + b * x^2 + a)) - (b^3 - 4 * a * b * c) * \log(c * x^4 + b * x^2 + a)) / (b^2 * c^2 - 4 * a * c^3), \frac{1}{4} * (2 * (b^2 * c - 4 * a * c^2) * x^2 - 2 * (b^2 - 2 * a * c) * \text{sqrt}(-b^2 + 4 * a * c) * \arctan(-(2 * c * x^2 + b) * \text{sqrt}(-b^2 + 4 * a * c) / (b^2 - 4 * a * c)) - (b^3 - 4 * a * b * c) * \log(c * x^4 + b * x^2 + a)) / (b^2 * c^2 - 4 * a * c^3) \right]$

Sympy [B] time = 1.54081, size = 316, normalized size = 3.9

$$\left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4c^2(4ac - b^2)} \right) \log \left(x^2 + \frac{-ab - 8ac^2 \left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4c^2(4ac - b^2)} \right) + 2b^2c \left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4c^2(4ac - b^2)} \right)}{2ac - b^2} \right) + \left(-\frac{b}{4c^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4c^2(4ac - b^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**5+b*x**3+a*x),x)

[Out] $(-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))* \log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*\log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) + \text{sqrt}(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)$

Giac [A] time = 1.09547, size = 101, normalized size = 1.25

$$\frac{x^2}{2c} - \frac{b \log(cx^4 + bx^2 + a)}{4c^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $1/2*x^2/c - 1/4*b*\log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*\arctan((2*c*x^2 + b)/\text{sqrt}(-b^2 + 4*a*c))/(\text{sqrt}(-b^2 + 4*a*c)*c^2)$

$$3.81 \quad \int \frac{x^5}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=179

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{x}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

Rubi [A] time = 0.22733, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1122, 1166, 205}

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right) - \left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right) + \frac{x}{c}}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}} - \sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x + b*x^3 + c*x^5),x]

[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
  :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
  x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
  2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
  ] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
  p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
  > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
  - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
  + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
  Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
  /b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{ax + bx^3 + cx^5} dx &= \int \frac{x^4}{a + bx^2 + cx^4} dx \\ &= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4} dx}{c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx}{2c} \\ &= \frac{x}{c} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.144767, size = 202, normalized size = 1.13

$$\frac{\left(b\sqrt{b^2-4ac} + 2ac - b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b\sqrt{b^2-4ac} - 2ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b^2-4ac+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b^2-4ac+b}} + \frac{x}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x + b*x^3 + c*x^5),x]

[Out] $x/c - ((-b^2 + 2*a*c + b*\sqrt{b^2 - 4*a*c})*\text{ArcTan}[(\sqrt{2}*\sqrt{c}*x)/\sqrt{b - \sqrt{b^2 - 4*a*c}}]) / (\sqrt{2}*c^{(3/2)}*\sqrt{b^2 - 4*a*c}*\sqrt{b - \sqrt{b^2 - 4*a*c}}) - ((b^2 - 2*a*c + b*\sqrt{b^2 - 4*a*c})*\text{ArcTan}[(\sqrt{2}*\sqrt{c}*x)/\sqrt{b + \sqrt{b^2 - 4*a*c}}]) / (\sqrt{2}*c^{(3/2)}*\sqrt{b^2 - 4*a*c}*\sqrt{b + \sqrt{b^2 - 4*a*c}})$

Maple [B] time = 0.013, size = 343, normalized size = 1.9

$$\frac{x}{c} - \frac{\sqrt{2}b}{2c} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} + \sqrt{2}a \arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^5+b*x^3+a*x),x)

[Out] $x/c - 1/2/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(cx*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b + 1/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(cx*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a - 1/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(cx*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2 + 1/2/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(cx*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b + 1/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(cx*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a - 1/2/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(cx*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] $x/c - \text{integrate}((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c$

Fricas [B] time = 1.41226, size = 2168, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5/(c*x^5+b*x^3+a*x),x, \text{algorithm}="fricas")$

[Out]
$$-1/2*(\sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(-2*(a*b^2 - a^2*c)*x + \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(-2*(a*b^2 - a^2*c)*x - \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})*\sqrt{-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) + \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(-2*(a*b^2 - a^2*c)*x + \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - \sqrt{1/2}*c*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(-2*(a*b^2 - a^2*c)*x - \sqrt{1/2}*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})*\sqrt{-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - 2*x)/c$$

Sympy [A] time = 1.60423, size = 129, normalized size = 0.72

$$\text{RootSum}\left(t^4(256a^2c^5 - 128ab^2c^4 + 16b^4c^3) + t^2(48a^2bc^2 - 28ab^3c + 4b^5) + a^3, \left(t \mapsto t \log\left(x + \frac{32t^3abc^4 - 8t^3b^3c^3 -}{a^2c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [C] time = 2.33583, size = 4593, normalized size = 25.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - (a*c^3)^(3/4)
*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 - 9*(a*c^3)^(3/4)*b*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))) + 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + 9*(a*c^
3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))^2 - 3*(a*c^3)^(3/4)*b*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)) ^2 - 3*(a*c^3)^(3/4)*b*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + (a*c^3)^(3
/4)*b*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + (a*c^3)^(1/4)*a*c^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - (a*c^3)^(1/4)*a*c^2*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
```


$$3.82 \quad \int \frac{x^4}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)}{4c}$$

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/(4*c)

Rubi [A] time = 0.0673125, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {1585, 1114, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{\log(a+bx^2+cx^4)}{4c}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x + b*x^3 + c*x^5), x]

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[a + b*x^2 + c*x^4]/(4*c)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{x^4}{ax + bx^3 + cx^5} dx &= \int \frac{x^3}{a + bx^2 + cx^4} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{a + bx + cx^2} dx, x, x^2 \right) \\
 &= \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4c} \\
 &= \frac{\log(a + bx^2 + cx^4)}{4c} + \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2c} \\
 &= \frac{b \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2 - 4ac}} \right)}{2c\sqrt{b^2 - 4ac}} + \frac{\log(a + bx^2 + cx^4)}{4c}
 \end{aligned}$$

Mathematica [A] time = 0.0279847, size = 62, normalized size = 0.98

$$\frac{\log(a + bx^2 + cx^4) - \frac{2b \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x + b*x^3 + c*x^5), x]

[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a + b*x^2 + c*x^4])/(4*c)

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln(cx^4 + bx^2 + a)}{4c} - \frac{b}{2c} \arctan\left((2cx^2 + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^5+b*x^3+a*x), x)

[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2/c*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x), x, algorithm="maxima")

[Out] integrate(x^4/(c*x^5 + b*x^3 + a*x), x)

Fricas [A] time = 1.31587, size = 443, normalized size = 7.03

$$\left[\frac{\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) + (b^2 - 4ac) \log(cx^4 + bx^2 + a)}{4(b^2c - 4ac^2)}, \frac{2\sqrt{-b^2 + 4ac} \arctan\left(-\frac{(2cx^2 + b)\sqrt{b^2 - 4ac}}{b^2 - 4ac}\right)}{4(b^2c - 4ac^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]

Sympy [B] time = 0.813335, size = 223, normalized size = 3.54

$$\left(\frac{b\sqrt{-4ac + b^2}}{4c(4ac - b^2)} + \frac{1}{4c} \right) \log \left(x^2 + \frac{-8ac \left(-\frac{b\sqrt{-4ac + b^2}}{4c(4ac - b^2)} + \frac{1}{4c} \right) + 2a + 2b^2 \left(-\frac{b\sqrt{-4ac + b^2}}{4c(4ac - b^2)} + \frac{1}{4c} \right)}{b} \right) + \left(\frac{b\sqrt{-4ac + b^2}}{4c(4ac - b^2)} + \frac{1}{4c} \right) \log \left(x^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**5+b*x**3+a*x),x)

[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)

Giac [A] time = 1.08947, size = 80, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{\log(cx^4 + bx^2 + a)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/4*log(c*x^4 + b*x^2 + a)/c
```

$$3.83 \quad \int \frac{x^3}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rubi [A] time = 0.0937336, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1585, 1130, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{b-\sqrt{b^2-4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x + b*x^3 + c*x^5), x]

[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1130

Int[((d_)*(x_))^(m_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2

+ q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ax + bx^3 + cx^5} dx &= \int \frac{x^2}{a + bx^2 + cx^4} dx \\ &= -\left(\frac{1}{2}\left(-1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx\right) + \frac{1}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx \\ &= -\frac{\sqrt{b - \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} + \frac{\sqrt{b + \sqrt{b^2 - 4ac}} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.0976082, size = 165, normalized size = 1.1

$$\frac{\left(\sqrt{b^2 - 4ac} - b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{\sqrt{b^2 - 4ac} + b} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3 + c*x^5),x]

[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])

Maple [A] time = 0.011, size = 208, normalized size = 1.4

$$\frac{\sqrt{2}}{2} \arctan \left(cx\sqrt{2} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} \right) \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{\sqrt{2}b}{2} \arctan \left(cx\sqrt{2} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} \right) \frac{1}{\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^5+b*x^3+a*x),x)

[Out] $\frac{1}{2} \cdot 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)})c)^{(1/2)} \cdot \arctan(cx \cdot 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)})c)^{(1/2)}) + \frac{1}{2} / (-4ac + b^2)^{(1/2)} \cdot 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)})c)^{(1/2)} \cdot \arctan(cx \cdot 2^{(1/2)} / ((b + (-4ac + b^2)^{(1/2)})c)^{(1/2)}) \cdot b - \frac{1}{2} \cdot 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)})c)^{(1/2)} \cdot \operatorname{arctanh}(cx \cdot 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)})c)^{(1/2)}) + \frac{1}{2} / (-4ac + b^2)^{(1/2)} \cdot 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)})c)^{(1/2)} \cdot \operatorname{arctanh}(cx \cdot 2^{(1/2)} / ((-b + (-4ac + b^2)^{(1/2)})c)^{(1/2)}) \cdot b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)

Fricas [B] time = 1.52182, size = 1206, normalized size = 8.04

$$\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{b + \frac{b^2c - 4ac^2}{\sqrt{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left(\frac{\sqrt{\frac{1}{2}}(b^2c - 4ac^2) \sqrt{\frac{b + \frac{b^2c - 4ac^2}{\sqrt{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}}}{\sqrt{b^2c^2 - 4ac^3}} + x \right) - \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{b + \frac{b^2c - 4ac^2}{\sqrt{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left(-\frac{\sqrt{\frac{1}{2}}(b^2c - 4ac^2) \sqrt{\frac{b + \frac{b^2c - 4ac^2}{\sqrt{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}}}{\sqrt{b^2c^2 - 4ac^3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}} + x) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b + (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}} + x) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}} + x) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}} + x) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}}/\sqrt{\frac{-(b - (b^2c - 4ac^2))}{(b^2c^2 - 4ac^3)}} + x)$

Sympy [A] time = 0.76821, size = 75, normalized size = 0.5

$\text{RootSum}\left(t^4(256a^2c^3 - 128ab^2c^2 + 16b^4c) + t^2(-16abc + 4b^3) + a, (t \mapsto t \log(64t^3ac^2 - 16t^3b^2c - 2tb + x))\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**5+b*x**3+a*x),x)

[Out] $\text{RootSum}(_t^{**4}(256*a^{**2}*c^{**3} - 128*a*b^{**2}*c^{**2} + 16*b^{**4}*c) + _t^{**2}*(-16*a*b*c + 4*b^{**3}) + a, \text{Lambda}(_t, _t*\log(64*_t^{**3}*a*c^{**2} - 16*_t^{**3}*b^{**2}*c - 2*_t*b + x)))$

Giac [C] time = 2.20602, size = 5350, normalized size = 35.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $\frac{1}{2}(3*((ac^3)^{3/4}b^2 - 4(ac^3)^{3/4}ac + (ac^3)^{3/4}\sqrt{b^2 - 4ac})\cos(5/4\pi + 1/2\text{real_part}(\arcsin(1/2\sqrt{ac}b/(a\text{abs}(c))))))^2 * \cosh(1/2\text{imag_part}(\arcsin(1/2\sqrt{ac}b/(a\text{abs}(c))))))^3 \sin(5/4\pi + 1/2\text{real_part}(\arcsin(1/2\sqrt{ac}b/(a\text{abs}(c)))))) - ((ac^3)^{3/4}b^2 - 4(a$

$$\begin{aligned}
&))^3 - 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - 4*a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) \\
&) * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^3 * \sin(1/4*\pi + 1/2 * \\
& 2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 - 3*((a*c^3)^{(3/4)}*b^2 - \\
& 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - 4*a*c}) * b) * \cos(1/4*\pi + 1/2 * \\
& \text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^3 * \cosh(1/2*\text{imag_part}(\arcsin(\\
& 1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(\\
& a*\text{abs}(c)))))) + 9*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)} * \\
& \sqrt{b^2 - 4*a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a * \\
& \text{abs}(c)))))) * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 * \sin(1/4 * \\
& \pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 * \sinh(1/2*\text{imag_par} \\
& \text{t}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) + 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(\\
& 3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - 4*a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\ar \\
& \text{csin}(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^3 * \cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c} \\
&) * b/(a*\text{abs}(c)))) * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 \\
& - 9*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - 4 * \\
& a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) * \cos \\
& h(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) * \sin(1/4*\pi + 1/2*\text{real_} \\
& \text{part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 * \sinh(1/2*\text{imag_part}(\arcsin(1/2*s \\
& \sqrt{a*c}*b/(a*\text{abs}(c))))))^2 - ((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a * \\
& c^3)^{(3/4)}*\sqrt{b^2 - 4*a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a} \\
& c)*b/(a*\text{abs}(c))))))^3 * \sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c) \\
&)))^3 + 3*((a*c^3)^{(3/4)}*b^2 - 4*(a*c^3)^{(3/4)}*a*c + (a*c^3)^{(3/4)}*\sqrt{b^2 - \\
& 4*a*c}) * b) * \cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) \\
&) * \sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^2 * \sinh(1/ \\
& 2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))^3 * \log(-2*x*(a/c)^{(1/4)} * \co \\
& s(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))) + x^2 + \sqrt{a/c}) / (a*b^ \\
& 2*c^3 - 4*a^2*c^4)
\end{aligned}$$

$$3.84 \quad \int \frac{x^2}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

[Out] $-(\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/\text{Sqrt}[b^2 - 4*a*c]$

Rubi [A] time = 0.0426447, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2/(a*x + b*x^3 + c*x^5), x]$

[Out] $-(\text{ArcTanh}[(b + 2*c*x^2)/\text{Sqrt}[b^2 - 4*a*c]])/\text{Sqrt}[b^2 - 4*a*c]$

Rule 1585

$\text{Int}[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] \rightarrow \text{Int}[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /;$ FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1107

$\text{Int}[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x] /;$ FreeQ[{a, b, c, p}, x]

Rule 618

$\text{Int}[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ax + bx^3 + cx^5} dx &= \int \frac{x}{a + bx^2 + cx^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right) \\ &= -\frac{\tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{\sqrt{b^2 - 4ac}} \end{aligned}$$

Mathematica [A] time = 0.0093676, size = 39, normalized size = 1.08

$$\frac{\tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right)}{\sqrt{4ac - b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0., size = 36, normalized size = 1.

$$\arctan \left((2cx^2 + b) \frac{1}{\sqrt{4ac - b^2}} \right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x),x)
```

[Out] $1/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `integrate(x^2/(c*x^5 + b*x^3 + a*x), x)`

Fricas [A] time = 1.53897, size = 290, normalized size = 8.06

$$\left[\frac{\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)}{2\sqrt{b^2-4ac}}, -\frac{\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{b^2-4ac} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")`

[Out] $[1/2*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*\sqrt{b^2 - 4*a*c}))/ (c*x^4 + b*x^2 + a))/\sqrt{b^2 - 4*a*c}, -\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c}/(b^2 - 4*a*c))/(b^2 - 4*a*c)]$

Sympy [B] time = 0.444931, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^2 + \frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2} + \frac{\sqrt{-\frac{1}{4ac-b^2}} \log\left(x^2 + \frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(c*x**5+b*x**3+a*x),x)`

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Giac [A] time = 1.11572, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```


$$3.85 \quad \int \frac{x}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.079619, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1585, 1093, 205}

$$\frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3 + c*x^5),x]

[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{ax + bx^3 + cx^5} dx &= \int \frac{1}{a + bx^2 + cx^4} dx \\ &= \frac{c \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{\sqrt{b^2 - 4ac}} - \frac{c \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{\sqrt{b^2 - 4ac}} \\ &= \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{b^2 - 4ac}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2}\sqrt{c} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{b^2 - 4ac}\sqrt{b + \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.0831481, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c} \left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{\sqrt{b^2 - 4ac} + b}} \right)}{\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5), x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]])/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]])/Sqrt[b + Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Maple [A] time = 0.01, size = 116, normalized size = 0.8

$$-c\sqrt{2} \arctan\left(cx\sqrt{2} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{-4ac + b^2}} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} - c\sqrt{2} \operatorname{Arctanh}\left(cx\sqrt{2} \frac{1}{\sqrt{(-b + \sqrt{-4ac + b^2})c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(c*x^5+b*x^3+a*x),x)`

[Out] $-c/(-4*a*c+b^2)^{(1/2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}-c/(-4*a*c+b^2)^{(1/2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2))}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^5 + bx^3 + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `integrate(x/(c*x^5 + b*x^3 + a*x), x)`

Fricas [B] time = 1.57106, size = 1323, normalized size = 8.82

$$-\frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{b + \frac{ab^2 - 4a^2c}{\sqrt{a^2b^2 - 4a^3c}}}{ab^2 - 4a^2c}} \log\left(2cx + \sqrt{\frac{1}{2}} \left(b^2 - 4ac - \frac{ab^3 - 4a^2bc}{\sqrt{a^2b^2 - 4a^3c}}\right) \sqrt{\frac{b + \frac{ab^2 - 4a^2c}{\sqrt{a^2b^2 - 4a^3c}}}{ab^2 - 4a^2c}}\right) + \frac{1}{2} \sqrt{\frac{1}{2}} \sqrt{\frac{b + \frac{ab^2 - 4a^2c}{\sqrt{a^2b^2 - 4a^3c}}}{ab^2 - 4a^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")`

[Out] $-1/2*\sqrt{1/2}*\sqrt{-(b + (a*b^2 - 4*a^2*c)/\sqrt{a^2*b^2 - 4*a^3*c})/(a*b^2 - 4*a^2*c)}*\log(2*c*x + \sqrt{1/2}*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/\sqrt{a^2*b^2 - 4*a^3*c}))$

```

a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))

```

Sympy [A] time = 0.896009, size = 87, normalized size = 0.58

$$\text{RootSum}\left(t^4\left(256a^3c^2 - 128a^2b^2c + 16ab^4\right) + t^2\left(-16abc + 4b^3\right) + c, \left(t \mapsto t \log\left(x + \frac{32t^3a^2bc - 8t^3ab^3 + 4tac - 2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [C] time = 1.35871, size = 1365, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*(((a*c^3)^(1/4)*b^2 - 4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))) - ((a*c^3)^(1/4)*b^2 -
4*(a*c^3)^(1/4)*a*c + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b)*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1
```

$$\begin{aligned}
& /2*\sqrt{a*c}*b/(a*abs(c))) - x)/((a/c)^{(1/4)}*\sin(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))))/(a*b^2*c - 4*a^2*c^2) + 1/2*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))))*\arctan(-((a/c)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))) - x)/((a/c)^{(1/4)}*\sin(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))))/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c))))))*\log(-2*x*(a/c)^{(1/4)}*\cos(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))) + x^2 + \sqrt{a/c})/(a*b^2*c - 4*a^2*c^2) - 1/4*(((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))) - ((a*c^3)^{(1/4)}*b^2 - 4*(a*c^3)^{(1/4)}*a*c + (a*c^3)^{(1/4)}*\sqrt{b^2 - 4*a*c})*b)*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c))))))*\log(-2*x*(a/c)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*abs(c)))) + x^2 + \sqrt{a/c})/(a*b^2*c - 4*a^2*c^2)
\end{aligned}$$

$$3.86 \quad \int \frac{1}{ax+bx^3+cx^5} dx$$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log(a+bx^2+cx^4)}{4a} + \frac{\log(x)}{a}$$

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)

Rubi [A] time = 0.0718082, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {1594, 1114, 705, 29, 634, 618, 206, 628}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} - \frac{\log(a+bx^2+cx^4)}{4a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(-1),x]

[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)

Rule 1594

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 705

Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d

$^2 - b*d*e + a*e^2$), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 628

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax + bx^3 + cx^5} dx &= \int \frac{1}{x(a + bx^2 + cx^4)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)} dx, x, x^2 \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x} dx, x, x^2 \right)}{2a} + \frac{\text{Subst} \left(\int \frac{-b-cx}{a+bx+cx^2} dx, x, x^2 \right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4a} - \frac{b \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4a} \\
&= \frac{\log(x)}{a} - \frac{\log(a + bx^2 + cx^4)}{4a} + \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{2a} \\
&= \frac{b \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log(a + bx^2 + cx^4)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0732088, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]

[Out] (4*sqrt[b^2 - 4*a*c]*Log[x] - (b + sqrt[b^2 - 4*a*c])*Log[b - sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b - sqrt[b^2 - 4*a*c])*Log[b + sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*sqrt[b^2 - 4*a*c])

Maple [A] time = 0.006, size = 66, normalized size = 1.

$$-\frac{\ln(cx^4 + bx^2 + a)}{4a} - \frac{b}{2a} \arctan\left((2cx^2 + b)\frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}} + \frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^5+b*x^3+a*x),x)

[Out] $-1/4*\ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^{(1/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})+\ln(x)/a$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} + \frac{1}{4} \log(cx^4 + bx^2 + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")`

[Out] `-integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a`

Fricas [A] time = 1.5878, size = 510, normalized size = 7.39

$$\left[\frac{\sqrt{b^2 - 4ac} b \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^2 - 4ac) \log(cx^4 + bx^2 + a) + 4(b^2 - 4ac) \log(x)}{4(ab^2 - 4a^2c)}, \frac{2\sqrt{-b^2 + 4ac}}{4(ab^2 - 4a^2c)} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")`

[Out] `[1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]`

Sympy [B] time = 2.86765, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right) \log\left(x^2 + \frac{-8a^2c \left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a} \right) + 2ab^2 \left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)} - \frac{1}{4a} \right) - 2ac + b^2}{bc} \right) + \left(\frac{b\sqrt{-4ac + b^2}}{4a(4ac - b^2)} - \frac{1}{4a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**5+b*x**3+a*x),x)

[Out] $(-b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a))\log(x^2 + (-8a^2c(-b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a)) + 2ab^2(-b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a)) - 2ac + b^2)/(bc)) + (b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a))\log(x^2 + (-8a^2c(b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a)) + 2ab^2(b\sqrt{-4ac + b^2}/(4a(4ac - b^2)) - 1/(4a)) - 2ac + b^2)/(bc)) + \log(x)/a$

Giac [A] time = 1.09554, size = 92, normalized size = 1.33

$$-\frac{b \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}} - \frac{\log(cx^4 + bx^2 + a)}{4a} + \frac{\log(x^2)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] $-1/2*b*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/(\sqrt{-b^2 + 4*a*c}*a) - 1/4*\log(c*x^4 + b*x^2 + a)/a + 1/2*\log(x^2)/a$

$$3.87 \quad \int \frac{1}{x(ax+bx^3+cx^5)} dx$$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2a}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2a}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

[Out] $-(1/(a*x)) - (\text{Sqrt}[c]*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rubi [A] time = 0.195319, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1123, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2a}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2a}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]

[Out] $-(1/(a*x)) - (\text{Sqrt}[c]*(1 + b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(1 - b/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[2]*a*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1123

```
Int[((d_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x(ax + bx^3 + cx^5)} dx &= \int \frac{1}{x^2(a + bx^2 + cx^4)} dx \\ &= -\frac{1}{ax} + \frac{\int \frac{-b-cx^2}{a+bx^2+cx^4} dx}{a} \\ &= -\frac{1}{ax} - \frac{\left(c\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2a} - \frac{\left(c\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2a} \\ &= -\frac{1}{ax} - \frac{\sqrt{c}\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2a}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2a}\sqrt{b+\sqrt{b^2-4ac}}} \end{aligned}$$

Mathematica [A] time = 0.422918, size = 191, normalized size = 1.1

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{2a} + \frac{2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]

[Out] $-(2/x + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-b + \text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/(2*a)$

Maple [A] time = 0.019, size = 232, normalized size = 1.3

$$-\frac{c\sqrt{2}}{2a} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} + \frac{c\sqrt{2}b}{2a} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{(b + \sqrt{-4ac + b^2})c}}\right) \frac{1}{\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^5+b*x^3+a*x),x)

[Out] $-1/2/a*c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})+1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b+1/2/a*c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})+1/2/a*c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b-1/a/x$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] $-\text{integrate}((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)$

Fricas [B] time = 1.62466, size = 2279, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out]
$$-1/2*(\sqrt{1/2}*a*x*\sqrt{-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x + \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))})*\sqrt{-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c)) - \sqrt{1/2}*a*x*\sqrt{-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x - \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))})*\sqrt{-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c)) + \sqrt{1/2}*a*x*\sqrt{-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x + \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))})*\sqrt{-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c)) - \sqrt{1/2}*a*x*\sqrt{-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c))*\log(-2*(b^2*c^2 - a*c^3)*x - \sqrt{1/2}*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))})*\sqrt{-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c))*\sqrt{(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))}/(a^3*b^2 - 4*a^4*c)) + 2)/(a*x)$$

Sympy [A] time = 1.80758, size = 148, normalized size = 0.85

$$\text{RootSum}\left(t^4(256a^5c^2 - 128a^4b^2c + 16a^3b^4) + t^2(48a^2bc^2 - 28ab^3c + 4b^5) + c^3, \left(t \mapsto t \log\left(x + \frac{-64t^3a^5c^2 + 48t^3a^4b^2c}{\dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**5+b*x**3+a*x),x)

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [C] time = 2.35354, size = 4581, normalized size = 26.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin(
5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - (a*c^3)^(3/4)
*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 - 9*(a*c^3)^(3/4)*a*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))) + 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + 9*(a*c^
3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^
2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c))))^2 - 3*(a*c^3)^(3/4)*a*cosh(1/2*imag_part(arcsin
(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)^2 - 3*(a*c^3)^(3/4)*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + (a*c^3)^(3
/4)*a*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))^3 + (a*c^3)^(1/4)*a*b*
c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - (a*c^3)^(1/4)*a*b*c*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1
/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2
*c - 4*a*c^2)*a^2) - 2*(3*(a*c^3)^(3/4)*a*cos(1/4*pi + 1/2*real_part(arcsin
```


$$\begin{aligned}
& /2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3 + 3*(a*c^3)^{(3/4)}*a*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sin(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3 + (a*c^3)^{(1/4)}*a*b*c*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) - (a*c^3)^{(1/4)}*a*b*c*\cos(5/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))))*\log(-2*x*(a/c)^{(1/4)}*\cos(5/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) + x^2 + \sqrt{a/c})/(\sqrt{b^2 - 4*a*c})*a*b*c*\text{abs}(a) - (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3 - 3*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2 - 3*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) + 9*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) + 3*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2 - 9*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2 - (a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3 + 3*(a*c^3)^{(3/4)}*a*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sin(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^2*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c))))^3 + (a*c^3)^{(1/4)}*a*b*c*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\cosh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) - (a*c^3)^{(1/4)}*a*b*c*\cos(1/4*\pi + 1/2*\text{real_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))*\sinh(1/2*\text{imag_part}(\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))))))*\log(-2*x*(a/c)^{(1/4)}*\cos(1/4*\pi + 1/2*\arcsin(1/2*\sqrt{a*c}*b/(a*\text{abs}(c)))) + x^2 + \sqrt{a/c})/(\sqrt{b^2 - 4*a*c})*a*b*c*\text{abs}(a) - (b^2*c - 4*a*c^2)*a^2) - 1/(a*x)
\end{aligned}$$

$$3.88 \quad \int \frac{1}{x^2(ax+bx^3+cx^5)} dx$$

Optimal. Leaf size=89

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rubi [A] time = 0.132696, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1585, 1114, 709, 800, 634, 618, 206, 628}

$$-\frac{(b^2 - 2ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{b \log(x)}{a^2} - \frac{1}{2ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]

[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a^2)

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
:> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[
1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m,
-1]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax + bx^3 + cx^5)} dx &= \int \frac{1}{x^3(a + bx^2 + cx^4)} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a + bx + cx^2)} dx, x, x^2 \right) \\
&= -\frac{1}{2ax^2} + \frac{\text{Subst} \left(\int \frac{-b-cx}{x(a+bx+cx^2)} dx, x, x^2 \right)}{2a} \\
&= -\frac{1}{2ax^2} + \frac{\text{Subst} \left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)} \right) dx, x, x^2 \right)}{2a} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{\text{Subst} \left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} dx, x, x^2 \right)}{2a^2} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4a^2} + \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{4a^2} \\
&= -\frac{1}{2ax^2} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^2 + cx^4)}{4a^2} - \frac{(b^2 - 2ac) \text{Subst} \left(\int \frac{1}{b^2-4ac-x^2} dx, x, b + 2cx^2 \right)}{2a^2} \\
&= -\frac{1}{2ax^2} - \frac{(b^2 - 2ac) \tanh^{-1} \left(\frac{b+2cx^2}{\sqrt{b^2-4ac}} \right)}{2a^2 \sqrt{b^2 - 4ac}} - \frac{b \log(x)}{a^2} + \frac{b \log(a + bx^2 + cx^4)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.124684, size = 135, normalized size = 1.52

$$\frac{\frac{(b\sqrt{b^2-4ac}-2ac+b^2) \log(-\sqrt{b^2-4ac}+b+2cx^2)}{\sqrt{b^2-4ac}} + \frac{(b\sqrt{b^2-4ac}+2ac-b^2) \log(\sqrt{b^2-4ac}+b+2cx^2)}{\sqrt{b^2-4ac}}}{4a^2} - \frac{2a}{x^2} - 4b \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]

[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)

Maple [A] time = 0.006, size = 119, normalized size = 1.3

$$\frac{b \ln(cx^4 + bx^2 + a)}{4a^2} - \frac{c}{a} \arctan\left((2cx^2 + b)\frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}} + \frac{b^2}{2a^2} \arctan\left((2cx^2 + b)\frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^5+b*x^3+a*x), x)

[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2-1/2/a/x^2-b*ln(x)/a^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b \log(x)}{a^2} + \frac{\frac{1}{4} b \log(cx^4 + bx^2 + a) + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}}}{a^2} - \frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x), x, algorithm="maxima")

[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x) /a^2 - 1/2/(a*x^2)

Fricas [A] time = 2.01744, size = 664, normalized size = 7.46

$$\left[\frac{(b^2 - 2ac)\sqrt{b^2 - 4ac}x^2 \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right) - (b^3 - 4abc)x^2 \log(cx^4 + bx^2 + a) + 4(b^3 - 4abc)x}{4(a^2b^2 - 4a^3c)x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x), x, algorithm="fricas")

[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4

$$*a*b*c)*x^2*\log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*\log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*\sqrt{-b^2 + 4*a*c})*x^2*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c})/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^2*\log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*\log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]$$

Sympy [B] time = 7.24562, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4a^2(4ac - b^2)} \right) \log \left(x^2 + \frac{-8a^3c \left(\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4a^2(4ac - b^2)} \right) + 2a^2b^2 \left(\frac{b}{4a^2} - \frac{\sqrt{-4ac + b^2}(2ac - b^2)}{4a^2(4ac - b^2)} \right) + 3abc - b^3}{2ac^2 - b^2c} \right) + \left(\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**5+b*x**3+a*x),x)

[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) - 1/(2*a*x**2) - b*log(x)/a**2

Giac [A] time = 1.09508, size = 127, normalized size = 1.43

$$\frac{b \log(cx^4 + bx^2 + a)}{4a^2} - \frac{b \log(x^2)}{2a^2} + \frac{(b^2 - 2ac) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2\sqrt{-b^2 + 4ac}a^2} + \frac{bx^2 - a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")

[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x^2 - a)/(a^2*x^2)

$$3.89 \quad \int \frac{x^{11}}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=166

$$-\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{x^2(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx^4}{2c(b^2-4ac)} - \frac{b \log(a+bx^2+cx^4)}{2c^3}$$

[Out] $((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)$

Rubi [A] time = 0.222667, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1585, 1114, 738, 800, 634, 618, 206, 628}

$$-\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}} + \frac{x^2(b^2-3ac)}{c^2(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{bx^4}{2c(b^2-4ac)} - \frac{b \log(a+bx^2+cx^4)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*(a+b*x+c*x^2)^p, x], x, x^2], x] /; Free

$Q[\{a, b, c, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 738

$\text{Int}[\frac{(d + e x)^m ((a + b x + c x^2)^p)}{x}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\frac{(d + e x)^{m-1} (d b - 2 a e + (2 c d - b e) x) (a + b x + c x^2)^{p+1}}{(p+1)(b^2 - 4 a c)}, x] + \text{Dist}[\frac{1}{(p+1)(b^2 - 4 a c)}, \text{Int}[\frac{(d + e x)^{m-2} (\text{Simp}[e(2 a e (m-1) + b d (2 p - m + 4)) - 2 c d^2 (2 p + 3) + e(b e - 2 d c)(m + 2 p + 2) x, x]}{(a + b x + c x^2)^{p+1}}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \ \text{NeQ}[2 c d - b e, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 800

$\text{Int}[\frac{(d + e x)^m ((f + g x))}{(a + b x + c x^2)}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{ExpandIntegrand}[\frac{(d + e x)^m (f + g x)}{(a + b x + c x^2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 634

$\text{Int}[\frac{(d + e x)}{(a + b x + c x^2)}, x_{\text{Symbol}}] \rightarrow \text{Dist}[\frac{(2 c d - b e)}{2 c}, \text{Int}[\frac{1}{(a + b x + c x^2)}, x], x] + \text{Dist}[\frac{e}{2 c}, \text{Int}[\frac{(b + 2 c x)}{(a + b x + c x^2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 c d - b e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4 a c]$

Rule 618

$\text{Int}[\frac{(a + b x + c x^2)^{-1}}{x}, x_{\text{Symbol}}] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[\frac{1}{\text{Simp}[b^2 - 4 a c - x^2, x]}, x], x, b + 2 c x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0]$

Rule 206

$\text{Int}[\frac{(a + b x + c x^2)^{-1}}{x}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\frac{(1 * \text{ArcTanh}[\frac{Rt[-b, 2] x}{Rt[a, 2]}])}{Rt[a, 2] * Rt[-b, 2]}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\frac{(d + e x)}{(a + b x + c x^2)}, x_{\text{Symbol}}] \rightarrow \text{Simp}[\frac{d * \text{Log}[\text{RemoveContent}[a + b x + c x^2, x]]}{b}, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 c d - b e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^9}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^4}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{x^6 (2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= \frac{x^6 (2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)} \right) dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2} dx, x, x^2 \right)}{c^2(b^2 - 4ac)} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{2c^3} \\
&= \frac{(b^2 - 3ac)x^2}{c^2(b^2 - 4ac)} - \frac{bx^4}{2c(b^2 - 4ac)} + \frac{x^6(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \log(a + bx^2 + cx^4)}{2c^3} - \frac{(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1}}{c^3(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.20864, size = 151, normalized size = 0.91

$$\frac{2(6a^2c^2 - 6ab^2c + b^4) \tan^{-1} \left(\frac{b+2cx^2}{\sqrt{4ac-b^2}} \right) + \frac{a^2c(3b-2cx^2) - ab^2(b-4cx^2) + b^4(-x^2)}{(b^2-4ac)(a+bx^2+cx^4)} - b \log(a + bx^2 + cx^4) + cx^2}{2c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (c*x^2 + (-b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +

$$\frac{2cx^2/\sqrt{-b^2+4ac}}{(-b^2+4ac)^{3/2}} - b\frac{\log[a+bx^2+cx^4]}{(2c^3)}$$

Maple [B] time = 0.013, size = 383, normalized size = 2.3

$$\frac{x^2}{2c^2} + \frac{a^2x^2}{c(cx^4+bx^2+a)(4ac-b^2)} - 2\frac{ax^2b^2}{c^2(cx^4+bx^2+a)(4ac-b^2)} + \frac{x^2b^4}{2c^3(cx^4+bx^2+a)(4ac-b^2)} - \frac{3a}{2c^2(cx^4+bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11/(c*x^5+b*x^3+a*x)^2,x)

[Out] $\frac{1}{2} \frac{1}{c^2} x^2 + \frac{1}{c} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 a^2 - 2 \frac{1}{c^2} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 a^2 b^2 + \frac{1}{2c^3} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 b^4 - \frac{3a}{2c^2} \frac{1}{(cx^4+bx^2+a)}$
 $+ \frac{1}{2} \frac{1}{c^3} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 b^4 - 3 \frac{1}{2} \frac{1}{c^2} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 a^2 b^2 + \frac{1}{2} \frac{1}{c^3} \frac{1}{(cx^4+bx^2+a)(4ac-b^2)} x^2 a^2 b^3 - 2 \frac{1}{c^2} \frac{1}{(4ac-b^2)} \ln(cx^4+bx^2+a) a^2 b + \frac{1}{2} \frac{1}{c^3} \frac{1}{(4ac-b^2)} \ln(cx^4+bx^2+a) a^2 b^3 - 6 \frac{1}{c} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) a^2 + 6 \frac{1}{c^2} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) a^2 b^2 - \frac{1}{c^3} \frac{1}{(4ac-b^2)^{3/2}} \arctan\left(\frac{2cx^2+b}{(4ac-b^2)^{1/2}}\right) a^2 b^4$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ab^3 - 3a^2bc + (b^4 - 4ab^2c + 2a^2c^2)x^2}{2(ab^2c^3 - 4a^2c^4 + (b^2c^4 - 4ac^5)x^4 + (b^3c^3 - 4abc^4)x^2)} + \frac{x^2}{2c^2} + \frac{-2 \int \frac{(b^3-4abc)x^3 + (ab^2-3a^2c)x}{cx^4+bx^2+a} dx}{b^2c^2 - 4ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $-\frac{1}{2} \frac{(ab^3 - 3a^2bc + (b^4 - 4ab^2c + 2a^2c^2)x^2)}{(ab^2c^3 - 4a^2c^4 + (b^2c^4 - 4ac^5)x^4 + (b^3c^3 - 4abc^4)x^2)} + \frac{1}{2} \frac{x^2}{c^2} + \frac{-2 \int \frac{(b^3-4abc)x^3 + (ab^2-3a^2c)x}{cx^4+bx^2+a} dx}{b^2c^2 - 4ac^3}$

Fricas [B] time = 1.67139, size = 1806, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x¹¹/(c*x⁵+b*x³+a*x)²,x, algorithm="fricas")

[Out]
$$\begin{aligned} & \left[\frac{1}{2} * \left((b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * x^6 - a * b^5 + 7 * a^2 * b^3 * c - 12 * a^3 * b * c^2 + (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * x^4 - (b^6 - 9 * a * b^4 * c + 26 * a^2 * b^2 * c^2 - 24 * a^3 * c^3) * x^2 - (a * b^4 - 6 * a^2 * b^2 * c + 6 * a^3 * c^2 + (b^4 * c - 6 * a * b^2 * c^2 + 6 * a^2 * c^3) * x^4 + (b^5 - 6 * a * b^3 * c + 6 * a^2 * b * c^2) * x^2) * \sqrt{b^2 - 4 * a * c} \right) * \log((2 * c^2 * x^4 + 2 * b * c * x^2 + b^2 - 2 * a * c + (2 * c * x^2 + b) * \sqrt{b^2 - 4 * a * c})) / (c * x^4 + b * x^2 + a) - (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2 + (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * x^4 + (b^6 - 8 * a * b^4 * c + 16 * a^2 * b^2 * c^2) * x^2) * \log(c * x^4 + b * x^2 + a) / (a * b^4 * c^3 - 8 * a^2 * b^2 * c^4 + 16 * a^3 * c^5 + (b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6) * x^4 + (b^5 * c^3 - 8 * a * b^3 * c^4 + 16 * a^2 * b * c^5) * x^2), \right. \\ & \left. \frac{1}{2} * \left((b^4 * c^2 - 8 * a * b^2 * c^3 + 16 * a^2 * c^4) * x^6 - a * b^5 + 7 * a^2 * b^3 * c - 12 * a^3 * b * c^2 + (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * x^4 - (b^6 - 9 * a * b^4 * c + 26 * a^2 * b^2 * c^2 - 24 * a^3 * c^3) * x^2 - 2 * (a * b^4 - 6 * a^2 * b^2 * c + 6 * a^3 * c^2 + (b^4 * c - 6 * a * b^2 * c^2 + 6 * a^2 * c^3) * x^4 + (b^5 - 6 * a * b^3 * c + 6 * a^2 * b * c^2) * x^2) * \sqrt{-b^2 + 4 * a * c} * \arctan(-(2 * c * x^2 + b) * \sqrt{-b^2 + 4 * a * c}) / (b^2 - 4 * a * c) - (a * b^5 - 8 * a^2 * b^3 * c + 16 * a^3 * b * c^2 + (b^5 * c - 8 * a * b^3 * c^2 + 16 * a^2 * b * c^3) * x^4 + (b^6 - 8 * a * b^4 * c + 16 * a^2 * b^2 * c^2) * x^2) * \log(c * x^4 + b * x^2 + a) / (a * b^4 * c^3 - 8 * a^2 * b^2 * c^4 + 16 * a^3 * c^5 + (b^4 * c^4 - 8 * a * b^2 * c^5 + 16 * a^2 * c^6) * x^4 + (b^5 * c^3 - 8 * a * b^3 * c^4 + 16 * a^2 * b * c^5) * x^2) \right] \end{aligned}$$

Sympy [B] time = 4.75116, size = 877, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)

[Out]
$$\begin{aligned} & \left(-b / (2 * c ** 3) - \sqrt{-(4 * a * c - b ** 2) ** 3} * (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) / (2 * c ** 3 * (64 * a ** 3 * c ** 3 - 48 * a ** 2 * b ** 2 * c ** 2 + 12 * a * b ** 4 * c - b ** 6)) \right) * \log(x ** 2 + (-5 * a ** 2 * b * c - 16 * a ** 2 * c ** 4 * (-b / (2 * c ** 3) - \sqrt{-(4 * a * c - b ** 2) ** 3} * (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) / (2 * c ** 3 * (64 * a ** 3 * c ** 3 - 48 * a ** 2 * b ** 2 * c ** 2 + 12 * a * b ** 4 * c - b ** 6))) + a * b ** 3 + 8 * a * b ** 2 * c ** 3 * (-b / (2 * c ** 3) - \sqrt{-(4 * a * c - b ** 2) ** 3} * (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) / (2 * c ** 3 * (64 * a ** 3 * c ** 3 - 48 * a ** 2 * b ** 2 * c ** 2 + 12 * a * b ** 4 * c - b ** 6))) - b ** 4 * c ** 2 * (-b / (2 * c ** 3) - \sqrt{-(4 * a * c - b ** 2) ** 3} * (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) / (2 * c ** 3 * (64 * a ** 3 * c ** 3 - 48 * a ** 2 * b ** 2 * c ** 2 + 12 * a * b ** 4 * c - b ** 6)))) / (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) + (-b / (2 * c ** 3) + \sqrt{-(4 * a * c - b ** 2) ** 3} * (6 * a ** 2 * c ** 2 - 6 * a * b ** 2 * c + b ** 4) / (2 * c ** 3 * (64 * a ** 3 * c ** 3 - 48 * a ** 2 * b ** 2 * c ** 2 + 12 * a * b ** 4 * c - b ** 6))) * \log(x ** 2 + \end{aligned}$$

$$\begin{aligned}
& (-5a^{**2}b*c - 16a^{**2}c^{**4}*(-b/(2*c^{**3}) + \text{sqrt}(-(4*a*c - b^{**2}))^{**3})*(6*a^{**2} \\
& *c^{**2} - 6*a*b^{**2}*c + b^{**4})/(2*c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b^{**2}*c^{**2} + 12*a \\
& *b^{**4}*c - b^{**6}))) + a*b^{**3} + 8*a*b^{**2}*c^{**3}*(-b/(2*c^{**3}) + \text{sqrt}(-(4*a*c - b \\
& *2)^{**3})*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(2*c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2}*b \\
& **2*c^{**2} + 12*a*b^{**4}*c - b^{**6}))) - b^{**4}*c^{**2}*(-b/(2*c^{**3}) + \text{sqrt}(-(4*a*c - \\
& b^{**2}))^{**3})*(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})/(2*c^{**3}*(64*a^{**3}*c^{**3} - 48*a^{**2} \\
& *b^{**2}*c^{**2} + 12*a*b^{**4}*c - b^{**6}))))/(6*a^{**2}*c^{**2} - 6*a*b^{**2}*c + b^{**4})) + (- \\
& 3*a^{**2}*b*c + a*b^{**3} + x^{**2}*(2*a^{**2}*c^{**2} - 4*a*b^{**2}*c + b^{**4}))/((8*a^{**2}*c^{**4} \\
& - 2*a*b^{**2}*c^{**3} + x^{**4}*(8*a*c^{**5} - 2*b^{**2}*c^{**4}) + x^{**2}*(8*a*b*c^{**4} - 2*b^{**3} \\
& *c^{**3})) + x^{**2}/(2*c^{**2}))
\end{aligned}$$

Giac [A] time = 21.8737, size = 217, normalized size = 1.31

$$\frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right) + \frac{x^2}{2c^2} + \frac{b^3x^4 - 4abcx^4 - 2a^2cx^2 - a^2b}{2(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} - \frac{b \log(cx^4 + bx^2 + a)}{2c^3}}{(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b*c*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/2*b*log(c*x^4 + b*x^2 + a)/c^3

$$3.90 \quad \int \frac{x^{10}}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=331

$$\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(3b^2}{2c^2(b^2-4ac)}$$

[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.702875, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(3b^2}{2c^2(b^2-4ac)}$$

Antiderivative was successfully verified.

[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^n, x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(
m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^8}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{x^4(10a + 3bx^2)}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{x^2(9ab + 3(3b^2 - 10ac)x^2)}{a + bx^2 + cx^4} dx}{6c(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{3a(3b^2 - 10ac) + 3b(3b^2 - 13ac)x^2}{a + bx^2 + cx^4} dx}{6c^2(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(3b^3 - 13abc - \frac{3b^4 - 19ab^2c + 20a^2c^2}{\sqrt{b^2 - 4ac}}\right)}{4c^2(b^2 - 4ac)} \\
&= \frac{(3b^2 - 10ac)x}{2c^2(b^2 - 4ac)} - \frac{bx^3}{2c(b^2 - 4ac)} + \frac{x^5(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(3b^3 - 13abc - \frac{3b^4 - 19ab^2c + 20a^2c^2}{\sqrt{b^2 - 4ac}}\right)}{2\sqrt{2}c^{5/2}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.713003, size = 327, normalized size = 0.99

$$\frac{\sqrt{2}(-20a^2c^2 + 3b^3\sqrt{b^2 - 4ac} + 19ab^2c - 13abc\sqrt{b^2 - 4ac} - 3b^4) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{2}(20a^2c^2 + 3b^3\sqrt{b^2 - 4ac} - 19ab^2c - 13abc\sqrt{b^2 - 4ac} + 3b^4) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{\sqrt{b^2 - 4ac} + b}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (4*sqrt[c]*x - (2*sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2 + 3*b^3*sqrt[b^2 - 4*a*c] - 13*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b - sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*sqrt[b - sqrt[b^2 - 4*a*c]]) - (sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*sqrt[b^2 - 4*a*c] - 13*a*b*c*sqrt[b^2 - 4*a*c])*ArcTan[(sqrt[2]*sqrt[c]*x)/sqrt[b + sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*sqrt[b + sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))

Maple [B] time = 0.033, size = 844, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{10}/(c*x^5+b*x^3+a*x)^2, x)$

[Out]
$$\begin{aligned} & x/c^2+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*x^2+a)*b^3 \\ & / (4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-1/2/c^2/(c*x^4+b*x^2 \\ & +a)*a/(4*a*c-b^2)*x*b^2-13/4/c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})* \\ & c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b+3/4/c^2/(\\ & 4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+ \\ & (-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/ \\ & ((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ & *c)^{(1/2)})*a^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^ \\ & 2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b \\ & ^2+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c \\ &)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^4+13/4/c/(4* \\ & a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b \\ & +(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b-3/4/c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ &)*b^3+5/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)} \\ & *\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a^2-19/4/c/(4* \\ & a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arcta} \\ & \operatorname{nh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b^2+3/4/c^2/(4*a*c-b^2) \\ & /(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2 \\ & ^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^4 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 - 3abc)x^3 + (ab^2 - 2a^2c)x}{2(b^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^4 + (b^3c^2 - 4abc^3)x^2)} + \frac{-\int \frac{3ab^2 - 10a^2c + (3b^3 - 13abc)x^2}{cx^4 + bx^2 + a} dx}{2(b^2c^2 - 4ac^3)} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{10}/(c*x^5+b*x^3+a*x)^2, x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b \\ & ^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*\text{integrate}(- (3*a*b^ \end{aligned}$$

$$2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3) + x/c^2$$

Fricas [B] time = 2.15809, size = 6311, normalized size = 19.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + \sqrt{1/2}*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x + 1/2*\sqrt{1/2}*(27*b^{10} - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) - \sqrt{1/2}*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*\log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x - 1/2*\sqrt{1/2}*(27*b^{10} - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^{10} - 12*a*b^4*c^{11} + 48*a^2*b^2*c^{12} - 64*a^3*c^{13})))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)) + \sqrt{1/2}*($

$$\begin{aligned}
& a^2b^2c^2 - 4a^2c^3 + (b^2c^3 - 4a^2c^4)x^4 + (b^3c^2 - 4ab^2c^3)x^2 \\
&)\sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - \\
& 12a^2b^4c^6 + 48a^2b^2c^7 - 64a^3c^8))\sqrt{((81b^8 - 918ab^6c + 3 \\
& 051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)/(b^6c^{10} - 12ab^4c^{11} \\
& + 48a^2b^2c^{12} - 64a^3c^{13})))/(b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 \\
& - 64a^3c^8))\log(-(189a^2b^6 - 1971a^3b^4c + 5625a^4b^2c^2 - 25 \\
& 00a^5c^3))x + 1/2\sqrt{1/2}(27b^{10} - 459ab^8c + 2961a^2b^6c^2 - 8 \\
& 818a^3b^4c^3 + 11360a^4b^2c^4 - 4000a^5c^5 + (3b^9c^5 - 52ab^7c^6 \\
& + 336a^2b^5c^7 - 960a^3b^3c^8 + 1024a^4b^2c^9)\sqrt{((81b^8 - 91 \\
& 8ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)/(b^6c^{10} - \\
& 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13}))}\sqrt{-(9b^7 - 105ab^5c \\
& + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 \\
& - 64a^3c^8))\sqrt{((81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 \\
& + 625a^4c^4)/(b^6c^{10} - 12ab^4c^{11} + 48a^2b^2c^{12} - 64a^3c^{13})))/(b^6c^5 - 12ab^4c^6 \\
& + 48a^2b^2c^7 - 64a^3c^8))} - \sqrt{1/2}(a^2b^2c^2 - 4a^2c^3 + (b^2c^3 - 4a^2c^4)x^4 + (b^3c^2 - 4ab^2c^3)x^2) \\
&)\sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 \\
& - 64a^3c^8))\sqrt{((81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)/(b^6c^{10} - 12ab^4c^{11} \\
& + 48a^2b^2c^{12} - 64a^3c^{13})))/(b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8))} \\
&)\log(-(189a^2b^6 - 1971a^3b^4c + 5625a^4b^2c^2 - 2500a^5c^3))x - 1/2\sqrt{1/2}(27b^{10} - 459ab^8c + 2961a^2b^6c^2 \\
& - 8818a^3b^4c^3 + 11360a^4b^2c^4 - 4000a^5c^5 + (3b^9c^5 - 52ab^7c^6 + 336a^2b^5c^7 - 960a^3b^3c^8 \\
& + 1024a^4b^2c^9)\sqrt{((81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)/(b^6c^{10} - 12ab^4c^{11} \\
& + 48a^2b^2c^{12} - 64a^3c^{13}))}\sqrt{-(9b^7 - 105ab^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (b^6c^5 - 12ab^4c^6 \\
& + 48a^2b^2c^7 - 64a^3c^8))\sqrt{((81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)/(b^6c^{10} - 12ab^4c^{11} \\
& + 48a^2b^2c^{12} - 64a^3c^{13})))/(b^6c^5 - 12ab^4c^6 + 48a^2b^2c^7 - 64a^3c^8))} + 2(3ab^2 - 10a^2c)x \\
&)/(a^2b^2c^2 - 4a^2c^3 + (b^2c^3 - 4a^2c^4)x^4 + (b^3c^2 - 4ab^2c^3)x^2)
\end{aligned}$$

Sympy [A] time = 6.66133, size = 450, normalized size = 1.36

$$\frac{x^3(3abc - b^3) + x(2a^2c - ab^2)}{8a^2c^3 - 2ab^2c^2 + x^4(8ac^4 - 2b^2c^3) + x^2(8abc^3 - 2b^3c^2)} + \text{RootSum}\left(t^4(1048576a^6c^{11} - 1572864a^5b^2c^{10} + 983040a^4b^4c^9 - 1048576a^3b^6c^8 + 1572864a^2b^8c^7 - 1048576ab^{10}c^6 + 1048576b^{12}c^5)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)

```
[Out] (x**3*(3*a*b*c - b**3) + x*(2*a**2*c - a*b**2))/(8*a**2*c**3 - 2*a*b**2*c**
2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2)) + Root
Sum(_t**4*(1048576*a**6*c**11 - 1572864*a**5*b**2*c**10 + 983040*a**4*b**4*
c**9 - 327680*a**3*b**6*c**8 + 61440*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 2
56*b**12*c**5) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840
*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**
11*c + 144*b**13) + 10000*a**7*c**2 - 4200*a**6*b**2*c + 441*a**5*b**4, Lam
bda(_t, _t*log(x + (65536*_t**3*a**4*b*c**9 - 61440*_t**3*a**3*b**3*c**8 +
21504*_t**3*a**2*b**5*c**7 - 3328*_t**3*a*b**7*c**6 + 192*_t**3*b**9*c**5 -
8000*_t*a**5*c**5 + 36160*_t*a**4*b**2*c**4 - 32476*_t*a**3*b**4*c**3 + 11
592*_t*a**2*b**6*c**2 - 1836*_t*a*b**8*c + 108*_t*b**10)/(2500*a**5*c**3 -
5625*a**4*b**2*c**2 + 1971*a**3*b**4*c - 189*a**2*b**6)))) + x/c**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.91 \quad \int \frac{x^9}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=132

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bx^2}{2c(b^2 - 4ac)} + \frac{\log(a + bx^2 + cx^4)}{4c^2}$$

[Out] $-(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^{(3/2)}) + Log[a + b*x^2 + c*x^4]/(4*c^2)$

Rubi [A] time = 0.151706, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1585, 1114, 738, 773, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{bx^2}{2c(b^2 - 4ac)} + \frac{\log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^{(3/2)}) + Log[a + b*x^2 + c*x^4]/(4*c^2)$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free

$Q[\{a, b, c, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 738

$\text{Int}[\{(d_.) + (e_.)(x_)\}^{(m_)} \{(a_.) + (b_.)(x_) + (c_.)(x_)^2\}^{(p_)}, x_Symbol] \rightarrow \text{Simp}[\{(d + e*x)^{(m - 1)}(d*b - 2*a*e + (2*c*d - b*e)*x)(a + b*x + c*x^2)^{(p + 1)}\} / \{(p + 1)(b^2 - 4*a*c)\}, x] + \text{Dist}[1 / \{(p + 1)(b^2 - 4*a*c)\}, \text{Int}[(d + e*x)^{(m - 2)} \text{Simp}[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x] * (a + b*x + c*x^2)^{(p + 1)}, x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 773

$\text{Int}[\{(d_.) + (e_.)(x_)\} \{(f_.) + (g_.)(x_)\} / \{(a_.) + (b_.)(x_) + (c_.)(x_)^2\}, x_Symbol] \rightarrow \text{Simp}[(e*g*x)/c, x] + \text{Dist}[1/c, \text{Int}[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x] / (a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 634

$\text{Int}[\{(d_.) + (e_.)(x_)\} / \{(a_.) + (b_.)(x_) + (c_.)(x_)^2\}, x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e) / (2*c), \text{Int}[1 / (a + b*x + c*x^2), x], x] + \text{Dist}[e / (2*c), \text{Int}[(b + 2*c*x) / (a + b*x + c*x^2), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ !\text{NiceSqrtQ}[b^2 - 4*a*c]$

Rule 618

$\text{Int}[\{(a_.) + (b_.)(x_) + (c_.)(x_)^2\}^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1 / \text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 206

$\text{Int}[\{(a_.) + (b_.)(x_)^2\}^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[(\text{Rt}[-b, 2]*x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[\{(d_.) + (e_.)(x_)\} / \{(a_.) + (b_.)(x_) + (c_.)(x_)^2\}, x_Symbol] \rightarrow \text{Simp}[(d * \text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]) / b, x] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^7}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^3}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{x(4a+bx)}{a+bx+cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2} dx, x, x^2 \right)}{2c(b^2 - 4ac)} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\text{Subst} \left(\int \frac{b+2cx}{a+bx+cx^2} dx, x, x^2 \right)}{4c^2} - \frac{(b(b^2 - 6ac))}{2c^2} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(a + bx^2 + cx^4)}{4c^2} + \frac{(b(b^2 - 6ac)) \text{Subst} \left(\int \frac{1}{a+bx+cx^2} dx, x, x^2 \right)}{2c^2} \\
&= -\frac{bx^2}{2c(b^2 - 4ac)} + \frac{x^4(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b(b^2 - 6ac) \tanh^{-1} \left(\frac{b+2cx}{\sqrt{b^2-4ac}} \right)}{2c^2(b^2 - 4ac)^{3/2}} + \frac{\log(a + bx^2 + cx^4)}{4c^2}
\end{aligned}$$

Mathematica [A] time = 0.185909, size = 121, normalized size = 0.92

$$\frac{2(-2a^2c+ab(b-3cx^2)+b^3x^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2b(b^2-6ac) \tan^{-1} \left(\frac{b+2cx}{\sqrt{4ac-b^2}} \right)}{(4ac-b^2)^{3/2}} + \log(a + bx^2 + cx^4)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)

Maple [A] time = 0.011, size = 222, normalized size = 1.7

$$\frac{1}{2cx^4 + 2bx^2 + 2a} \left(\frac{b(3ac - b^2)x^2}{c^2(4ac - b^2)} + \frac{a(2ac - b^2)}{c^2(4ac - b^2)} \right) + \frac{\ln(cx^4 + bx^2 + a)a}{c(4ac - b^2)} - \frac{\ln(cx^4 + bx^2 + a)b^2}{4c^2(4ac - b^2)} - 3 \frac{ab}{c(4ac - b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ab^2 - 2a^2c + (b^3 - 3abc)x^2}{2(ab^2c^2 - 4a^2c^3 + (b^2c^3 - 4ac^4)x^4 + (b^3c^2 - 4abc^3)x^2)} - \frac{-\int \frac{(b^2-4ac)x^3+abx}{cx^4+bx^2+a} dx}{b^2c - 4ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c)*x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)

Fricas [B] time = 1.38481, size = 1412, normalized size = 10.7

$$\left[\frac{2ab^4 - 12a^2b^2c + 16a^3c^2 + 2(b^5 - 7ab^3c + 12a^2bc^2)x^2 + ((b^3c - 6abc^2)x^4 + ab^3 - 6a^2bc + (b^4 - 6ab^2c)x^2)\sqrt{b^2 - 4ac}}{4(ab^4c^2 - 8a^2b^2c^3 + 16a^3c^4 + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [B] time = 3.50461, size = 745, normalized size = 5.64

$$\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) \log \left(x^2 + \frac{-32a^2c^3 \left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} + \frac{1}{4c^2} \right) + 8a^2c + 16ab^2c}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a*c - b**2)
```



```

**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x**2*(3*a
*b*c - b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3)
+ x**2*(8*a*b*c**3 - 2*b**3*c**2))

```

Giac [A] time = 25.6046, size = 205, normalized size = 1.55

$$-\frac{(b^3 - 6abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(b^2c^2 - 4ac^3)\sqrt{-b^2 + 4ac}} - \frac{b^2cx^4 - 4ac^2x^4 - b^3x^2 + 2abcx^2 - ab^2}{4(cx^4 + bx^2 + a)(b^2c^2 - 4ac^3)} + \frac{\log(cx^4 + bx^2 + a)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

$$3.92 \quad \int \frac{x^8}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

[Out] $-(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c)))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{(3/2)}*(b^2 - 4*a*c)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c)))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{(3/2)}*(b^2 - 4*a*c)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rubi [A] time = 0.525544, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1120, 1279, 1166, 205}

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}} - 6ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}} + \frac{x^3(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c)))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{(3/2)}*(b^2 - 4*a*c)*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c)))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*c^{(3/2)}*(b^2 - 4*a*c)*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,

$x] /; \text{FreeQ}\{a, b, c, m, p, q, r\}, x\} \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{PosQ}[q - p] \ \&\& \ \text{PosQ}[r - p]$

Rule 1120

$\text{Int}[(d \cdot x)^m \cdot (a + b \cdot x^2 + c \cdot x^4)^p, x_Symbol] \rightarrow -\text{Simp}[d^3 \cdot (d \cdot x)^{m-3} \cdot (2a + b \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1} / (2 \cdot (p+1) \cdot (b^2 - 4ac)), x] + \text{Dist}[d^4 / (2 \cdot (p+1) \cdot (b^2 - 4ac)), \text{Int}[(d \cdot x)^{m-4} \cdot (2a \cdot (m-3) + b \cdot (m+4p+3) \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m, 3] \ \&\& \ \text{IntegerQ}[2p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1279

$\text{Int}[(f \cdot x)^m \cdot (d + e \cdot x^2) \cdot (a + b \cdot x^2 + c \cdot x^4)^p, x_Symbol] \rightarrow \text{Simp}[(e \cdot f \cdot (f \cdot x)^{m-1} \cdot (a + b \cdot x^2 + c \cdot x^4)^{p+1}) / (c \cdot (m+4p+3)), x] - \text{Dist}[f^2 / (c \cdot (m+4p+3)), \text{Int}[(f \cdot x)^{m-2} \cdot (a + b \cdot x^2 + c \cdot x^4)^p \cdot \text{Simp}[a \cdot e \cdot (m-1) + (b \cdot e \cdot (m+2p+1) - c \cdot d \cdot (m+4p+3)) \cdot x^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{GtQ}[m, 1] \ \&\& \ \text{NeQ}[m+4p+3, 0] \ \&\& \ \text{IntegerQ}[2p] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{IntegerQ}[m])$

Rule 1166

$\text{Int}[(d + e \cdot x^2) / (a + b \cdot x^2 + c \cdot x^4), x_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4ac, 2]\}, \text{Dist}[e/2 + (2cd - be)/(2q), \text{Int}[1/(b/2 - q/2 + c \cdot x^2), x], x] + \text{Dist}[e/2 - (2cd - be)/(2q), \text{Int}[1/(b/2 + q/2 + c \cdot x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4ac]$

Rule 205

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^6}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{x^2(6a + bx^2)}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{ab + (b^2 - 6ac)x^2}{a + bx^2 + cx^4} dx}{2c(b^2 - 4ac)} \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2 - 6ac - \frac{b(b^2 - 8ac)}{\sqrt{b^2 - 4ac}}\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4c(b^2 - 4ac)} + \\
&= -\frac{bx}{2c(b^2 - 4ac)} + \frac{x^3(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\left(b^2 - 6ac - \frac{b(b^2 - 8ac)}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}} +
\end{aligned}$$

Mathematica [A] time = 0.572439, size = 282, normalized size = 1.04

$$\frac{2\sqrt{cx}(a(b-2cx^2)+b^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $\frac{(-2\sqrt{c}x(b^2x^2 + a(b - 2cx^2)))}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(\sqrt{2}(-b^3 + 8abc + b^2\sqrt{b^2 - 4ac} - 6ac\sqrt{b^2 - 4ac}))\text{ArcTan}[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}] + (\sqrt{2}(b^3 - 8abc + b^2\sqrt{b^2 - 4ac} - 6ac\sqrt{b^2 - 4ac}))\text{ArcTan}[\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}]})}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}$

Maple [B] time = 0.029, size = 602, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^8/(c*x^5+b*x^3+a*x)^2,x)$

[Out] $(-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+3/2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a-1/4/(4*a*c-b^2)/c*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3-3/2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a+1/4/(4*a*c-b^2)/c*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2+2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*a*b-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(b^2 - 2ac)x^3 + abx}{2((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4abc^2)x^2)} - \frac{\int \frac{(b^2 - 6ac)x^2 + ab}{cx^4 + bx^2 + a} dx}{2(b^2c - 4ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^8/(c*x^5+b*x^3+a*x)^2,x, \text{algorithm}="maxima")$

[Out] $-1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*\text{integrate}(-((b^2 - 6*a*c)*x^2 + a*b)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)$

Fricas [B] time = 1.72308, size = 4806, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\frac{(b^4 - 18ab^2c + 81a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))\sqrt{-(b^5 - 15ab^3c + 60a^2b^2c^2 - (b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6))\sqrt{(b^4 - 18ab^2c + 81a^2c^2)/(b^6c^6 - 12ab^4c^7 + 48a^2b^2c^8 - 64a^3c^9))}}{(b^6c^3 - 12ab^4c^4 + 48a^2b^2c^5 - 64a^3c^6)))/((b^2c^2 - 4ac^3)x^4 + ab^2c - 4a^2c^2 + (b^3c - 4ab^2c^2)x^2)$$

Sympy [A] time = 4.63166, size = 379, normalized size = 1.4

$$\frac{-abx + x^3(2ac - b^2)}{8a^2c^2 - 2ab^2c + x^4(8ac^3 - 2b^2c^2) + x^2(8abc^2 - 2b^3c)} + \text{RootSum}\left(t^4(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)

[Out]
$$\frac{-(-a*b*x + x**3*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + \text{RootSum}(_t**4*(1048576*a**6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4*c**7 - 327680*a**3*b**6*c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c + 25*a**3*b**4, \text{Lambda}(_t, _t*\log(x + (49152*_t**3*a**4*c**7 - 40960*_t**3*a**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b**6*c**4 + 64*_t**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*b**3*c**2 - 88*_t*a*b**5*c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*b**2*c + 5*a*b**4))))$$

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.93

$$\int \frac{x^7}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=78

$$\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rubi [A] time = 0.0719037, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1114, 722, 618, 206}

$$\frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 722


```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol]
:> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 - b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol]
:> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^5}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{a \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{b^2 - 4ac} \\
&= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2a) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
&= \frac{x^2(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2a \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0911856, size = 93, normalized size = 1.19

$$\frac{a(b - 2cx^2) + b^2x^2}{2c(4ac - b^2)(a + bx^2 + cx^4)} + \frac{2a \tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)

Maple [A] time = 0.009, size = 104, normalized size = 1.3

$$\frac{1}{2cx^4 + 2bx^2 + 2a} \left(-\frac{(2ac - b^2)x^2}{c(4ac - b^2)} + \frac{ab}{c(4ac - b^2)} \right) + 2 \frac{a}{(4ac - b^2)^{3/2}} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x)

[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.36568, size = 864, normalized size = 11.08

$$\left[\frac{ab^3 - 4a^2bc + (b^4 - 6ab^2c + 8a^2c^2)x^2 + 2(ac^2x^4 + abcx^2 + a^2c)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4c - 8a^2b^2c^2 + 16a^3c^3 + (b^4c^2 - 8ab^2c^3 + 16a^2c^4)x^4 + (b^5c - 8ab^3c^2 + 16a^2bc^3)x^2)} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]

Sympy [B] time = 1.74624, size = 282, normalized size = 3.62

$$-a \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x^2 + \frac{-16a^3c^2 \sqrt{\frac{1}{(4ac - b^2)^3}} + 8a^2b^2c \sqrt{\frac{1}{(4ac - b^2)^3}} - ab^4 \sqrt{\frac{1}{(4ac - b^2)^3}} + ab}{2ac} \right) + a \sqrt{\frac{1}{(4ac - b^2)^3}} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)

[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) - (-a*b + x**2*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))

Giac [A] time = 29.8736, size = 130, normalized size = 1.67

$$\frac{2 a \arctan\left(\frac{2 c x^2+b}{\sqrt{-b^2+4 a c}}\right)}{(b^2-4 a c) \sqrt{-b^2+4 a c}} - \frac{b^2 x^2-2 a c x^2+a b}{2\left(c x^4+b x^2+a\right)\left(b^2 c-4 a c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))

$$3.94 \quad \int \frac{x^6}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.357826, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1120, 1166, 205}

$$\frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b - \frac{4ac+b^2}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b\sqrt{b^2-4ac} + 4ac + b^2\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2, x]

[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,

x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1120

Int[((d_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{x^6}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^4}{(a + bx^2 + cx^4)^2} dx \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{2a - bx^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 + 4ac - b\sqrt{b^2 - 4ac}) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4(b^2 - 4ac)^{3/2}} + \frac{(b^2 + 4ac + b\sqrt{b^2 - 4ac})}{4(b^2 - 4ac)^{3/2}} \\ &= \frac{x(2a + bx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 + 4ac - b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{(b^2 + 4ac + b\sqrt{b^2 - 4ac})}{2\sqrt{2}\sqrt{c}(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.440714, size = 235, normalized size = 0.99

$$\frac{1}{4} \left(\frac{2(2ax + bx^3)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} - 4ac - b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}(b\sqrt{b^2 - 4ac} + 4ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{\sqrt{c}(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4

Maple [B] time = 0.026, size = 452, normalized size = 1.9

$$\frac{1}{cx^4 + bx^2 + a} \left(-\frac{bx^3}{8ac - 2b^2} - \frac{ax}{4ac - b^2} \right) - \frac{\sqrt{2}b}{16ac - 4b^2} \arctan \left(cx\sqrt{2} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} \right) \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x)

[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) + \frac{1}{2}*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

Fricas [B] time = 1.39542, size = 3584, normalized size = 15.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*b*x^3 + \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)}})/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*\log((3*b^2 + 4*a*c)*x + \sqrt{1/2}*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)})*\sqrt{-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)}})/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) - \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)}})/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*\log((3*b^2 + 4*a*c)*x - \sqrt{1/2}*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)})*\sqrt{-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)}})/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)) + \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)}})/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))/\sqrt{(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))}$

$$\frac{a^3c^5}{(b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)} \log((3b^2 + 4ac)x + \sqrt{1/2}(b^4 - 8ab^2c + 16a^2c^2 - 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)/\sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})) \sqrt{-(b^3 + 12ab^2c - (b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)/\sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})} - \sqrt{1/2} \left((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4ab^2c)x^2 \right) \sqrt{-(b^3 + 12ab^2c - (b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)/\sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})} / (b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4) \log((3b^2 + 4ac)x - \sqrt{1/2}(b^4 - 8ab^2c + 16a^2c^2 - 2(b^7c - 12ab^5c^2 + 48a^2b^3c^3 - 64a^3b^2c^4)/\sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})) \sqrt{-(b^3 + 12ab^2c - (b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4)/\sqrt{b^6c^2 - 12ab^4c^3 + 48a^2b^2c^4 - 64a^3c^5})} / (b^6c - 12ab^4c^2 + 48a^2b^2c^3 - 64a^3c^4) \right) + 4ax / ((b^2c - 4ac^2)x^4 + ab^2 - 4a^2c + (b^3 - 4ab^2c)x^2)$$

Sympy [A] time = 3.35899, size = 294, normalized size = 1.24

$$\frac{2ax + bx^3}{8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)} + \text{RootSum}\left(t^4(1048576a^6c^7 - 1572864a^5b^2c^6 + 983040a^4b^4c^5 - 3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)

[Out] $-(2ax + bx^3)/(8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)) + \text{RootSum}(_t^4(1048576a^6c^7 - 1572864a^5b^2c^6 + 983040a^4b^4c^5 - 327680a^3b^6c^4 + 61440a^2b^8c^3 - 6144ab^{10}c^2 + 256b^{12}c) + _t^2(-12288a^4b^2c^4 + 8192a^3b^3c^3 - 1536a^2b^5c^2 + 16b^9) + 16a^3c^2 + 24a^2b^2c + 9ab^4, \text{Lambda}(_t, _t \log(x + (16384_t^3a^3b^2c^4 - 12288_t^3a^2b^3c^3 + 3072_t^3ab^5c^2 - 256_t^3b^7c + 64_t^3a^2c^2 - 128_t^3ab^2c - 4_t^3b^4)/(4ac + 3b^2))))$

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.95 \quad \int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=75

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rubi [A] time = 0.0689017, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1114, 638, 618, 206}

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^3}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{2(b^2 - 4ac)} \\
&= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
&= \frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0693446, size = 79, normalized size = 1.05

$$\frac{2a + bx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{b \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{(4ac - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $(2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^{(3/2)}$

Maple [A] time = 0.004, size = 77, normalized size = 1.

$$\frac{-bx^2 - 2a}{(8ac - 2b^2)(cx^4 + bx^2 + a)} - b \arctan\left((2cx^2 + b)\frac{1}{\sqrt{4ac - b^2}}\right)(4ac - b^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x)

[Out] $1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.28242, size = 778, normalized size = 10.37

$$\left[\frac{2ab^2 - 8a^2c + (b^3 - 4abc)x^2 - (bcx^4 + b^2x^2 + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)}, \frac{2ab^2 - 8a^2c + (b^3 - 4abc)x^2 - (bcx^4 + b^2x^2 + ab)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac + (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*sqrt(b^2 - 4*a*c))*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]

Sympy [B] time = 1.66677, size = 267, normalized size = 3.56

$$\frac{b \sqrt{-\frac{1}{(4ac-b^2)^3}} \log \left(x^2 + \frac{-16a^2bc^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} + 8ab^3c \sqrt{-\frac{1}{(4ac-b^2)^3}} - b^5 \sqrt{-\frac{1}{(4ac-b^2)^3} + b^2}}{2bc} \right)}{2} - \frac{b \sqrt{-\frac{1}{(4ac-b^2)^3}} \log \left(x^2 + \frac{16a^2bc^2 \sqrt{-\frac{1}{(4ac-b^2)^3}} - 8ab^3c}{2} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)

[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - (2*a + b*x**2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))

Giac [A] time = 25.7126, size = 111, normalized size = 1.48

$$\frac{b \arctan \left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}} \right)}{(b^2 - 4ac)\sqrt{-b^2+4ac}} + \frac{bx^2 + 2a}{2(cx^4 + bx^2 + a)(b^2 - 4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

$$3.96 \quad \int \frac{x^4}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=221

$$\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

[Out] $-(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(2*b - \text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rubi [A] time = 0.240247, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1119, 1166, 205}

$$\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}(2b-\sqrt{b^2-4ac})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}(\sqrt{b^2-4ac}+2b)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(2*b - \text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])*ArcTan[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 1585

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n_.], x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
  :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x^2}{(a + bx^2 + cx^4)^2} dx \\
&= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\int \frac{b - 2cx^2}{a + bx^2 + cx^4} dx}{2(b^2 - 4ac)} \\
&= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\left(c \left(1 + \frac{2b}{\sqrt{b^2 - 4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{2(b^2 - 4ac)} + \frac{c(2b - \sqrt{b^2 - 4ac})}{2(b^2 - 4ac)} \\
&= -\frac{x(b + 2cx^2)}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(2b - \sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(2b + \sqrt{b^2 - 4ac})}{\sqrt{2}(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.461759, size = 222, normalized size = 1.

$$\frac{-bx - 2cx^3}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\sqrt{c}(\sqrt{b^2 - 4ac} - 2b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(\sqrt{b^2 - 4ac} + 2b) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2 - 4ac} + b}}\right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{\sqrt{b^2 - 4ac} + b}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $(-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (\text{Sqrt}[c]*(-2*b + \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(2*b + \text{Sqrt}[b^2 - 4*a*c])* \text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]) / (\text{Sqrt}[2]*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Maple [A] time = 0.063, size = 342, normalized size = 1.6

$$\frac{x}{8ac - 2b^2} \left(x^2 + \frac{1}{2c} \sqrt{-4ac + b^2} + \frac{b}{2c} \right)^{-1} + \frac{c\sqrt{2}b}{4ac - b^2} \arctan \left(cx\sqrt{2} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}} \right) \frac{1}{\sqrt{-4ac + b^2}} \frac{1}{\sqrt{(b + \sqrt{-4ac + b^2})c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x)

[Out] $1/2/(4*a*c - b^2)*x/(x^2 + 1/2*(-4*a*c + b^2)^{(1/2)}/c + 1/2/c*b) + c/(4*a*c - b^2)/(-4*a*c + b^2)^{(1/2)}*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}) + b + 1/2*c/(4*a*c - b^2)*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}) + 1/2/(4*a*c - b^2)*x/(x^2 + 1/2/c*b - 1/2*(-4*a*c + b^2)^{(1/2)}/c) + c/(4*a*c - b^2)/(-4*a*c + b^2)^{(1/2)}*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}) + b - 1/2*c/(4*a*c - b^2)*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x*2^{(1/2)}/((-b + (-4*a*c + b^2)^{(1/2)})*c)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $-1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*\text{integrate}((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)$

Fricas [B] time = 1.61236, size = 3623, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $-1/4*(4*c*x^3 + \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*\log((3*b^2*c + 4*a*c^2)*x + 1/2*\sqrt{1/2}*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}))*\sqrt{-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)) - \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*\log((3*b^2*c + 4*a*c^2)*x - 1/2*\sqrt{1/2}*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}))*\sqrt{-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)) + \sqrt{1/2}*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*\sqrt{-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*\log((3*b^2*c + 4*a*c^2)*x + 1/2*\sqrt{1/2}*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}))*\sqrt{-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))/\sqrt{a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3}})/((a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))$

$$a^4c^3)) - \sqrt{1/2} * ((b^2c - 4a^2c^2) * x^4 + a^2b^2 - 4a^2c + (b^3 - 4a^2b^2c) * x^2) * \sqrt{-(b^3 + 12a^2bc - (a^2b^6 - 12a^2b^4c + 48a^3b^2c^2 - 64a^4c^3)) / \sqrt{(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3))} / ((a^2b^6 - 12a^2b^4c + 48a^3b^2c^2 - 64a^4c^3)) * \log((3b^2c + 4a^2c^2) * x - 1/2 * \sqrt{1/2} * (b^5 - 8a^2b^3c + 16a^2b^2c^2 + (a^2b^8 - 8a^2b^6c + 128a^4b^2c^3 - 256a^5c^4) / \sqrt{(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3))} * \sqrt{-(b^3 + 12a^2bc - (a^2b^6 - 12a^2b^4c + 48a^3b^2c^2 - 64a^4c^3)) / \sqrt{(a^2b^6 - 12a^3b^4c + 48a^4b^2c^2 - 64a^5c^3))} / (a^2b^6 - 12a^2b^4c + 48a^3b^2c^2 - 64a^4c^3))) + 2bx) / ((b^2c - 4a^2c^2) * x^4 + a^2b^2 - 4a^2c + (b^3 - 4a^2b^2c) * x^2)$$

Sympy [A] time = 3.48862, size = 298, normalized size = 1.35

$$\frac{bx + 2cx^3}{8a^2c - 2ab^2 + x^4(8ac^2 - 2b^2c) + x^2(8abc - 2b^3)} + \text{RootSum}\left(t^4(1048576a^7c^6 - 1572864a^6b^2c^5 + 983040a^5b^4c^4 - 327680a^4b^6c^3 + 61440a^3b^8c^2 - 6144a^2b^{10}c + 256a^2b^{12}) + t^2(-12288a^4b^2c^4 + 8192a^3b^3c^3 - 1536a^2b^5c^2 + 16b^9) + 16a^2c^3 + 24a^2b^2c^2 + 9b^4c, \text{Lambda}(t, t \log(x + (16384t^3a^5c^4 - 8192t^3a^4b^2c^3 + 512t^3a^2b^6c - 64t^3a^2b^8 - 128t^2a^2b^2c^2 - 16t^2a^2b^3c - 4t^2b^5) / (4a^2c^2 + 3b^2c))))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)

[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 - 1572864*a**6*b**2*c**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**6*c**3 + 61440*a**3*b**8*c**2 - 6144*a**2*b**10*c + 256*a*b**12) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lambda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*b**2*c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a**2*b**8 - 128*_t**2*a**2*b**2*c**2 - 16*_t**2*a**2*b**3*c - 4*_t**2*b**5)/(4*a**2*c**2 + 3*b**2*c))))

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.97 \quad \int \frac{x^3}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=74

$$\frac{2c \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

[Out] $-(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(3/2)}$

Rubi [A] time = 0.0649478, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1107, 614, 618, 206}

$$\frac{2c \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^{(3/2)}$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(b + 2*c*x)
*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{x}{(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{c \text{Subst} \left(\int \frac{1}{a + bx + cx^2} dx, x, x^2 \right)}{b^2 - 4ac} \\
&= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(2c) \text{Subst} \left(\int \frac{1}{b^2 - 4ac - x^2} dx, x, b + 2cx^2 \right)}{b^2 - 4ac} \\
&= -\frac{b + 2cx^2}{2(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{2c \tanh^{-1} \left(\frac{b + 2cx^2}{\sqrt{b^2 - 4ac}} \right)}{(b^2 - 4ac)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0853882, size = 79, normalized size = 1.07

$$-\frac{\frac{4c \tan^{-1} \left(\frac{b + 2cx^2}{\sqrt{4ac - b^2}} \right)}{\sqrt{4ac - b^2}} + \frac{b + 2cx^2}{a + bx^2 + cx^4}}{2(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $-\left(\frac{b + 2cx^2}{a + bx^2 + cx^4} + \frac{4c \operatorname{ArcTan}\left[\frac{b + 2cx^2}{\sqrt{-b^2 + 4ac}}\right]}{\sqrt{-b^2 + 4ac}}\right) / (2(b^2 - 4ac))$

Maple [A] time = 0.004, size = 75, normalized size = 1.

$$\frac{2cx^2 + b}{(8ac - 2b^2)(cx^4 + bx^2 + a)} + 2 \frac{c}{(4ac - b^2)^{3/2}} \arctan\left(\frac{2cx^2 + b}{\sqrt{4ac - b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x)

[Out] $1/2 * (2cx^2 + b) / (4ac - b^2) / (cx^4 + bx^2 + a) + 2c / (4ac - b^2)^{(3/2)} * \arctan((2cx^2 + b) / (4ac - b^2)^{(1/2)})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.37787, size = 783, normalized size = 10.58

$$\left[\frac{b^3 - 4abc + 2(b^2c - 4ac^2)x^2 + 2(c^2x^4 + bcx^2 + ac)\sqrt{b^2 - 4ac} \log\left(\frac{2c^2x^4 + 2bcx^2 + b^2 - 2ac - (2cx^2 + b)\sqrt{b^2 - 4ac}}{cx^4 + bx^2 + a}\right)}{2(ab^4 - 8a^2b^2c + 16a^3c^2 + (b^4c - 8ab^2c^2 + 16a^2c^3)x^4 + (b^5 - 8ab^3c + 16a^2bc^2)x^2)}, -\frac{b^3 - 4abc + \dots}{2(ab^4 - \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $[-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c)*\sqrt{b^2 - 4*a*c}*\log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*\sqrt{b^2 - 4*a*c}))/c*x^4 + b*x^2 + a))/a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2, -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x^2 + a*c)*\sqrt{-b^2 + 4*a*c}*\arctan(-(2*c*x^2 + b)*\sqrt{-b^2 + 4*a*c}/(b^2 - 4*a*c)))/a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2]$

Sympy [B] time = 1.57392, size = 267, normalized size = 3.61

$$-c \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x^2 + \frac{-16a^2c^3 \sqrt{\frac{1}{(4ac - b^2)^3}} + 8ab^2c^2 \sqrt{\frac{1}{(4ac - b^2)^3}} - b^4c \sqrt{\frac{1}{(4ac - b^2)^3}} + bc}{2c^2} \right) + c \sqrt{\frac{1}{(4ac - b^2)^3}} \log \left(x^2 + \frac{-16a^2c^3 \sqrt{\frac{1}{(4ac - b^2)^3}} + 8ab^2c^2 \sqrt{\frac{1}{(4ac - b^2)^3}} - b^4c \sqrt{\frac{1}{(4ac - b^2)^3}} + bc}{2c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)

[Out] $-c*\sqrt{-1/(4*a*c - b**2)**3}*\log(x**2 + (-16*a**2*c**3*\sqrt{-1/(4*a*c - b**2)**3} + 8*a*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**3} - b**4*c*\sqrt{-1/(4*a*c - b**2)**3} + b*c)/(2*c**2)) + c*\sqrt{-1/(4*a*c - b**2)**3}*\log(x**2 + (16*a**2*c**3*\sqrt{-1/(4*a*c - b**2)**3} - 8*a*b**2*c**2*\sqrt{-1/(4*a*c - b**2)**3} + b**4*c*\sqrt{-1/(4*a*c - b**2)**3} + b*c)/(2*c**2)) + (b + 2*c*x**2)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))$

Giac [A] time = 31.5121, size = 111, normalized size = 1.5

$$\frac{2c \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{(b^2-4ac)\sqrt{-b^2+4ac}} - \frac{2cx^2+b}{2(cx^4+bx^2+a)(b^2-4ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")


```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*  
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

$$3.98 \quad \int \frac{x^2}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=252

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(-b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rubi [A] time = 0.46056, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1585, 1092, 1166, 205}

$$\frac{x(-2ac + b^2 + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(-b\sqrt{b^2 - 4ac} - 12ac + b^2) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,

$x] /; \text{FreeQ}\{a, b, c, m, p, q, r\}, x] \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p] \&\& \text{PosQ}[r - p]$

Rule 1092

$\text{Int}[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4]^{(p_)} , x_Symbol] :> -\text{Simp}[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^{(p + 1)})/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2*p]$

Rule 1166

$\text{Int}[(d_) + (e_)*(x_)^2]/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[e/2 + (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 - q/2 + c*x^2), x], x] + \text{Dist}[e/2 - (2*c*d - b*e)/(2*q), \text{Int}[1/(b/2 + q/2 + c*x^2), x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[b^2 - 4*a*c]$

Rule 205

$\text{Int}[(a_) + (b_)*(x_)^2]^{(-1)}, x_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{(a + bx^2 + cx^4)^2} dx \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\int \frac{b^2 - 2ac - 2(b^2 - 4ac) - bcx^2}{a + bx^2 + cx^4} dx}{2a(b^2 - 4ac)} \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(c(b^2 - 12ac - b\sqrt{b^2 - 4ac})) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)^{3/2}} + \frac{(c(b^2 - 12ac + b\sqrt{b^2 - 4ac})) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2 - 4ac} + cx^2} dx}{4a(b^2 - 4ac)^{3/2}} \\ &= \frac{x(b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{c}(b^2 - 12ac + b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} - \frac{\sqrt{c}(b^2 - 12ac - b\sqrt{b^2 - 4ac}) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{2\sqrt{2}a(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} \end{aligned}$$

Mathematica [A] time = 0.460747, size = 243, normalized size = 0.96

$$\frac{\frac{2x(-2ac+b^2+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]

[Out] $\left(\frac{2*x*(b^2 - 2*a*c + b*c*x^2)}{(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)} + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2 - 12*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-b^2 + 12*a*c + b*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])\right)/(4*a)$

Maple [B] time = 0.044, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(c*x^5+b*x^3+a*x)^2,x)

[Out] $-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2/c*b)*b-c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2/c*b)+1/4/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^{(1/2)}/c+1/2/c*b)*b^2-1/4*c/(4*a*c-b^2)/a^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b-3*c^2/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})+1/4*c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x^2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^{(1/2)}/c)*b+c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^{(1/2)}/c)-1/4/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a*x/(x^2+1/2/c*b-1/2*(-4*a*c+b^2)^{(1/2)}/c)*b^2+1/4*c/(4*a*c-b^2)/a^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b-3*c^2/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\operatorname{arctanh}(c*x^2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})+1/4*c/(-4*a*c+b^2)^{(1/2)}/(4*a*c-b^2)/a^2^{(1/2)}/((-b+$

$$-4*a*c+b^2)^{(1/2))*c)^{(1/2)*\operatorname{arctanh}(c*x*2^{(1/2)/((-b+(-4*a*c+b^2)^{(1/2))*c)^{(1/2))}*b^2}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] $\frac{1}{2}*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + \frac{1}{2}*\operatorname{integrate}((b*c*x^2 + b^2 - 6*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)$

Fricas [B] time = 1.76653, size = 4918, normalized size = 19.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] $\frac{1}{4}*(2*b*c*x^3 + \sqrt{1/2}*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\sqrt{(b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))})/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*\sqrt{1/2}*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*\sqrt{(b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))})*\sqrt{-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\sqrt{(b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))})/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)) - \sqrt{1/2}*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*\sqrt{-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*\sqrt{(b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))})/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*\log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*$

```

x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3)) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)

```

Sympy [A] time = 4.47327, size = 394, normalized size = 1.56

$$\frac{bcx^3 + x(-2ac + b^2)}{8a^3c - 2a^2b^2 + x^4(8a^2c^2 - 2ab^2c) + x^2(8a^2bc - 2ab^3)} + \text{RootSum}\left(t^4(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
- 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a**
9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**6*
c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**2*(
-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**
2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 +
25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*_t**3
*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c + 64*_
t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_t*a**2*b
**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*c**3 + 5*
b**4*c**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.99 \quad \int \frac{x}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=122

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)

Rubi [A] time = 0.187625, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(b^2 - 6ac) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2 - 4ac)^{3/2}} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{\log(x)}{a^2} + \frac{-2ac + b^2 + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x(a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-b^2 + 4ac - bcx}{x(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{-b^2 + 4ac}{ax} + \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{b(b^2 - 5ac) + c(b^2 - 4ac)x}{a + bx + cx^2} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{4a^2} - \frac{(b(b^2 - 6ac)) \text{Subst} \left(\int \frac{1}{b^2 - 4ac} dx, x, x^2 \right)}{4a^2} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\log(x)}{a^2} - \frac{\log(a + bx^2 + cx^4)}{4a^2} + \frac{(b(b^2 - 6ac)) \text{Subst} \left(\int \frac{1}{b^2 - 4ac} dx, x, x^2 \right)}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{b(b^2 - 6ac) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{2a^2(b^2 - 4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a + bx^2 + cx^4)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.32239, size = 207, normalized size = 1.7

$$\frac{2a(-2ac + b^2 + bcx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(b^2 \sqrt{b^2 - 4ac} - 4ac \sqrt{b^2 - 4ac} - 6abc + b^3) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-b^2 \sqrt{b^2 - 4ac} + 4ac \sqrt{b^2 - 4ac} - 6abc + b^3) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + 4$$

$$4a^2$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^3 - 6*a*b*c + b^2*sqrt[b^2 - 4*a*c] - 4*a*c*sqrt[b^2 - 4*a*c])*Log

$$\frac{[b - \sqrt{b^2 - 4ac}] + 2cx^2}{(b^2 - 4ac)^{3/2}} + \frac{((b^3 - 6abc - b^2\sqrt{b^2 - 4ac} + 4ac\sqrt{b^2 - 4ac})\log[b + \sqrt{b^2 - 4ac}] + 2cx^2)}{(b^2 - 4ac)^{3/2}} \frac{1}{(4a^2)}$$

Maple [B] time = 0.014, size = 253, normalized size = 2.1

$$\frac{bcx^2}{2a(cx^4 + bx^2 + a)(4ac - b^2)} + \frac{c}{(cx^4 + bx^2 + a)(4ac - b^2)} - \frac{b^2}{2a(cx^4 + bx^2 + a)(4ac - b^2)} - \frac{c \ln(cx^4 + bx^2 + a)}{a(4ac - b^2)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^5+b*x^3+a*x)^2,x)

[Out]
$$-1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)+1/4/a^2/(4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^2-3/a/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b*c+1/2/a^2/(4*a*c-b^2)^{(3/2)}*\arctan((2*c*x^2+b)/(4*a*c-b^2)^{(1/2)})*b^3+\ln(x)/a^2$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^2 + b^2 - 2ac}{2((ab^2c - 4a^2c^2)x^4 + a^2b^2 - 4a^3c + (ab^3 - 4a^2bc)x^2)} + \frac{-\int \frac{(b^2c - 4ac^2)x^3 + (b^3 - 5abc)x}{cx^4 + bx^2 + a} dx}{a^2b^2 - 4a^3c} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out]
$$1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + \text{integrate}(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + \log(x)/a^2$$

Fricas [B] time = 1.78416, size = 1728, normalized size = 14.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]

Sympy [B] time = 40.9679, size = 772, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)

[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2 - b**3*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 4

```

8*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(b*sqrt(-(4*a*c -
b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(6*a*b*c**2
- b**3*c)) - (-2*a*c + b**2 + b*c*x**2)/(8*a**3*c - 2*a**2*b**2 + x**4*(8*
a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + log(x)/a**2

```

Giac [A] time = 22.1448, size = 224, normalized size = 1.84

$$\frac{(b^3 - 6abc) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{2(a^2b^2 - 4a^3c)\sqrt{-b^2 + 4ac}} + \frac{b^2cx^4 - 4ac^2x^4 + b^3x^2 - 2abcx^2 + 3ab^2 - 8a^2c}{4(cx^4 + bx^2 + a)(a^2b^2 - 4a^3c)} - \frac{\log(cx^4 + bx^2 + a)}{4a^2} + \frac{\log(x^2)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] $-1/2*(b^3 - 6*a*b*c)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^2*b^2 - 4*a^3*c)*\sqrt{-b^2 + 4*a*c}) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/4*\log(c*x^4 + b*x^2 + a)/a^2 + 1/2*\log(x^2)/a^2$

$$3.100 \quad \int \frac{1}{(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=308

$$\frac{3b^2 - 10ac}{2a^2x(b^2 - 4ac)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

[Out] $-(3b^2 - 10ac)/(2a^2(b^2 - 4ac)x) + (b^2 - 2ac + bcx^2)/(2a(b^2 - 4ac)x(a + bx^2 + cx^4)) - (\text{Sqrt}[c]*(3b^3 - 16abc + (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{3/2}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]) + (\text{Sqrt}[c]*(3b^3 - 16abc - (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{3/2}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])$

Rubi [A] time = 1.35189, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1594, 1121, 1281, 1166, 205}

$$\frac{3b^2 - 10ac}{2a^2x(b^2 - 4ac)} - \frac{\sqrt{c} \left((3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{c} \left(-(3b^2 - 10ac) \sqrt{b^2 - 4ac} - 16abc + 3b^3 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^2(b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(-2), x]

[Out] $-(3b^2 - 10ac)/(2a^2(b^2 - 4ac)x) + (b^2 - 2ac + bcx^2)/(2a(b^2 - 4ac)x(a + bx^2 + cx^4)) - (\text{Sqrt}[c]*(3b^3 - 16abc + (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{3/2}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4ac]]) + (\text{Sqrt}[c]*(3b^3 - 16abc - (3b^2 - 10ac)*\text{Sqrt}[b^2 - 4ac])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]]])/(2*\text{Sqrt}[2]*a^2*(b^2 - 4ac)^{3/2}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4ac]])$

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^n, x
_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 := -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1)
)/ (2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] := Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^2 (a + bx^2 + cx^4)^2} dx \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\int \frac{-3b^2 + 10ac - 3bcx^2}{x^2(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} + \frac{\int \frac{-b(3b^2 - 13ac) - c(3b^2 - 10ac)x^2}{a + bx^2 + cx^4} dx}{2a^2(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\left(c\left(3b^2 - 10ac + \frac{3b^3}{\sqrt{b^2 - 4ac}} - \frac{16abc}{\sqrt{b^2 - 4ac}}\right)\right) \int}{4a^2(b^2 - 4ac)} \\
&= -\frac{3b^2 - 10ac}{2a^2(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x(a + bx^2 + cx^4)} - \frac{\sqrt{c}\left(3b^2 - 10ac + \frac{3b^3}{\sqrt{b^2 - 4ac}} - \frac{16abc}{\sqrt{b^2 - 4ac}}\right) \tan^{-1}}{2\sqrt{2}a^2(b^2 - 4ac)\sqrt{b - \sqrt{b^2 - 4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.668442, size = 302, normalized size = 0.98

$$\frac{2x(-3abc - 2ac^2x^2 + b^2cx^2 + b^3)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} + 16abc - 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2), x]

[Out] $\left(-\frac{4}{x} - \frac{(2bx^3 - 3abc + b^2cx^2 - 2a^2cx^2)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{(\sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} + 16abc - 3b^3) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right) + \sqrt{2}\sqrt{c}(-3b^2\sqrt{b^2 - 4ac} + 10ac\sqrt{b^2 - 4ac} - 16abc + 3b^3) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}}\right))}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}}\right) / (4a^2)$

Maple [B] time = 0.03, size = 712, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(c*x^5+b*x^3+a*x)^2,x)$

[Out]
$$\begin{aligned} & -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b \\ & ^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2/a^2/(c*x^4+b*x^2+a) \\ & *b^3/(4*a*c-b^2)*x-5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c) \\ & ^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})+3/4/a^2*c/(4*a* \\ & c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4* \\ & a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)} \\ &)/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)} \\ &))*c)^{(1/2)}*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c \\ & +b^2)^{(1/2)})*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})* \\ & b^3+5/2/a*c^2/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh \\ & (c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})-3/4/a^2*c/(4*a*c-b^2)*2^{(1/2)} \\ &)/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)})*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(- \\ & 4*a*c+b^2)^{(1/2)})*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})* \\ & b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)} \\ &)*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2)})*c)^{(1/2)})*b^3 \\ & -1/a^2/x \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3b^2c - 10ac^2)x^4 + 2ab^2 - 8a^2c + (3b^3 - 11abc)x^2}{2((a^2b^2c - 4a^3c^2)x^5 + (a^2b^3 - 4a^3bc)x^3 + (a^3b^2 - 4a^4c)x)} + \frac{-\int \frac{3b^3 - 13abc + (3b^2c - 10ac^2)x^2}{cx^4 + bx^2 + a} dx}{2(a^2b^2 - 4a^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(c*x^5+b*x^3+a*x)^2,x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2 \\ &)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a \\ & ^4*c)*x) + 1/2*\text{integrate}(- (3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c* \\ & x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) \end{aligned}$$

Fricas [B] time = 2.10783, size = 6460, normalized size = 20.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out]
$$-1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*x^2 - \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))}/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*\sqrt{1/2}*(27*b^{11} - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))}) + \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))}/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*\sqrt{1/2}*(27*b^{11} - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^{10} - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^{10}*c^5)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3)))*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))}) - \sqrt{1/2}*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*\sqrt{-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*\sqrt{(81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^{10}*b^6 - 12*a^{11}*b^4*c + 48*a^{12}*b^2*c^2 - 64*a^{13}*c^3))}/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*\log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*\sqrt{1/2}*(27*b^{11} - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^{10}$$

$$\begin{aligned}
& - 55a^6b^8c + 392a^7b^6c^2 - 1344a^8b^4c^3 + 2176a^9b^2c^4 - 1280a^{10}c^5) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3) \\
&) \sqrt{-(9b^7 - 105a^2b^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))} \\
&) / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3) + \sqrt{(1/2) * ((a^2b^2c - 4a^3c^2) * x^5 + (a^2b^3 - 4a^3b^2c) * x^3 + (a^3b^2 - 4a^4c) * x)) \sqrt{-(9b^7 - 105a^2b^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))} / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3) * \log(-(189b^6c^3 - 1971ab^4c^4 + 5625a^2b^2c^5 - 2500a^3c^6) * x - 1/2 \sqrt{(1/2) * (27b^11 - 486ab^9c + 3330a^2b^7c^2 - 10549a^3b^5c^3 + 14408a^4b^3c^4 - 5200a^5b^2c^5 + (3a^5b^{10} - 55a^6b^8c + 392a^7b^6c^2 - 1344a^8b^4c^3 + 2176a^9b^2c^4 - 1280a^{10}c^5) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3)) \sqrt{-(9b^7 - 105a^2b^5c + 385a^2b^3c^2 - 420a^3b^2c^3 - (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) \sqrt{(81b^8 - 918ab^6c + 3051a^2b^4c^2 - 2550a^3b^2c^3 + 625a^4c^4)} / (a^{10}b^6 - 12a^{11}b^4c + 48a^{12}b^2c^2 - 64a^{13}c^3))} / (a^5b^6 - 12a^6b^4c + 48a^7b^2c^2 - 64a^8c^3)) / ((a^2b^2c - 4a^3c^2) * x^5 + (a^2b^3 - 4a^3b^2c) * x^3 + (a^3b^2 - 4a^4c) * x)
\end{aligned}$$

Sympy [A] time = 7.14549, size = 481, normalized size = 1.56

$$\text{RootSum}\left(t^4 (1048576a^{11}c^6 - 1572864a^{10}b^2c^5 + 983040a^9b^4c^4 - 327680a^8b^6c^3 + 61440a^7b^8c^2 - 6144a^6b^{10}c + 256a^5b^{12})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)

[Out] RootSum(_t**4*(1048576*a**11*c**6 - 1572864*a**10*b**2*c**5 + 983040*a**9*b**4*c**4 - 327680*a**8*b**6*c**3 + 61440*a**7*b**8*c**2 - 6144*a**6*b**10*c + 256*a**5*b**12) + _t**2*(430080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840*a**4*b**5*c**4 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a**b**11*c + 144*b**13) + 10000*a**2*c**7 - 4200*a*b**2*c**6 + 441*b**4*c**5, Lambda(_t, _t*log(x + (-81920*_t**3*a**10*c**5 + 139264*_t**3*a**9*b**2*c**4 - 86016*_t**3*a**8*b**4*c**3 + 25088*_t**3*a**7*b**6*c**2 - 3520*_t**3*a**6*b**8*c + 192*_t**3*a**5*b**10 - 27200*_t*a**5*b*c**5 + 60176*_t*a**4*b

$$\begin{aligned} & *3*c**4 - 42448*_t*a**3*b**5*c**3 + 13320*_t*a**2*b**7*c**2 - 1944*_t*a*b** \\ & 9*c + 108*_t*b**11)/(2500*a**3*c**6 - 5625*a**2*b**2*c**5 + 1971*a*b**4*c** \\ & 4 - 189*b**6*c**3))) - (8*a**2*c - 2*a*b**2 + x**4*(10*a*c**2 - 3*b**2*c) \\ & + x**2*(11*a*b*c - 3*b**3))/(x**5*(8*a**3*c**2 - 2*a**2*b**2*c) + x**3*(8*a \\ & **3*b*c - 2*a**2*b**3) + x*(8*a**4*c - 2*a**3*b**2)) \end{aligned}$$

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.101 \quad \int \frac{1}{x(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=162

$$\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{b^2-3ac}{a^2x^2(b^2-4ac)} + \frac{b \log(a+bx^2+cx^4)}{2a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac+b^2+bc}{2ax^2(b^2-4ac)(a+b^2+bc)}$$

[Out] $-\left(\frac{b^2-3ac}{a^2(b^2-4ac)x^2}\right) + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \left(\frac{b^4-6ab^2c+6a^2c^2}{a^3(b^2-4ac)^{3/2}}\right) \operatorname{ArcTan}\left[\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right] - \frac{2b \log(x)}{a^3} + \frac{b \log(a+bx^2+cx^4)}{2a^3} + \frac{-2ac+b^2+bc}{2ax^2(b^2-4ac)(a+b^2+bc)}$

Rubi [A] time = 0.250251, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{(6a^2c^2 - 6ab^2c + b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{b^2-3ac}{a^2x^2(b^2-4ac)} + \frac{b \log(a+bx^2+cx^4)}{2a^3} - \frac{2b \log(x)}{a^3} + \frac{-2ac+b^2+bc}{2ax^2(b^2-4ac)(a+b^2+bc)}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $-\left(\frac{b^2-3ac}{a^2(b^2-4ac)x^2}\right) + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)} - \left(\frac{b^4-6ab^2c+6a^2c^2}{a^3(b^2-4ac)^{3/2}}\right) \operatorname{ArcTan}\left[\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right] - \frac{2b \log(x)}{a^3} + \frac{b \log(a+bx^2+cx^4)}{2a^3} + \frac{-2ac+b^2+bc}{2ax^2(b^2-4ac)(a+b^2+bc)}$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1114

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m-1)/2)*(a+b*x+c*x^2)^p, x], x, x^2], x] /; Free

$Q[\{a, b, c, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 740

$\text{Int}[\left(\frac{(d + e x)^m \left((a + b x + c x^2)^p\right)}{x}, x_{\text{Symbol}}\right) \rightarrow \text{Simp}\left[\frac{(d + e x)^{m+1} (b c d - b^2 e + 2 a c e + c(2 c d - b e) x) (a + b x + c x^2)^{p+1}}{(p+1)(b^2 - 4 a c)(c d^2 - b d e + a e^2)}\right], x] + \text{Dist}\left[\frac{1}{(p+1)(b^2 - 4 a c)(c d^2 - b d e + a e^2)}, \text{Int}\left[(d + e x)^m \text{Simp}\left[b c d e (2 p - m + 2) + b^2 e^2 (m + p + 2) - 2 c^2 d^2 (2 p + 3) - 2 a c e^2 (m + 2 p + 3) - c e (2 c d - b e) (m + 2 p + 4) x, x\right] (a + b x + c x^2)^{p+1}, x\right], x\right] /;$ $\text{FreeQ}[\{a, b, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \ \text{NeQ}[2 c d - b e, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntQuadraticQ}[a, b, c, d, e, m, p, x]$

Rule 800

$\text{Int}\left[\frac{(d + e x)^m \left((f + g x)\right)}{(a + b x + c x^2)}, x_{\text{Symbol}}\right] \rightarrow \text{Int}\left[\text{ExpandIntegrand}\left[\frac{(d + e x)^m (f + g x)}{(a + b x + c x^2)}, x\right], x\right] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{NeQ}[c d^2 - b d e + a e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 634

$\text{Int}\left[\frac{(d + e x)}{(a + b x + c x^2)}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\frac{2 c d - b e}{2 c}, \text{Int}\left[\frac{1}{(a + b x + c x^2)}, x\right], x\right] + \text{Dist}\left[\frac{e}{2 c}, \text{Int}\left[\frac{(b + 2 c x)}{(a + b x + c x^2)}, x\right], x\right] /;$ $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2 c d - b e, 0] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4 a c]$

Rule 618

$\text{Int}\left[\frac{1}{(a + b x + c x^2)^{-1}}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}[-2, \text{Subst}\left[\text{Int}\left[\frac{1}{\text{Simp}[b^2 - 4 a c - x^2, x]}, x\right], x, b + 2 c x\right], x] /;$ $\text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4 a c, 0]$

Rule 206

$\text{Int}\left[\frac{1}{(a + b x + c x^2)^{-1}}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{(1 * \text{ArcTanh}[\text{Rt}[-b, 2] x] / \text{Rt}[a, 2])}{\text{Rt}[a, 2] * \text{Rt}[-b, 2]}, x\right] /;$ $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}\left[\frac{(d + e x)}{(a + b x + c x^2)}, x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{d * \text{Log}[\text{RemoveContent}[a + b x + c x^2, x]]}{b}, x\right] /;$ $\text{FreeQ}[\{a, b, c, d\}, x]$

e}, x] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^3(a + bx^2 + cx^4)^2} dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2(a + bx + cx^2)^2} dx, x, x^2 \right) \\
 &= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-2(b^2 - 3ac) - 2bcx}{x^2(a + bx + cx^2)} dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
 &= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{2(-b^2 + 3ac)}{ax^2} - \frac{2b(-b^2 + 4ac)}{a^2x} + \frac{2(-b^4 + 5ab^2c - 3a^2c^2 - bc)}{a^2(a + bx + cx^2)} \right) dx, x, x^2 \right)}{2a(b^2 - 4ac)} \\
 &= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} - \frac{\text{Subst} \left(\int \frac{-b^4 + 5ab^2c - 3a^2c^2}{a + bx + cx^2} dx, x, x^2 \right)}{a^3(b^2 - 4ac)} \\
 &= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} + \frac{b \text{Subst} \left(\int \frac{b + 2cx}{a + bx + cx^2} dx, x, x^2 \right)}{2a^3} \\
 &= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{2b \log(x)}{a^3} + \frac{b \log(a + bx^2 + cx^4)}{2a^3} \\
 &= -\frac{b^2 - 3ac}{a^2(b^2 - 4ac)x^2} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^2(a + bx^2 + cx^4)} - \frac{(b^4 - 6ab^2c + 6a^2c^2) \tanh^{-1} \left(\frac{b + 2cx}{\sqrt{b^2 - 4ac}} \right)}{a^3(b^2 - 4ac)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.274994, size = 248, normalized size = 1.53

$$\frac{(6a^2c^2 + b^3\sqrt{b^2 - 4ac} - 6ab^2c - 4abc\sqrt{b^2 - 4ac} + b^4) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-6a^2c^2 + b^3\sqrt{b^2 - 4ac} + 6ab^2c - 4abc\sqrt{b^2 - 4ac} - b^4) \log(\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} - \frac{a(-3ab^2 + b^3 + 2c^2)}{2a^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2), x]

[Out]
$$\frac{-(a/x^2) - (a(b^3 - 3ab^2c + b^2c^2x^2 - 2ac^2x^2))/((b^2 - 4ac)(a + bx^2 + cx^4)) - 4b \operatorname{Log}[x] + ((b^4 - 6ab^2c + 6a^2c^2 + b^3\sqrt{b^2 - 4ac}) - 4ab^2c\sqrt{b^2 - 4ac})\operatorname{Log}[b - \sqrt{b^2 - 4ac} + 2cx^2]}{(b^2 - 4ac)^{3/2}} + ((-b^4 + 6ab^2c - 6a^2c^2 + b^3\sqrt{b^2 - 4ac}) - 4ab^2c\sqrt{b^2 - 4ac})\operatorname{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2]}{(b^2 - 4ac)^{3/2}}/(2a^3)$$

Maple [B] time = 0.017, size = 352, normalized size = 2.2

$$-\frac{c^2x^2}{a(cx^4 + bx^2 + a)(4ac - b^2)} + \frac{cx^2b^2}{2a^2(cx^4 + bx^2 + a)(4ac - b^2)} - \frac{3bc}{2a(cx^4 + bx^2 + a)(4ac - b^2)} + \frac{b^3}{2a^2(cx^4 + bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(c*x^5+b*x^3+a*x)^2,x)`

[Out]
$$-1/a/(c^2x^4 + b^2x^2 + a)c^2/(4ac - b^2)x^2 + 1/2/a^2/(c^2x^4 + b^2x^2 + a)c/(4ac - b^2)x^2 + b^2 - 3/2/a/(c^2x^4 + b^2x^2 + a)b/(4ac - b^2)c + 1/2/a^2/(c^2x^4 + b^2x^2 + a)b^3/(4ac - b^2) + 2/a^2/(4ac - b^2)c \ln(c^2x^4 + b^2x^2 + a)b - 1/2/a^3/(4ac - b^2) \ln(c^2x^4 + b^2x^2 + a)b^3 - 6/a/(4ac - b^2)^{3/2} \arctan((2cx^2 + b)/(4ac - b^2)^{1/2})c^2 + 6/a^2/(4ac - b^2)^{3/2} \arctan((2cx^2 + b)/(4ac - b^2)^{1/2})b^2c - 1/a^3/(4ac - b^2)^{3/2} \arctan((2cx^2 + b)/(4ac - b^2)^{1/2})b^4 - 1/2/a^2/x^2 - 2b \ln(x)/a^3$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2(b^2c - 3ac^2)x^4 + ab^2 - 4a^2c + (2b^3 - 7abc)x^2}{2((a^2b^2c - 4a^3c^2)x^6 + (a^2b^3 - 4a^3bc)x^4 + (a^3b^2 - 4a^4c)x^2)} - \frac{-2 \int \frac{(b^3c - 4abc^2)x^3 + (b^4 - 5ab^2c + 3a^2c^2)x}{cx^4 + bx^2 + a} dx}{a^3b^2 - 4a^4c} - \frac{2b \log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")`

[Out]
$$-1/2*(2*(b^2c - 3a^2c^2)x^4 + ab^2 - 4a^2c + (2b^3 - 7ab^2c)x^2)/((a^2b^2c - 4a^3c^2)x^6 + (a^2b^3 - 4a^3b^2c)x^4 + (a^3b^2 - 4a^4c)x^2) - 2*\integrate(-((b^3c - 4ab^2c^2)x^3 + (b^4 - 5ab^2c + 3a^2c^2)x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3$$

Fricas [B] time = 2.19399, size = 2103, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12 \\ & *a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a \\ & *b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - \\ & 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*\text{sqrt}(b^2 - 4*a*c)*\log((2*c^2*x^4 + 2*b*c*x^2 \\ & + b^2 - 2*a*c + (2*c*x^2 + b)*\text{sqrt}(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b \\ & ^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2) \\ & *x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\log(c*x^4 + b*x^2 + a) + 4 \\ & *((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2) \\ & *x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\log(x))/((a^3*b^4*c - \\ & 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^ \\ & 4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c \\ & + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15 \\ & *a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + \\ & (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^ \\ & 2)*\text{sqrt}(-b^2 + 4*a*c)*\arctan(-(2*c*x^2 + b)*\text{sqrt}(-b^2 + 4*a*c)/(b^2 - 4*a*c \\ &)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2* \\ & b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\log(c*x^4 + b*x^2 \\ & + a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16* \\ & a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*\log(x))/((a^3* \\ & b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b \\ & *c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)] \end{aligned}$$

Sympy [B] time = 72.9524, size = 906, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)

[Out]
$$\begin{aligned} & (b/(2*a**3) - \text{sqrt}(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2 \\ & *a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*\log(x**2 + \end{aligned}$$

$$\begin{aligned}
& (-16a^{5c^2}(b/(2a^3) - \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) + 8a^4b^2c(b/(2a^3) - \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) - a^3b^4(b/(2a^3) - \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) + 11a^2b^2c^2 - 7ab^3c + b^5)/(6a^2c^3 - 6ab^2c^2 + b^4c)) + (b/(2a^3) + \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) * \log(x^2 + (-16a^{5c^2}(b/(2a^3) + \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) + 8a^4b^2c(b/(2a^3) + \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) - a^3b^4(b/(2a^3) + \sqrt{-(4ac - b^2)^3})(6a^2c^2 - 6ab^2c + b^4)/(2a^3(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6))) + 11a^2b^2c^2 - 7ab^3c + b^5)/(6a^2c^3 - 6ab^2c^2 + b^4c)) - (4a^2c - ab^2 + x^4(6ac^2 - 2b^2c) + x^2(7abc - 2b^3))/(x^6(8a^3c^2 - 2a^2b^2c) + x^4(8a^3bc - 2a^2b^3) + x^2(8a^4c - 2a^3b^2)) - 2b \log(x)/a^3
\end{aligned}$$

Giac [A] time = 25.5326, size = 246, normalized size = 1.52

$$\frac{(b^4 - 6ab^2c + 6a^2c^2) \arctan\left(\frac{2cx^2 + b}{\sqrt{-b^2 + 4ac}}\right)}{(a^3b^2 - 4a^4c)\sqrt{-b^2 + 4ac}} - \frac{2b^2cx^4 - 6ac^2x^4 + 2b^3x^2 - 7abcx^2 + ab^2 - 4a^2c}{2(cx^6 + bx^4 + ax^2)(a^2b^2 - 4a^3c)} + \frac{b \log(cx^4 + bx^2 + a)}{2a^3} - \frac{b \log(x^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out] (b^4 - 6a*b^2*c + 6a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3

$$3.102 \quad \int \frac{1}{x^2(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=361

$$\frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) - \sqrt{c} \left(28a^2c^2 - 29ab^2c - b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}} - 2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

[Out] $-(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rubi [A] time = 3.07713, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1585, 1121, 1281, 1166, 205}

$$\frac{\sqrt{c} \left(28a^2c^2 - 29ab^2c + b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}} \right) - \sqrt{c} \left(28a^2c^2 - 29ab^2c - b(5b^2 - 19ac) \sqrt{b^2 - 4ac} + 5b^4 \right) \tan^{-1} \left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b + \sqrt{b^2 - 4ac}}} \right)}{2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b - \sqrt{b^2 - 4ac}} - 2\sqrt{2}a^3 (b^2 - 4ac)^{3/2} \sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $-(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) + (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) - (\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/(2*\text{Sqrt}[2]*a^3*(b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])$

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^4(a + bx^2 + cx^4)^2} dx \\
&= \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} - \frac{\int \frac{-5b^2 + 14ac - 5bcx^2}{x^4(a + bx^2 + cx^4)} dx}{2a(b^2 - 4ac)} \\
&= -\frac{5b^2 - 14ac}{6a^2(b^2 - 4ac)x^3} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} + \frac{\int \frac{-3b(5b^2 - 19ac) - 3c(5b^2 - 14ac)x^2}{x^2(a + bx^2 + cx^4)} dx}{6a^2(b^2 - 4ac)} \\
&= -\frac{5b^2 - 14ac}{6a^2(b^2 - 4ac)x^3} + \frac{b(5b^2 - 19ac)}{2a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} - \frac{\int \frac{-3(5b^4 - 24ab^2c)}{6a^2(b^2 - 4ac)} dx}{6a^2(b^2 - 4ac)} \\
&= -\frac{5b^2 - 14ac}{6a^2(b^2 - 4ac)x^3} + \frac{b(5b^2 - 19ac)}{2a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} - \frac{c(5b^4 - 29ab^2c)}{6a^2(b^2 - 4ac)} \\
&= -\frac{5b^2 - 14ac}{6a^2(b^2 - 4ac)x^3} + \frac{b(5b^2 - 19ac)}{2a^3(b^2 - 4ac)x} + \frac{b^2 - 2ac + bcx^2}{2a(b^2 - 4ac)x^3(a + bx^2 + cx^4)} + \frac{\sqrt{c}(5b^4 - 29ab^2c)}{6a^2(b^2 - 4ac)}
\end{aligned}$$

Mathematica [A] time = 0.788114, size = 344, normalized size = 0.95

$$\frac{6x(2a^2c^2 - 4ab^2c - 3abc^2x^2 + b^3cx^2 + b^4)}{(b^2 - 4ac)(a + bx^2 + cx^4)} + \frac{3\sqrt{2}\sqrt{c}(28a^2c^2 + 5b^3\sqrt{b^2 - 4ac} - 29ab^2c - 19abc\sqrt{b^2 - 4ac} + 5b^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)}{(b^2 - 4ac)^{3/2}\sqrt{b - \sqrt{b^2 - 4ac}}} + \frac{3\sqrt{2}\sqrt{c}(-28a^2c^2 + 5b^3\sqrt{b^2 - 4ac} + 29ab^2c - 19abc\sqrt{b^2 - 4ac} - 5b^4)}{(b^2 - 4ac)^{3/2}\sqrt{b + \sqrt{b^2 - 4ac}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*\text{Sqrt}[2]*\text{Sqrt}[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*\text{Sqrt}[b^2 - 4*a*c] - 19*a*b*c*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b - \text{Sqrt}[b^2 - 4*a*c]]) + (3*\text{Sqrt}[2]*\text{Sqrt}[c]*(-5*b^4 + 29*a*b^2*c - 28*a^2*c^2 + 5*b^3*\text{Sqrt}[b^2 - 4*a*c] - 19*a*b*c*\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[c]*x)/\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]])]/((b^2 - 4*a*c)^{(3/2)}*\text{Sqrt}[b + \text{Sqrt}[b^2 - 4*a*c]]))/(12*a^3)$

Maple [B] time = 0.036, size = 913, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/x^2/(c*x^5+b*x^3+a*x)^2,x)$

[Out]
$$\begin{aligned} & 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*x^4+b*x^2+a)*b^3*c \\ & / (4*a*c-b^2)*x^3-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2+2/a^2/(c*x^4+b*x^2+a) \\ &)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^4+19/4/a^2*c^ \\ & 2/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/ \\ & (b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b-5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)}/((b+(-4*a* \\ & c+b^2)^{(1/2}))*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)} \\ &)*b^3+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2} \\ &)*c)^{(1/2)}*\arctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}-29/4/a^2*c^2 \\ & / (4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\ar \\ & ctan(c*x*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b^2+5/4/a^3*c/(4*a*c-b^2 \\ &)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\arctan(c*x*2^{(1/2) \\ &)}/((b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b^4-19/4/a^2*c^2/(4*a*c-b^2)*2^{(1/2) \\ &)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2) \\ &)*c)^{(1/2)}*b+5/4/a^3*c/(4*a*c-b^2)*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2) \\ &)*\arctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b^3+7/a*c^3/(\\ & 4*a*c-b^2)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\ar \\ & ctanh(c*x*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}-29/4/a^2*c^2/(4*a*c-b^2 \\ &)/(-4*a*c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\arctanh(c*x* \\ & 2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b^2+5/4/a^3*c/(4*a*c-b^2)/(-4*a* \\ & c+b^2)^{(1/2)}*2^{(1/2)}/((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*\arctanh(c*x*2^{(1/2)}/ \\ & ((-b+(-4*a*c+b^2)^{(1/2}))*c)^{(1/2)}*b^4-1/3/a^2/x^3+2/a^3*b/x \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3(5b^3c - 19abc^2)x^6 + (15b^4 - 62ab^2c + 14a^2c^2)x^4 - 2a^2b^2 + 8a^3c + 10(ab^3 - 4a^2bc)x^2}{6((a^3b^2c - 4a^4c^2)x^7 + (a^3b^3 - 4a^4bc)x^5 + (a^4b^2 - 4a^5c)x^3)} - \int \frac{5b^4 - 24ab^2c + 14a^2c^2 + (5b^3c - 19abc^2)}{cx^4 + bx^2 + a} dx}{2(a^3b^2 - 4a^4c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x^2/(c*x^5+b*x^3+a*x)^2,x, \text{algorithm}="maxima")$

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
- 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrate(
(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [B] time = 2.94168, size = 7960, normalized size = 22.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x
^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x
^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
+ 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*
b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c +
700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a
^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
8*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
- 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c
- 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a
^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a
```


$$5a^3b^3c^3 + 1260a^4b^2c^4 - (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) \sqrt{(625b^{12} - 8250ab^{10}c + 39525a^2b^8c^2 - 83630a^3b^6c^3 + 76686a^4b^4c^4 - 24108a^5b^2c^5 + 2401a^6c^6)} / (a^{14}b^6 - 12a^{15}b^4c + 48a^{16}b^2c^2 - 64a^{17}c^3) / (a^7b^6 - 12a^8b^4c + 48a^9b^2c^2 - 64a^{10}c^3) / ((a^3b^2c - 4a^4c^2)x^7 + (a^3b^3 - 4a^4bc)x^5 + (a^4b^2 - 4a^5c)x^3)$$

Sympy [A] time = 12.0217, size = 566, normalized size = 1.57

$$\text{RootSum}\left(t^4 (1048576a^{13}c^6 - 1572864a^{12}b^2c^5 + 983040a^{11}b^4c^4 - 327680a^{10}b^6c^3 + 61440a^9b^8c^2 - 6144a^8b^{10}c + 256a^7b^{12})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)

[Out] RootSum(_t**4*(1048576*a**13*c**6 - 1572864*a**12*b**2*c**5 + 983040*a**11*b**4*c**4 - 327680*a**10*b**6*c**3 + 61440*a**9*b**8*c**2 - 6144*a**8*b**10*c + 256*a**7*b**12) + _t**2*(-1290240*a**7*b*c**7 + 3440640*a**6*b**3*c**6 - 3515904*a**5*b**5*c**5 + 1870848*a**4*b**7*c**4 - 572272*a**3*b**9*c**3 + 101856*a**2*b**11*c**2 - 9840*a*b**13*c + 400*b**15) + 38416*a**2*c**9 - 17640*a*b**2*c**8 + 2025*b**4*c**7, Lambda(_t, _t*log(x + (-212992*_t**3*a**12*b*c**5 + 299008*_t**3*a**11*b**3*c**4 - 164864*_t**3*a**10*b**5*c**3 + 44800*_t**3*a**9*b**7*c**2 - 6016*_t**3*a**8*b**9*c + 320*_t**3*a**7*b**11 - 21952*_t*a**7*c**7 + 289856*_t*a**6*b**2*c**6 - 682820*_t*a**5*b**4*c**5 + 642828*_t*a**4*b**6*c**4 - 302316*_t*a**3*b**8*c**3 + 75760*_t*a**2*b**10*c**2 - 9700*_t*a*b**12*c + 500*_t*b**14)/(9604*a**4*c**8 - 50421*a**3*b**2*c**7 + 43410*a**2*b**4*c**6 - 12325*a*b**6*c**5 + 1125*b**8*c**4))) + (-8*a**3*c + 2*a**2*b**2 + x**6*(57*a*b*c**2 - 15*b**3*c) + x**4*(-14*a**2*c**2 + 62*a*b**2*c - 15*b**4) + x**2*(40*a**2*b*c - 10*a*b**3))/(x**7*(24*a**4*c**2 - 6*a**3*b**2*c) + x**5*(24*a**4*b*c - 6*a**3*b**3) + x**3*(24*a**5*c - 6*a**4*b**2))

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

```
[Out] Exception raised: NotImplementedError
```

$$3.103 \quad \int \frac{1}{x^3(ax+bx^3+cx^5)^2} dx$$

Optimal. Leaf size=219

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}} + \frac{b(3b^2-11ac)}{2a^3x^2(b^2-4ac)} - \frac{3b^2-8ac}{4a^2x^4(b^2-4ac)} - \frac{(3b^2-2ac)\log(a+bx^2+cx^4)}{4a^4} +$$

[Out] $-(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)$

Rubi [A] time = 0.312461, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {1585, 1114, 740, 800, 634, 618, 206, 628}

$$\frac{b(30a^2c^2 - 20ab^2c + 3b^4) \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}} + \frac{b(3b^2-11ac)}{2a^3x^2(b^2-4ac)} - \frac{3b^2-8ac}{4a^2x^4(b^2-4ac)} - \frac{(3b^2-2ac)\log(a+bx^2+cx^4)}{4a^4} +$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2), x]

[Out] $-(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)$

Rule 1585

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_) + (c_)*(x_)^(r_))^(n_), x_Symbol] := Int[u*x^(m+n*p)*(a+b*x^(q-p)+c*x^(r-p))^n, x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q-p] && PosQ[r-p]

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (ax + bx^3 + cx^5)^2} dx &= \int \frac{1}{x^5 (a + bx^2 + cx^4)^2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^3 (a + bx + cx^2)^2} dx, x, x^2 \right) \\
&= \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \frac{-3b^2 + 8ac - 3bcx}{x^3 (a + bx + cx^2)} dx, x, x^2 \right)}{2a (b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} - \frac{\text{Subst} \left(\int \left(\frac{-3b^2 + 8ac}{ax^3} + \frac{3b^3 - 11abc}{a^2 x^2} + \frac{(b^2 - 4ac)(-3b^2 + 2ac)}{a^3 x} + \frac{b(3b^2 - 2ac)}{a^4} \right) dx, x, x^2 \right)}{2a (b^2 - 4ac)} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{(3b^2 - 2ac)}{a^4} \\
&= -\frac{3b^2 - 8ac}{4a^2 (b^2 - 4ac) x^4} + \frac{b(3b^2 - 11ac)}{2a^3 (b^2 - 4ac) x^2} + \frac{b^2 - 2ac + bcx^2}{2a (b^2 - 4ac) x^4 (a + bx^2 + cx^4)} + \frac{b(3b^4 - 20ab^2 + 15a^2c)}{4a^4}
\end{aligned}$$

Mathematica [A] time = 0.365738, size = 328, normalized size = 1.5

$$\frac{2a(2a^2c^2 - 4ab^2c - 3abc^2x^2 + b^3cx^2 + b^4)}{(b^2 - 4ac)(a + bx^2 + cx^4)} - \frac{(8a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 + 3b^4\sqrt{b^2 - 4ac} - 20ab^3c - 14ab^2c\sqrt{b^2 - 4ac} + 3b^5) \log(-\sqrt{b^2 - 4ac} + b + 2cx^2)}{(b^2 - 4ac)^{3/2}} + \frac{(-8a^2c^2\sqrt{b^2 - 4ac} + 30a^2bc^2 + 3b^4\sqrt{b^2 - 4ac} - 20ab^3c - 14ab^2c\sqrt{b^2 - 4ac} + 3b^5)}{4a^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]

[Out]
$$\begin{aligned} & -(a^2/x^4) + (4ab)/x^2 + (2a(b^4 - 4ab^2c + 2a^2c^2 + b^3cx^2 - \\ & 3ab^2c^2x^2))/((b^2 - 4ac)(a + bx^2 + cx^4)) + 4(3b^2 - 2ac) \operatorname{Log}[x] - ((3b^5 - 20ab^3c + 30a^2b^2c^2 + 3b^4\sqrt{b^2 - 4ac} - 14a \\ & b^2c\sqrt{b^2 - 4ac} + 8a^2c^2\sqrt{b^2 - 4ac})\operatorname{Log}[b - \sqrt{b^2 - 4ac} + 2cx^2])/(b^2 - 4ac)^{3/2} + ((3b^5 - 20ab^3c + 30a^2b^2c^2 - 3b^4\sqrt{b^2 - 4ac} + 14a \\ & b^2c\sqrt{b^2 - 4ac} - 8a^2c^2\sqrt{b^2 - 4ac})\operatorname{Log}[b + \sqrt{b^2 - 4ac} + 2cx^2])/(b^2 - 4ac)^{3/2})/(4a^4) \end{aligned}$$

Maple [B] time = 0.019, size = 443, normalized size = 2.

$$\frac{3c^2bx^2}{2a^2(cx^4 + bx^2 + a)(4ac - b^2)} - \frac{b^3cx^2}{2a^3(cx^4 + bx^2 + a)(4ac - b^2)} - \frac{c^2}{a(cx^4 + bx^2 + a)(4ac - b^2)} + 2\frac{b^2c}{a^2(cx^4 + bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x)

[Out]
$$\begin{aligned} & 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^2-1/2/a^3/(c*x^4+b*x^2+a)*b^3*c \\ & / (4*a*c-b^2)*x^2-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^2+2/a^2/(c*x^4+b*x^2+a)/ \\ & (4*a*c-b^2)*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^4+2/a^2/(4*a*c-b^2) \\ & *c^2*\ln(c*x^4+b*x^2+a)-7/2/a^3/(4*a*c-b^2)*c*\ln(c*x^4+b*x^2+a)*b^2+3/4/a^4/ \\ & (4*a*c-b^2)*\ln(c*x^4+b*x^2+a)*b^4+15/a^2/(4*a*c-b^2)^{3/2}*\arctan((2*c*x^2+ \\ & b)/(4*a*c-b^2)^{1/2})*b*c^2-10/a^3/(4*a*c-b^2)^{3/2}*\arctan((2*c*x^2+b)/(4* \\ & a*c-b^2)^{1/2})*b^3*c+3/2/a^4/(4*a*c-b^2)^{3/2}*\arctan((2*c*x^2+b)/(4*a*c-b \\ & ^2)^{1/2})*b^5-1/4/a^2/x^4-2/a^3*\ln(x)*c+3/a^4*\ln(x)*b^2+1/a^3*b/x^2 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2(3b^3c - 11abc^2)x^6 + (6b^4 - 25ab^2c + 8a^2c^2)x^4 - a^2b^2 + 4a^3c + 3(ab^3 - 4a^2bc)x^2}{4((a^3b^2c - 4a^4c^2)x^8 + (a^3b^3 - 4a^4bc)x^6 + (a^4b^2 - 4a^5c)x^4)} - \frac{1}{4}(3b^4 - 14ab^2c + 8a^2c^2) \log(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8
+ (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*b^4
*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)
/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

Fricas [B] time = 2.7237, size = 2627, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3
*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a
*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^
7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8
+ (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 2
3*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3
*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^
2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*
a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2
*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3
2*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^
2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3
*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*l
og(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [B] time = 131.412, size = 1074, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)

[Out]
$$\begin{aligned} & (-b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) \log(x^2 + (32a^6c^2(-b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) - 16a^5b^2c(-b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) + 2a^4b^4(-b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) - 16a^3c^3 + 47a^2b^2c^2 - 23ab^4c + 3b^6) / (30a^2b^3c^3 - 20ab^3c^2 + 3b^5c)) + (b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) \log(x^2 + (32a^6c^2(b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) - 16a^5b^2c(b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) + 2a^4b^4(b\sqrt{-(4ac - b^2)^3} (30a^2c^2 - 20ab^2c + 3b^4) / (4a^4(64a^3c^3 - 48a^2b^2c^2 + 12ab^4c - b^6)) + (2ac - 3b^2) / (4a^4)) - 16a^3c^3 + 47a^2b^2c^2 - 23ab^4c + 3b^6) / (30a^2b^3c^3 - 20ab^3c^2 + 3b^5c)) + (-4a^3c + a^2b^2 + x^6(22ab^2c^2 - 6b^3c) + x^4(-8a^2c^2 + 25ab^2c - 6b^4) + x^2(12a^2b^2c - 3ab^3)) / (x^8(16a^4c^2 - 4a^3b^2c) + x^6(16a^4b^2c - 4a^3b^3) + x^4(16a^5c - 4a^4b^2)) - (2ac - 3b^2) \log(x) / a^4 \end{aligned}$$

Giac [A] time = 23.7638, size = 370, normalized size = 1.69

$$-\frac{(3b^5 - 20ab^3c + 30a^2bc^2) \arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2(a^4b^2 - 4a^5c)\sqrt{-b^2+4ac}} + \frac{3b^4cx^4 - 14ab^2c^2x^4 + 8a^2c^3x^4 + 3b^5x^2 - 12ab^3cx^2 + 2a^2bc^2x^2 + 5a^5}{4(a^4b^2 - 4a^5c)(cx^4 + bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")

[Out]
$$-1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*\arctan((2*c*x^2 + b)/\sqrt{-b^2 + 4*a*c})/((a^4*b^2 - 4*a^5*c)*\sqrt{-b^2 + 4*a*c}) + 1/4*(3*b^4*c*x^4 - 14*a*b^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^2 + 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 + a)) - 1/4*(3*b^2 - 2*a*c)*\log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2*a*c)*\log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)$$

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}} dx$

Optimal. Leaf size=142

$$\frac{2x^2 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + {}_1F_1\left(\frac{3}{4}; \frac{1}{2}, \frac{1}{2}; \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.170831, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1930, 1141, 510}

$$\frac{2x^2 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + {}_1F_1\left(\frac{3}{4}; \frac{1}{2}, \frac{1}{2}; \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1930

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^p, x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q))))^p, Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] && ! IntegerQ[p] && PosQ[n - q]

Rule 1141

Int[((d_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b

+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{ax + bx^3 + cx^5}} dx &= \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\left(\sqrt{x}\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right) \int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} dx}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{2x^2\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4}; \frac{1}{2}, \frac{1}{2}, \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax + bx^3 + cx^5}} \end{aligned}$$

Mathematica [A] time = 0.0951912, size = 170, normalized size = 1.2

$$\frac{2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4}; \frac{1}{2}, \frac{1}{2}, \frac{7}{4}; -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{3\sqrt{x(a + bx^2 + cx^4)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)

[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.105 $\int x^{3/2} \sqrt{ax + bx^3 + cx^5} dx$

Optimal. Leaf size=380

$$\frac{\sqrt[4]{a}\sqrt{x}(\sqrt{ab}\sqrt{c} - 6ac + 2b^2)(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax + bx^3 + cx^5}} - \frac{2x^{3/2}(b^2 - 3ac)(a + bx^2 + cx^4)}{15c^{3/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}}$$

[Out] $(-2*(b^2 - 3*a*c)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(15*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) + (\text{Sqrt}[x]*(b + 3*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(15*c) + (2*a^{(1/4)}*(b^2 - 3*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(15*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (a^{(1/4)}*(2*b^2 + \text{Sqrt}[a]*b*\text{Sqrt}[c] - 6*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(30*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rubi [A] time = 0.287345, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1919, 1953, 1197, 1103, 1195}

$$\frac{2x^{3/2}(b^2 - 3ac)(a + bx^2 + cx^4)}{15c^{3/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} - \frac{\sqrt[4]{a}\sqrt{x}(\sqrt{ab}\sqrt{c} - 6ac + 2b^2)(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(3/2)}*\text{Sqrt}[a*x + b*x^3 + c*x^5], x]$

[Out] $(-2*(b^2 - 3*a*c)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(15*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) + (\text{Sqrt}[x]*(b + 3*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(15*c) + (2*a^{(1/4)}*(b^2 - 3*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(15*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (a^{(1/4)}*(2*b^2 + \text{Sqrt}[a]*b*\text{Sqrt}[c] - 6*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(30*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rule 1919

```
Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_
), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)
*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*
n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q
))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_)*((A_) + (B_)*(x_)^(j_)))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x
_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4
]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

$4*a*c, 0]$ && PosQ[c/a]

Rubi steps

$$\begin{aligned}
 \int x^{3/2} \sqrt{ax + bx^3 + cx^5} dx &= \frac{\sqrt{x}(b + 3cx^2) \sqrt{ax + bx^3 + cx^5}}{15c} + \frac{\int \frac{\sqrt{x}(-ab - 2(b^2 - 3ac)x^2)}{\sqrt{ax + bx^3 + cx^5}} dx}{15c} \\
 &= \frac{\sqrt{x}(b + 3cx^2) \sqrt{ax + bx^3 + cx^5}}{15c} + \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{-ab - 2(b^2 - 3ac)x^2}{\sqrt{a + bx^2 + cx^4}} dx}{15c \sqrt{ax + bx^3 + cx^5}} \\
 &= \frac{\sqrt{x}(b + 3cx^2) \sqrt{ax + bx^3 + cx^5}}{15c} + \frac{\left(2\sqrt{a}(b^2 - 3ac) \sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} dx}{15c^{3/2} \sqrt{ax + bx^3 + cx^5}} + \left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} dx \\
 &= -\frac{2(b^2 - 3ac)x^{3/2}(a + bx^2 + cx^4)}{15c^{3/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} + \frac{\sqrt{x}(b + 3cx^2) \sqrt{ax + bx^3 + cx^5}}{15c} + \frac{2^4 \sqrt{a}(b^2 - 3ac)}{15c}
 \end{aligned}$$

Mathematica [C] time = 1.51973, size = 486, normalized size = 1.28

$$\sqrt{x} \left(i \left(b^2 \sqrt{b^2 - 4ac} - 3ac \sqrt{b^2 - 4ac} + 4abc - b^3 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left(i \sinh^{-1} \left(\sqrt{2x} \sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]

[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*x*(b + 3*c*x^2)*(a + b*x^2 + c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [B] time = 0.069, size = 1042, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^{3/2}*(c*x^5+b*x^3+a*x)^{1/2}, x)$

[Out]
$$\begin{aligned} & -1/30*(x*(c*x^4+b*x^2+a))^{1/2}*(-6*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2})^{1/2}*x^7 \\ & *b*c^2-6*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*(-4*a*c+b^2)^{1/2}*x^7*c^2-8*(\\ & 1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*x^5*b^2*c-8*(1/a*(-b+(-4*a*c+b^2)^{1/2})) \\ &)^{1/2}*(-4*a*c+b^2)^{1/2}*x^5*b*c-6*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*x^ \\ & 3*a*b*c-6*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*(-4*a*c+b^2)^{1/2}*x^3*a*c-2* \\ & (1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*x^3*b^3-2*(1/a*(-b+(-4*a*c+b^2)^{1/2})) \\ &)^{1/2}*(-4*a*c+b^2)^{1/2}*x^3*b^2+12*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a) \\ & /a)^{1/2}*((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticF(1/2*x^2^{1/2} \\ & (1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}- \\ & 2*a*c+b^2)/a/c)^{1/2})*a^2*c-3*b^2*a*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a) \\ & /a)^{1/2}*((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticF(1/2*x^2^{1/2} \\ & (1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}- \\ & 2*a*c+b^2)/a/c)^{1/2})*b*a*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a)/a)^{1/2}* \\ & ((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticF(1/2*x^2^{1/2}*(1/a*(\\ & -b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}-2*a*c+b^2) \\ & /a/c)^{1/2})*(-4*a*c+b^2)^{1/2}-12*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a)/a) \\ &)^{1/2}*((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticE(1/2*x^2^{1/2} \\ & (1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}-2* \\ & a*c+b^2)/a/c)^{1/2})*a^2*c+4*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a)/a)^{1/2} \\ & *((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticE(1/2*x^2^{1/2}*(1/a* \\ & (-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}-2*a*c+b^2) \\ & /a/c)^{1/2})*a*b^2-2*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*x*a*b^2-2*(1/a*(\\ & -b+(-4*a*c+b^2)^{1/2}))^{1/2}*(-4*a*c+b^2)^{1/2}*x*a*b/x^{1/2}/(c*x^4+b*x^ \\ & 2+a)/c/(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}/(b+(-4*a*c+b^2)^{1/2}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + axx^2}^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^{3/2}*(c*x^5+b*x^3+a*x)^{1/2}, x, \text{algorithm}=\text{"maxima"})$

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^5 + bx^3 + axx^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}} \sqrt{x(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + axx^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)

3.106 $\int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx$

Optimal. Leaf size=129

$$\frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\sqrt{x}(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax + bx^3 + cx^5}}$$

[Out] $((b + 2*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(8*c*\text{Sqrt}[x]) - ((b^2 - 4*a*c)*\text{Sqrt}[x]*\text{Sqrt}[a + b*x^2 + c*x^4]*\text{ArcTanh}[(b + 2*c*x^2)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4])])/(16*c^(3/2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rubi [A] time = 0.0933199, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1918, 1914, 1107, 621, 206}

$$\frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\sqrt{x}(b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[x]*\text{Sqrt}[a*x + b*x^3 + c*x^5], x]$

[Out] $((b + 2*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(8*c*\text{Sqrt}[x]) - ((b^2 - 4*a*c)*\text{Sqrt}[x]*\text{Sqrt}[a + b*x^2 + c*x^4]*\text{ArcTanh}[(b + 2*c*x^2)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4])])/(16*c^(3/2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rule 1918

$\text{Int}[(x_)^(m_)*(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]^(p_)$, $x_Symbol] \rightarrow \text{Simp}[(x^(m - n + q + 1)*(b + 2*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(2*c*(n - q)*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \text{Int}[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /;$ $\text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{EqQ}[r, 2*n - q] \ \&\& \ \text{PosQ}[n - q] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{RationalQ}[m, q] \ \&\& \ \text{EqQ}[m + p*q + 1, n - q]$

Rule 1914

$\text{Int}[(x_)^(m_)/\text{Sqrt}[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]]$, $x_Symbol] \rightarrow \text{Dist}[(x^(q/2)*\text{Sqrt}[a + b*x^(n - q) + c*x^(2*(n - q))])/\text{Sqrt}[$

$a*x^q + b*x^n + c*x^{(2*n - q)}$, Int[$x^{(m - q/2)}/\text{Sqrt}[a + b*x^{(n - q)} + c*x^{(2*(n - q))}$], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 621

Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{(b^2 - 4ac) \int \frac{x^{3/2}}{\sqrt{ax + bx^3 + cx^5}} dx}{8c} \\
 &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \int \frac{x}{\sqrt{a + bx^2 + cx^4}} dx}{8c\sqrt{ax + bx^3 + cx^5}} \\
 &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, x^2 \right)}{16c\sqrt{ax + bx^3 + cx^5}} \\
 &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{\left((b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \right) \text{Subst} \left(\int \frac{1}{4c - x^2} dx, x, \frac{b + 2cx^2}{\sqrt{a + bx^2 + cx^4}} \right)}{8c\sqrt{ax + bx^3 + cx^5}} \\
 &= \frac{(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{8c\sqrt{x}} - \frac{(b^2 - 4ac) \sqrt{x} \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right)}{16c^{3/2} \sqrt{ax + bx^3 + cx^5}}
 \end{aligned}$$

Mathematica [A] time = 0.0775967, size = 126, normalized size = 0.98

$$\frac{\sqrt{x(a+bx^2+cx^4)} \left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c} - \frac{(b^2-4ac) \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}} \right)}{2\sqrt{x}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (Sqrt[x*(a + b*x^2 + c*x^4)]*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4])

Maple [A] time = 0.016, size = 157, normalized size = 1.2

$$\frac{1}{16}\sqrt{x(cx^4+bx^2+a)}\left(4x^2c^{3/2}\sqrt{cx^4+bx^2+a}+4\ln\left(\frac{1}{2}\frac{2cx^2+2\sqrt{cx^4+bx^2+a}\sqrt{c}+b}{\sqrt{c}}\right)\right)ac-\ln\left(\frac{1}{2}\left(2cx^2+2\sqrt{cx^4+bx^2+a}\sqrt{c}+b\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2), x)

[Out] 1/16*(x*(c*x^4+b*x^2+a))^(1/2)/c^(3/2)*(4*x^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)+4*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*c-ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^2+2*b*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5+bx^3+ax}\sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)

Fricas [A] time = 1.39673, size = 547, normalized size = 4.24

$$\left[\frac{(b^2 - 4ac)\sqrt{cx} \log\left(-\frac{8c^2x^5 + 8bcx^3 + 4\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{c}\sqrt{x} + (b^2 + 4ac)x}{x}\right) - 4\sqrt{cx^5 + bx^3 + ax}(2c^2x^2 + bc)\sqrt{x}}{32c^2x}, \frac{(b^2 - 4ac)\sqrt{cx}}{32c^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x} \sqrt{x(a + bx^2 + cx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)

Giac [A] time = 1.19448, size = 171, normalized size = 1.33

$$\frac{1}{8} \sqrt{cx^4 + bx^2 + a} \left(2x^2 + \frac{b}{c}\right) + \frac{(b^2 - 4ac) \log\left(\left|-2\left(\sqrt{cx^2 - \sqrt{cx^4 + bx^2 + a}}\sqrt{c} - b\right)\right|\right)}{16c^{\frac{3}{2}}} - \frac{b^2 \log\left(\left|-b + 2\sqrt{a}\sqrt{c}\right|\right) - 4ac \log\left(\left|\sqrt{a}\sqrt{c}\right|\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - 1/16*(b^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*b*sqrt(c))/c^(3/2)
```

$$3.107 \quad \int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}} dx$$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{a}\sqrt{x}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{ab}\sqrt{x}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}$$

[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.224418, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1921, 1953, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\sqrt{x}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{ab}\sqrt{x}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x], x]

[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1921


```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(
2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

```

Rule 1953

```

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2
)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]

```

Rule 1197

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]

```

Rule 1103

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4
]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

```

Rule 1195

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}} dx &= \frac{1}{3} \sqrt{x} \sqrt{ax + bx^3 + cx^5} + \frac{1}{3} \int \frac{\sqrt{x}(2a + bx^2)}{\sqrt{ax + bx^3 + cx^5}} dx \\
&= \frac{1}{3} \sqrt{x} \sqrt{ax + bx^3 + cx^5} + \frac{\left(\sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{2a + bx^2}{\sqrt{a + bx^2 + cx^4}} dx}{3\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{1}{3} \sqrt{x} \sqrt{ax + bx^3 + cx^5} + \frac{\left(\sqrt{a} \left(2\sqrt{a} + \frac{b}{\sqrt{c}}\right) \sqrt{x} \sqrt{a + bx^2 + cx^4}\right) \int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx}{3\sqrt{ax + bx^3 + cx^5}} - \frac{\left(\sqrt{ab} \sqrt{x} \sqrt{a + bx^2 + cx^4}\right)}{3\sqrt{c} \sqrt{ax + bx^3 + cx^5}} \\
&= \frac{bx^{3/2} (a + bx^2 + cx^4)}{3\sqrt{c} (\sqrt{a} + \sqrt{cx^2}) \sqrt{ax + bx^3 + cx^5}} + \frac{1}{3} \sqrt{x} \sqrt{ax + bx^3 + cx^5} - \frac{\sqrt[4]{ab} \sqrt{x} (\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{cx^2})}}}{3c^{3/4} \sqrt{ax + bx^3 + cx^5}}
\end{aligned}$$

Mathematica [C] time = 0.970893, size = 452, normalized size = 1.3

$$\frac{\sqrt{x} \left(-i \left(b \sqrt{b^2 - 4ac} + 4ac - b^2 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac}}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left(i \sinh^{-1} \left(\sqrt{2x} \sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}} \right), \frac{\sqrt{b^2 - 4ac} + b}{b - \sqrt{b^2 - 4ac}} \right) + 4 \right)}{12c \sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x], x]

[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.019, size = 508, normalized size = 1.5

$$\frac{1}{3cx^4 + 3bx^2 + 3a} \sqrt{x(cx^4 + bx^2 + a)} \left(\sqrt{-4ac + b^2} \sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} x^5 c + \sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} x^5 bc + \sqrt{-4ac + b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x)

[Out] $\frac{1}{3} \frac{(x(c x^4 + b x^2 + a))^{1/2}}{x^{1/2}} \left((-4ac + b^2)^{1/2} \frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} x^5 c + (-4ac + b^2)^{1/2} \frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} x^5 bc + (-4ac + b^2)^{1/2} x^3 b + \frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} x^3 b^2 + a \frac{-2(x^2(-4ac + b^2)^{1/2} - b x^2 - 2a)}{a} \right)^{1/2} \left(\frac{x^2(-4ac + b^2)^{1/2} + b x^2 + 2a}{a} \right)^{1/2} \text{EllipticF}\left(\frac{1}{2} x^2 \sqrt{\frac{1}{a}(-b + (-4ac + b^2)^{1/2})}, \frac{1}{2} \sqrt{\frac{1}{a}(-b + (-4ac + b^2)^{1/2})} \frac{(b(-4ac + b^2)^{1/2} - 2ac + b^2)}{a/c} \right)^{1/2} \left(\frac{x^2(-4ac + b^2)^{1/2} + b x^2 + 2a}{a} \right)^{1/2} \text{EllipticE}\left(\frac{1}{2} x^2 \sqrt{\frac{1}{a}(-b + (-4ac + b^2)^{1/2})}, \frac{1}{2} \sqrt{\frac{1}{a}(-b + (-4ac + b^2)^{1/2})} \frac{(b(-4ac + b^2)^{1/2} - 2ac + b^2)}{a/c} \right)^{1/2} + (-4ac + b^2)^{1/2} \frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} x^5 a + \frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} x^5 a b \Big/ (c x^4 + b x^2 + a) \Big/ \left(\frac{1}{a} (-b + (-4ac + b^2)^{1/2})^{1/2} \right)^{1/2} \Big/ (b + (-4ac + b^2)^{1/2})^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a + bx^2 + cx^4)}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)

[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4)))/sqrt(x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)

$$3.108 \quad \int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}} dx$$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.208964, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1921, 1953, 1251, 843, 621, 206, 724}

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}} + \frac{b\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2), x]

[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1921

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q)))^p]/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q

+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

Rule 1953

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 843

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax + bx^3 + cx^5}}{x^{3/2}} dx &= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{1}{2} \int \frac{2a + bx^2}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} dx \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{(\sqrt{x}\sqrt{a + bx^2 + cx^4}) \int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}} dx}{2\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{(\sqrt{x}\sqrt{a + bx^2 + cx^4}) \text{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{4\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} + \frac{(a\sqrt{x}\sqrt{a + bx^2 + cx^4}) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{2\sqrt{ax + bx^3 + cx^5}} + \frac{(b\sqrt{x}\sqrt{a + bx^2 + cx^4}) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{4\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} - \frac{(a\sqrt{x}\sqrt{a + bx^2 + cx^4}) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} + \frac{(b\sqrt{x}\sqrt{a + bx^2 + cx^4}) \text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{ax + bx^3 + cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\sqrt{x}\sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax + bx^3 + cx^5}} + \frac{b\sqrt{x}\sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax + bx^3 + cx^5}}
\end{aligned}$$

Mathematica [A] time = 0.0663108, size = 155, normalized size = 0.8

$$\frac{\sqrt{x}\sqrt{a + bx^2 + cx^4} \left(2\sqrt{c}\sqrt{a + bx^2 + cx^4} - 2\sqrt{a}\sqrt{c} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right) + b \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) \right)}{4\sqrt{c}\sqrt{x}(a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2), x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] - 2*Sqrt[a]*Sqrt[c]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])) + b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.017, size = 136, normalized size = 0.7

$$-\frac{1}{4}\sqrt{x(c x^4 + b x^2 + a)}\left(2\sqrt{a}\ln\left(\frac{2a + bx^2 + 2\sqrt{a}\sqrt{cx^4 + bx^2 + a}}{x^2}\right)\sqrt{c} - b\ln\left(\frac{1}{2}\left(2cx^2 + 2\sqrt{cx^4 + bx^2 + a}\sqrt{c} + b\right)\frac{1}{\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x)`

[Out]
$$-1/4*(x*(c*x^4+b*x^2+a))^{(1/2)}*(2*a^{(1/2)}*\ln((2*a+b*x^2+2*a^{(1/2)}*(c*x^4+b*x^2+a)^{(1/2)})/x^2)*c^{(1/2)}-b*\ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})-2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}/x^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}/c^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)`

Fricas [A] time = 1.64065, size = 1575, normalized size = 8.12

$$\left[\frac{b\sqrt{cx} \log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}(2cx^2+b)\sqrt{c}\sqrt{x+(b^2+4ac)x}}{x}\right) + 2\sqrt{acx} \log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)}{8cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [1/8*(b*\sqrt{c})*x*\log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*\sqrt{c*x^5 + b*x^3 + a*x}) \\ & *(2*c*x^2 + b)*\sqrt{c}*\sqrt{x} + (b^2 + 4*a*c)*x)/x) + 2*\sqrt{a}*c*x*\log(- \\ & (b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*\sqrt{c*x^5 + b*x^3 + a*x}*(b*x^2 \\ & + 2*a)*\sqrt{a}*\sqrt{x})/x^5) + 4*\sqrt{c*x^5 + b*x^3 + a*x}*c*\sqrt{x})/(c*x), \\ & -1/4*(b*\sqrt{-c})*x*\arctan(1/2*\sqrt{c*x^5 + b*x^3 + a*x}*(2*c*x^2 + b)* \\ & \sqrt{-c}*\sqrt{x})/(c^2*x^5 + b*c*x^3 + a*c*x) - \sqrt{a}*c*x*\log(-((b^2 + 4*a \\ & *c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*\sqrt{c*x^5 + b*x^3 + a*x}*(b*x^2 + 2*a)* \\ & \sqrt{a}*\sqrt{x})/x^5) - 2*\sqrt{c*x^5 + b*x^3 + a*x}*c*\sqrt{x})/(c*x), 1/8*(4 \end{aligned}$$


```
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x(a + bx^2 + cx^4)}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)

[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)

3.109 $\int x^{3/2} (ax + bx^3 + cx^5)^{3/2} dx$

Optimal. Leaf size=244

$$\frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(4cx^2(5b^2 - 16ac) + b(5b^2 - 4ac))\sqrt{ax + bx^3 + cx^5}}{640c^2} - \frac{3b\sqrt{x}(b^2 - 4ac)}{640c^2}$$

[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.357206, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1919, 1945, 1949, 12, 1914, 1107, 621, 206}

$$\frac{(128a^2c^2 - 100ab^2c + 15b^4)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(4cx^2(5b^2 - 16ac) + b(5b^2 - 4ac))\sqrt{ax + bx^3 + cx^5}}{640c^2} - \frac{3b\sqrt{x}(b^2 - 4ac)}{640c^2}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]

[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1919

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p]/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q

```
)*)Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
.)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^
(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p
*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1
)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
(n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
+ (n - q)*(2*p + 1) + 1, 0]
```

Rule 1949

```
Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_
.)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(B*x^(m - n + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
+ p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
```

$a*x^q + b*x^n + c*x^{(2*n - q)}$, $\text{Int}[x^{(m - q/2)}/\text{Sqrt}[a + b*x^{(n - q)} + c*x^{(2*(n - q))}]$, x , x /; $\text{FreeQ}\{a, b, c, m, n, q\}, x$ && $\text{EqQ}[r, 2*n - q]$ && $\text{PosQ}[n - q]$ && $((\text{EqQ}[m, 1] \&\& \text{EqQ}[n, 3] \&\& \text{EqQ}[q, 2]) \|\| ((\text{EqQ}[m + 1/2] \|\| \text{EqQ}[m, 3/2] \|\| \text{EqQ}[m, 1/2] \|\| \text{EqQ}[m, 5/2]) \&\& \text{EqQ}[n, 3] \&\& \text{EqQ}[q, 1]))$

Rule 1107

$\text{Int}[(x_*)*((a_*) + (b_*)*(x_*)^2 + (c_*)*(x_*)^4)^{(p_*)}]$, x_Symbol \rightarrow $\text{Dist}[1/2,$
 $\text{Subst}[\text{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x$ /; $\text{FreeQ}\{a, b, c, p\}, x$

Rule 621

$\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2], x_Symbol]$ \rightarrow $\text{Dist}[2,$ $\text{Subst}[\text{Int}[1/(4*c - x^2), x], x, (b + 2*c*x)/\text{Sqrt}[a + b*x + c*x^2]], x$ /; $\text{FreeQ}\{a, b, c\}, x$ && $\text{NeQ}[b^2 - 4*a*c, 0]$

Rule 206

$\text{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x_Symbol]$ \rightarrow $\text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x]$ /; $\text{FreeQ}\{a, b\}, x$ && $\text{NegQ}[a/b]$ && $(\text{GtQ}[a, 0] \|\| \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int x^{3/2} (ax + bx^3 + cx^5)^{3/2} dx &= \frac{\sqrt{x} (3b + 8cx^2) (ax + bx^3 + cx^5)^{3/2}}{80c} + \frac{3 \int \sqrt{x} (-2ab - (5b^2 - 16ac)x^2) \sqrt{ax + bx^3 + cx^5}}{80c} \\
&= -\frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2) \sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x} (3b + 8cx^2) (ax + bx^3 + cx^5)^{3/2}}{80c} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2} \\
&= \frac{(15b^4 - 100ab^2c + 128a^2c^2) \sqrt{ax + bx^3 + cx^5}}{1280c^3 \sqrt{x}} - \frac{x^{3/2} (b(5b^2 - 4ac) + 4c(5b^2 - 16ac)x^2)}{640c^2}
\end{aligned}$$

Mathematica [A] time = 0.212016, size = 192, normalized size = 0.79

$$\frac{(x(a + bx^2 + cx^4))^{3/2} \left(-\frac{3b(b^2 - 4ac) \left((b^2 - 4ac) \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c}\sqrt{a + bx^2 + cx^4}} \right) - 2\sqrt{c}(b + 2cx^2)\sqrt{a + bx^2 + cx^4} \right)}{256c^{7/2}} - \frac{b(b + 2cx^2)(a + bx^2 + cx^4)^{3/2}}{16c^2} + \frac{(a + bx^2 + cx^4)^{5/2}}{5c} \right)}{2x^{3/2}(a + bx^2 + cx^4)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] ((x*(a + b*x^2 + c*x^4))^(3/2)*(-(b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c^2) + (a + b*x^2 + c*x^4)^(5/2)/(5*c) - (3*b*(b^2 - 4*a*c)*(-2*sqrt[c]*(b + 2*c*x^2)*sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*sqrt[c]*sqrt[a + b*x^2 + c*x^4])]))/(256*c^(7/2)))/(2*x^(3/2)*(a +

$$b*x^2 + c*x^4)^{(3/2)}$$

Maple [A] time = 0.026, size = 369, normalized size = 1.5

$$-\frac{1}{2560}\sqrt{x(cx^4+bx^2+a)}\left(-256x^8c^{9/2}\sqrt{cx^4+bx^2+a}-352x^6bc^{7/2}\sqrt{cx^4+bx^2+a}-512x^4ac^{7/2}\sqrt{cx^4+bx^2+a}-16x^4b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x)

[Out]
$$-1/2560*(x*(c*x^4+b*x^2+a))^{(1/2)}/c^{(7/2)}*(-256*x^8*c^{(9/2)}*(c*x^4+b*x^2+a)^{(1/2)}-352*x^6*b*c^{(7/2)}*(c*x^4+b*x^2+a)^{(1/2)}-512*x^4*a*c^{(7/2)}*(c*x^4+b*x^2+a)^{(1/2)}-16*x^4*b^2*c^{(5/2)}*(c*x^4+b*x^2+a)^{(1/2)}-112*x^2*a*b*c^{(5/2)}*(c*x^4+b*x^2+a)^{(1/2)}+20*x^2*b^3*c^{(3/2)}*(c*x^4+b*x^2+a)^{(1/2)}+240*\ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})*a^2*b*c^2-120*\ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})*a*b^3*c+15*\ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^{(1/2)}*c^{(1/2)}+b)/c^{(1/2)})*b^5-256*a^2*c^{(5/2)}*(c*x^4+b*x^2+a)^{(1/2)}+200*a*b^2*c^{(3/2)}*(c*x^4+b*x^2+a)^{(1/2)}-30*b^4*c^{(1/2)}*(c*x^4+b*x^2+a)^{(1/2)})/x^{(1/2)}/(c*x^4+b*x^2+a)^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)

Fricas [A] time = 1.44359, size = 921, normalized size = 3.77

$$\frac{15(b^5 - 8ab^3c + 16a^2bc^2)\sqrt{cx} \log\left(-\frac{8c^2x^5 + 8bcx^3 - 4\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{c}\sqrt{x} + (b^2 + 4ac)x}{x}\right) + 4(128c^5x^8 + 176bc^4x^6 + 15b^4c - 5120c^4x}{5120c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*
b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2
+ 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2
+ 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2
)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^
4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4
- 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*
x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

3.110 $\int \sqrt{x} (ax + bx^3 + cx^5)^{3/2} dx$

Optimal. Leaf size=487

$$\frac{\sqrt[4]{a}\sqrt{x}(84a^2c^2 - 57ab^2c + 4\sqrt{ab}\sqrt{c}(b^2 - 6ac) + 8b^4)(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{ax + bx^3 + cx^5}}$$

[Out] $((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(315*c^{(5/2)})*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (\text{Sqrt}[x]*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(315*c^2) + ((3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^{(3/2)})/(63*c*\text{Sqrt}[x]) - (a^{(1/4)}*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(315*c^{(11/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5]) + (a^{(1/4)}*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*\text{Sqrt}[a]*b*\text{Sqrt}[c]*(b^2 - 6*a*c))*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(630*c^{(11/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rubi [A] time = 0.457044, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1919, 1945, 1953, 1197, 1103, 1195}

$$\frac{x^{3/2}(84a^2c^2 - 57ab^2c + 8b^4)(a + bx^2 + cx^4)}{315c^{5/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} + \frac{\sqrt[4]{a}\sqrt{x}(84a^2c^2 - 57ab^2c + 4\sqrt{ab}\sqrt{c}(b^2 - 6ac) + 8b^4)(\sqrt{a} + \sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[x]*(a*x + b*x^3 + c*x^5)^{(3/2)}, x]$

[Out] $((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(315*c^{(5/2)})*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (\text{Sqrt}[x]*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(315*c^2) + ((3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^{(3/2)})/(63*c*\text{Sqrt}[x]) - (a^{(1/4)}*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4])/(315*c^{(11/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

$$+ (a^{1/4}*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*\sqrt{a}*b*\sqrt{c}*(b^2 - 6*a*c))*\sqrt{x}*(\sqrt{a} + \sqrt{c}*x^2)*\sqrt{(a + b*x^2 + c*x^4)/(\sqrt{a} + \sqrt{c}*x^2)^2}*\text{EllipticF}[2*\text{ArcTan}[(c^{1/4}*x)/a^{1/4}], (2 - b/(\sqrt{a}*\sqrt{c}))]/4)/(630*c^{11/4}*\sqrt{a*x + b*x^3 + c*x^5})$$

Rule 1919

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] := Simp[(x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*(2*p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)), Int[x^(m - (n - 2*q))*Simp[-(a*b*(m + p*q - n + q + 1)) + (2*a*c*(m + p*q + (n - q)*(2*p - 1) + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1945

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)*(A_.) + (B_.)*(x_)^(r_.), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1953

```
Int[((x_)^(m_.)*((A_.) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x]
- Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /;
FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]},
Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /;
FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol]
:= With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x]
+ Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /;
EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x} (ax + bx^3 + cx^5)^{3/2} dx &= \frac{(3b + 7cx^2)(ax + bx^3 + cx^5)^{3/2}}{63c\sqrt{x}} + \frac{\int \frac{(-ab - 2(2b^2 - 7ac)x^2)\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}} dx}{21c} \\
&= -\frac{\sqrt{x}(b(4b^2 - 9ac) + 6c(2b^2 - 7ac)x^2)\sqrt{ax + bx^3 + cx^5}}{315c^2} + \frac{(3b + 7cx^2)(ax + bx^3 + cx^5)^{3/2}}{63c\sqrt{x}} \\
&= -\frac{\sqrt{x}(b(4b^2 - 9ac) + 6c(2b^2 - 7ac)x^2)\sqrt{ax + bx^3 + cx^5}}{315c^2} + \frac{(3b + 7cx^2)(ax + bx^3 + cx^5)^{3/2}}{63c\sqrt{x}} \\
&= -\frac{\sqrt{x}(b(4b^2 - 9ac) + 6c(2b^2 - 7ac)x^2)\sqrt{ax + bx^3 + cx^5}}{315c^2} + \frac{(3b + 7cx^2)(ax + bx^3 + cx^5)^{3/2}}{63c\sqrt{x}} \\
&= \frac{(8b^4 - 57ab^2c + 84a^2c^2)x^{3/2}(a + bx^2 + cx^4)}{315c^{5/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} - \frac{\sqrt{x}(b(4b^2 - 9ac) + 6c(2b^2 - 7ac)x^2)\sqrt{ax + bx^3 + cx^5}}{315c^2}
\end{aligned}$$

Mathematica [C] time = 2.32099, size = 609, normalized size = 1.25

$$\sqrt{x} \left(-i \left(84a^2c^2\sqrt{b^2 - 4ac} - 132a^2bc^2 + 8b^4\sqrt{b^2 - 4ac} + 65ab^3c - 57ab^2c\sqrt{b^2 - 4ac} - 8b^5 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + b}{b - \sqrt{b^2 - 4ac}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*x*(-4*b^4*x^2 - b^3*c*x^4 + 5*3*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2 - 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sq

$$2*(-4*a*c+b^2)^{(1/2)}-b*x^2-2*a)/a)^{(1/2)}*((x^2*(-4*a*c+b^2)^{(1/2)}+b*x^2+2*a)/a)^{(1/2)}*EllipticF(1/2*x^2^{(1/2)}*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}, 1/2*2^{(1/2)}*((b*(-4*a*c+b^2)^{(1/2)}-2*a*c+b^2)/a/c)^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*a*b^3-27*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^3*a*b^2*c-24*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x*a^2*b*c-85*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^9*b^2*c^3-53*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^7*b^3*c^2+(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^5*b^4*c+4*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^3*b^4+4*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x*a*b^4-35*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^11*c^4-35*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^11*b*c^4)/x^{(1/2)})/(c*x^4+b*x^2+a)/c^2/(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^5 + bx^3 + ax)^{\frac{3}{2}} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")

[Out] integral((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x} (x(a + bx^2 + cx^4))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)`

[Out] `Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^5 + bx^3 + ax)^{\frac{3}{2}} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")`

[Out] `integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)`

$$3.111 \quad \int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}} dx$$

Optimal. Leaf size=177

$$\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{3\sqrt{x}(b^2 - 4ac)^2\sqrt{a + bx^2 + cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax + bx^3 + cx^5}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^2}$$

[Out] $(-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(128*c^2*\text{Sqrt}[x]) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^{(3/2)})/(16*c*x^{(3/2)}) + (3*(b^2 - 4*a*c)^2*\text{Sqrt}[x]*\text{Sqrt}[a + b*x^2 + c*x^4]*\text{ArcTanh}[(b + 2*c*x^2)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4])])/(256*c^{(5/2)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rubi [A] time = 0.138344, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1918, 1914, 1107, 621, 206}

$$\frac{3(b^2 - 4ac)(b + 2cx^2)\sqrt{ax + bx^3 + cx^5}}{128c^2\sqrt{x}} + \frac{3\sqrt{x}(b^2 - 4ac)^2\sqrt{a + bx^2 + cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax + bx^3 + cx^5}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a*x + b*x^3 + c*x^5)^{(3/2)}/\text{Sqrt}[x], x]$

[Out] $(-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(128*c^2*\text{Sqrt}[x]) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^{(3/2)})/(16*c*x^{(3/2)}) + (3*(b^2 - 4*a*c)^2*\text{Sqrt}[x]*\text{Sqrt}[a + b*x^2 + c*x^4]*\text{ArcTanh}[(b + 2*c*x^2)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x^2 + c*x^4])])/(256*c^{(5/2)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rule 1918

$\text{Int}[(x_)^{(m_.)}*((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{(p_.)}, x_Symbol] :> \text{Simp}[(x^{(m - n + q + 1)}*(b + 2*c*x^{(n - q)})*(a*x^q + b*x^n + c*x^{(2*n - q)})^p)/(2*c*(n - q)*(2*p + 1)), x] - \text{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), \text{Int}[x^{(m + q)}*(a*x^q + b*x^n + c*x^{(2*n - q)})^{(p - 1)}, x], x] /;$ FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, n - q]

Rule 1914

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1107

```
Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2], x_Symbol] := Dist[2, Subst[Int
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3 + cx^5)^{3/2}}{\sqrt{x}} dx &= \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} - \frac{(3(b^2 - 4ac)) \int \sqrt{x} \sqrt{ax + bx^3 + cx^5} dx}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{128c^2 \sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{(3(b^2 - 4ac))}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{128c^2 \sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{(3(b^2 - 4ac))}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{128c^2 \sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{(3(b^2 - 4ac))}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{128c^2 \sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{(3(b^2 - 4ac))}{16c} \\
&= -\frac{3(b^2 - 4ac)(b + 2cx^2) \sqrt{ax + bx^3 + cx^5}}{128c^2 \sqrt{x}} + \frac{(b + 2cx^2)(ax + bx^3 + cx^5)^{3/2}}{16cx^{3/2}} + \frac{3(b^2 - 4ac)^2}{16c}
\end{aligned}$$

Mathematica [A] time = 0.113543, size = 152, normalized size = 0.86

$$\frac{\sqrt{x} \sqrt{a + bx^2 + cx^4} \left(2\sqrt{c} (b + 2cx^2) \sqrt{a + bx^2 + cx^4} (4c(5a + 2cx^4) - 3b^2 + 8bcx^2) + 3(b^2 - 4ac)^2 \tanh^{-1} \left(\frac{b + 2cx^2}{2\sqrt{c} \sqrt{a + bx^2 + cx^4}} \right) \right)}{256c^{5/2} \sqrt{x} (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x], x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[x]*(a + b*x^2 + c*x^4))

Maple [A] time = 0.02, size = 295, normalized size = 1.7

$$\frac{1}{256} \sqrt{x} \sqrt{cx^4 + bx^2 + a} \left(32x^6 c^{7/2} \sqrt{cx^4 + bx^2 + a} + 48x^4 bc^{5/2} \sqrt{cx^4 + bx^2 + a} + 80x^2 ac^{5/2} \sqrt{cx^4 + bx^2 + a} + 4x^2 b^2 c^{3/2} \sqrt{cx^4 + bx^2 + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)`

[Out] $\frac{1}{256} * (x * (c * x^4 + b * x^2 + a))^{1/2} / c^{5/2} * (32 * x^6 * c^{7/2} * (c * x^4 + b * x^2 + a)^{1/2} + 48 * x^4 * b * c^{5/2} * (c * x^4 + b * x^2 + a)^{1/2} + 80 * x^2 * a * c^{5/2} * (c * x^4 + b * x^2 + a)^{1/2} + 4 * x^2 * b^2 * c^{3/2} * (c * x^4 + b * x^2 + a)^{1/2} + 48 * \ln(1/2 * (2 * c * x^2 + 2 * (c * x^4 + b * x^2 + a)^{1/2} * c^{1/2} + b) / c^{1/2}) * a^2 * c^2 - 24 * \ln(1/2 * (2 * c * x^2 + 2 * (c * x^4 + b * x^2 + a)^{1/2} * c^{1/2} + b) / c^{1/2}) * a * b^2 * c + 3 * \ln(1/2 * (2 * c * x^2 + 2 * (c * x^4 + b * x^2 + a)^{1/2} * c^{1/2} + b) / c^{1/2}) * b^4 + 40 * a * b * c^{3/2} * (c * x^4 + b * x^2 + a)^{1/2} - 6 * b^3 * c^{1/2} * (c * x^4 + b * x^2 + a)^{1/2}) / x^{1/2} / (c * x^4 + b * x^2 + a)^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")`

[Out] `integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)`

Fricas [A] time = 1.51134, size = 768, normalized size = 4.34

$$\frac{3(b^4 - 8ab^2c + 16a^2c^2)\sqrt{cx} \log\left(-\frac{8c^2x^5 + 8bcx^3 + 4\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{c}\sqrt{x} + (b^2 + 4ac)x}{x}\right) + 4(16c^4x^6 + 24bc^3x^4 - 3b^3c + 20abc^2)}{512c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{512} * (3 * (b^4 - 8 * a * b^2 * c + 16 * a^2 * c^2) * \sqrt{c} * x * \log(- (8 * c^2 * x^5 + 8 * b * c * x^3 + 4 * \sqrt{c * x^5 + b * x^3 + a * x} * (2 * c * x^2 + b) * \sqrt{c} * \sqrt{x} + (b^2 + 4 * a * c) * x) / x) + 4 * (16 * c^4 * x^6 + 24 * b * c^3 * x^4 - 3 * b^3 * c + 20 * a * b * c^2 + 2 * (b^2 * c^2 + 20 * a * c^3) * x^2) * \sqrt{c * x^5 + b * x^3 + a * x} * \sqrt{x}) / (c^3 * x), -1/256 * (3 * ($

$$b^4 - 8ab^2c + 16a^2c^2) \sqrt{-c} x \arctan(1/2 \sqrt{c} x^5 + b x^3 + a x) (2cx^2 + b) \sqrt{-c} \sqrt{x} / (c^2 x^5 + b c x^3 + a c x) - 2(16c^4 x^6 + 24b c^3 x^4 - 3b^3 c + 20a b c^2 + 2(b^2 c^2 + 20a c^3) x^2) \sqrt{t(c x^5 + b x^3 + a x) \sqrt{x}} / (c^3 x)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)

[Out] Timed out

Giac [B] time = 1.5184, size = 624, normalized size = 3.53

$$\frac{1}{16} \left(2 \sqrt{cx^4 + bx^2 + a} \left(2x^2 + \frac{b}{c} \right) + \frac{(b^2 - 4ac) \log \left(\left| -2 \left(\sqrt{cx^2 - \sqrt{cx^4 + bx^2 + a}} \sqrt{c} - b \right) \right| \right)}{c^{\frac{3}{2}}} - \frac{b^2 \log \left(\left| -b + 2 \sqrt{a} \sqrt{c} \right| \right) - 4a}{c^{\frac{3}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")

[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - (b^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))) + 2*sqrt(a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/384*(sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c^7 - 12*a*c^8)/c^9)*x^2 + (15*b^3*c^6 - 52*a*b*c^7)/c^9 - (15*sqrt(a)*b^3 - 52*a^(3/2)*b*c)/c^3)*c + 1/256*(5*b^4*c^6 - 24*a*b^2*c^7 + 16*a^2*c^8)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(17/2)

$$3.112 \quad \int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}} dx$$

Optimal. Leaf size=425

$$\frac{\sqrt[4]{a}\sqrt{x}(\sqrt{a}\sqrt{c}(b^2-20ac)+2b(b^2-8ac))(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}} - \frac{2bx^3}{35c^{3/2}}$$

[Out] $(-2*b*(b^2 - 8*a*c)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(35*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) + (\text{Sqrt}[x]*(b^2 + 10*a*c + 3*b*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^{(3/2)}/(7*\text{Sqrt}[x]) + (2*a^{(1/4)}*b*(b^2 - 8*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4)]/(35*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (a^{(1/4)}*(\text{Sqrt}[a]*\text{Sqrt}[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4)]/(70*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

Rubi [A] time = 0.44855, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1921, 1945, 1953, 1197, 1103, 1195}

$$\frac{2bx^{3/2}(b^2-8ac)(a+bx^2+cx^4)}{35c^{3/2}(\sqrt{a}+\sqrt{cx^2})\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}\sqrt{x}(\sqrt{a}\sqrt{c}(b^2-20ac)+2b(b^2-8ac))(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x]

[Out] $(-2*b*(b^2 - 8*a*c)*x^{(3/2)}*(a + b*x^2 + c*x^4))/(35*c^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5]) + (\text{Sqrt}[x]*(b^2 + 10*a*c + 3*b*c*x^2)*\text{Sqrt}[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^{(3/2)}/(7*\text{Sqrt}[x]) + (2*a^{(1/4)}*b*(b^2 - 8*a*c)*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticE}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4)]/(35*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5]) - (a^{(1/4)}*(\text{Sqrt}[a]*\text{Sqrt}[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*\text{Sqrt}[x]*(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)*\text{Sqrt}[(a + b*x^2 + c*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[c]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(c^{(1/4)}*x)/a^{(1/4)}], (2 - b/(\text{Sqrt}[a]*\text{Sqrt}[c]))/4)]/(70*c^{(7/4)}*\text{Sqrt}[a*x + b*x^3 + c*x^5])$

ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]/(70*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1921

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(m + p*(2*n - q) + 1), x] + Dist[((n - q)*p)/(m + p*(2*n - q) + 1), Int[x^(m + q)*(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]

Rule 1945

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*((A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(x^(m + 1)*(b*B*(n - q)*p + A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), x] + Dist[((n - q)*p)/(c*(m + p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n - q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n - q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]

Rule 1953

Int[((x_)^(m_)*((A_) + (B_)*(x_)^(j_)))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[

c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax + bx^3 + cx^5)^{3/2}}{x^{3/2}} dx &= \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}} + \frac{3}{7} \int \frac{(2a + bx^2)\sqrt{ax + bx^3 + cx^5}}{\sqrt{x}} dx \\
&= \frac{\sqrt{x}(b^2 + 10ac + 3bcx^2)\sqrt{ax + bx^3 + cx^5}}{35c} + \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}} + \frac{\int \frac{\sqrt{x}(-a(b^2 - 20ac) - 2b(b^2 - 8ac)x^2)}{\sqrt{ax + bx^3 + cx^5}} dx}{35c} \\
&= \frac{\sqrt{x}(b^2 + 10ac + 3bcx^2)\sqrt{ax + bx^3 + cx^5}}{35c} + \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \int \frac{-a}{\sqrt{ax + bx^3 + cx^5}} dx}{35c\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{\sqrt{x}(b^2 + 10ac + 3bcx^2)\sqrt{ax + bx^3 + cx^5}}{35c} + \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(2\sqrt{ab}(b^2 - 8ac)\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \int \frac{-a}{\sqrt{ax + bx^3 + cx^5}} dx}{35c^{3/2}\sqrt{ax + bx^3 + cx^5}} \\
&= -\frac{2b(b^2 - 8ac)x^{3/2}(a + bx^2 + cx^4)}{35c^{3/2}(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} + \frac{\sqrt{x}(b^2 + 10ac + 3bcx^2)\sqrt{ax + bx^3 + cx^5}}{35c} + \frac{(ax + bx^3 + cx^5)^{3/2}}{7\sqrt{x}}
\end{aligned}$$

Mathematica [C] time = 1.7485, size = 540, normalized size = 1.27

$$\sqrt{x} \left(i \left(-20a^2c^2 + b^3\sqrt{b^2 - 4ac} + 9ab^2c - 8abc\sqrt{b^2 - 4ac} - b^4 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left(i \sinh^{-1} \left(\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x]

[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]) + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [B] time = 0.023, size = 1394, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2), x)

[Out] -1/70*(x*(c*x^4+b*x^2+a))^(1/2)*(-10*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^9*b*c^3-10*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^9*c^3-26*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^7*b^2*c^2-26*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^7*b*c^2-40*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*a*b*c^2-40*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*a*c^2-18*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^3*c-18*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^5*b^2*c-46*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*a*b^2*c-46*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*a*b*c-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^4-2*(-4*

```

a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^3+12*EllipticF(1/2
*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*b*c-20*(-4*a*c+b^2)^(1/
2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*c-3*Ell
ipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-
4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2
*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^3+(-4*a*c+b^2
)^(1/2)*EllipticF(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(
1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1
/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^2-
32*EllipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*
((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a^2*b*c+4*E
llipticE(1/2*x*2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*
(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2
-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*b^3-30*(1/a*(
-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a^2*b*c-30*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*
a*c+b^2)^(1/2)))^(1/2)*x*a^2*c-2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b^
3-2*(-4*a*c+b^2)^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b^2/x^(1/2)
/(c*x^4+b*x^2+a)/c/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2
))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}(cx^4 + bx^2 + a)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)/sqrt(x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(a + bx^2 + cx^4))^{\frac{3}{2}}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)`

[Out] `Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^5 + bx^3 + ax)^{\frac{3}{2}}}{x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")`

[Out] `integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)`

$$3.113 \quad \int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.0618114, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1914, 1107, 621, 206}

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1107

Int[(x_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2,
Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^{3/2}}{\sqrt{ax + bx^3 + cx^5}} dx &= \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \int \frac{x}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} dx, x, x^2\right)}{2\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \text{Subst}\left(\int \frac{1}{4c-x^2} dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{\sqrt{x}\sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax + bx^3 + cx^5}} \end{aligned}$$

Mathematica [A] time = 0.0182489, size = 82, normalized size = 1.

$$\frac{\sqrt{x}\sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5], x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.012, size = 72, normalized size = 0.9

$$\frac{1}{2} \sqrt{x(cx^4 + bx^2 + a)} \ln \left(\frac{1}{2} \left(2cx^2 + 2\sqrt{cx^4 + bx^2 + a}\sqrt{c} + b \right) \frac{1}{\sqrt{c}} \right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{cx^4 + bx^2 + a}} \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)

[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)

Fricas [A] time = 1.46885, size = 325, normalized size = 3.96

$$\left[\frac{\log \left(-\frac{8c^2x^5 + 8bcx^3 + 4\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{c}\sqrt{x} + (b^2 + 4ac)x}{x} \right)}{4\sqrt{c}}, -\frac{\sqrt{-c} \arctan \left(\frac{\sqrt{cx^5 + bx^3 + ax}(2cx^2 + b)\sqrt{-c}\sqrt{x}}{2(c^2x^5 + bcx^3 + acx)} \right)}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3

+ a*c*x))/c]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2), x)

[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)

Giac [A] time = 1.31361, size = 54, normalized size = 0.66

$$\frac{\log\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{1024c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2), x, algorithm="giac")

[Out] -1/1024*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)

$$3.114 \quad \int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=121

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.0463985, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1914, 1103}

$$\frac{\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1914

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)] , x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] || EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)]/(2*q*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{ax + bx^3 + cx^5}} dx = \frac{\left(\sqrt{x}\sqrt{a + bx^2 + cx^4}\right) \int \frac{1}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax + bx^3 + cx^5}}$$

$$= \frac{\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax + bx^3 + cx^5}}$$

Mathematica [C] time = 0.126265, size = 193, normalized size = 1.6

$$\frac{i\sqrt{x}\sqrt{\frac{\sqrt{b^2-4ac+b+2cx^2}}{\sqrt{b^2-4ac+b}}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1\text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac+b}}}\right), \frac{\sqrt{b^2-4ac+b}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac+b}}}\sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.016, size = 177, normalized size = 1.5

$$\frac{1}{2cx^4 + 2bx^2 + 2a} \sqrt{x(cx^4 + bx^2 + a)} \sqrt{-2 \frac{x^2\sqrt{-4ac + b^2} - bx^2 - 2a}{a}} \sqrt{\frac{1}{a} (x^2\sqrt{-4ac + b^2} + bx^2 + 2a)} \text{EllipticF}\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)`

[Out] $\frac{1}{2}x^{1/2}*(x*(c*x^4+b*x^2+a))^{1/2}/(c*x^4+b*x^2+a)/(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2}*(-2*(x^2*(-4*a*c+b^2)^{1/2}-b*x^2-2*a)/a)^{1/2}*((x^2*(-4*a*c+b^2)^{1/2}+b*x^2+2*a)/a)^{1/2}*EllipticF(1/2*x*2^{1/2}*(1/a*(-b+(-4*a*c+b^2)^{1/2}))^{1/2},1/2*2^{1/2}*((b*(-4*a*c+b^2)^{1/2}-2*a*c+b^2)/a/c)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5 + bx^3 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

$$3.115 \quad \int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rubi [A] time = 0.0293618, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}} dx = -\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Mathematica [A] time = 0.0193418, size = 83, normalized size = 1.63

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4} \tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x}(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]), x]

[Out] -(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.014, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x}(cx^4+bx^2+a)\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx^4+bx^2+a}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2), x)

[Out] -1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)

Fricas [A] time = 1.39996, size = 327, normalized size = 6.41

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

$$3.116 \quad \int \frac{1}{x^{3/2} \sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=330

$$\frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.184284, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1929, 12, 1914, 1139, 1103, 1195}

$$\frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right)\right) \Big|_{\frac{1}{4}} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt{a}}\right)\right) \Big|_{\frac{1}{4}}}{a^{3/4}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1929

```

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] := Simp[(x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
)/(a*(m + p*q + 1)), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
+ p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]

```

Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

Rule 1914

```

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[
a*x^q + b*x^n + c*x^(2*n - q)], Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))

```

Rule 1139

```

Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q =
Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]

```

Rule 1103

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

```

Rule 1195

```

Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]

```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}} dx &= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\int \frac{cx^{5/2}}{\sqrt{ax+bx^3+cx^5}} dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{c \int \frac{x^{5/2}}{\sqrt{ax+bx^3+cx^5}} dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\left(c\sqrt{x}\sqrt{a+bx^2+cx^4}\right) \int \frac{x^2}{\sqrt{a+bx^2+cx^4}} dx}{a\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right) \int \frac{1}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{c}\sqrt{x}\sqrt{a+bx^2+cx^4}\right) \int \frac{1}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{a}\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{c}x^{3/2}(a+bx^2+cx^4)}{a(\sqrt{a}+\sqrt{cx^2})\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} - \frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [C] time = 0.48716, size = 303, normalized size = 0.92

$$\frac{-4(a+bx^2+cx^4) + \frac{i\sqrt{2x}(\sqrt{b^2-4ac}-b)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}+1\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)x\right)\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2x}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right)\right)}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]

[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])) - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c]))]/(4*a*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.023, size = 508, normalized size = 1.5

$$\frac{1}{(cx^4 + bx^2 + a)a} \left(-\sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} \sqrt{-4ac + b^2} x^4 c - \sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} x^4 bc - c \sqrt{-2 \frac{x^2 \sqrt{-4ac + b^2} - b}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)`

[Out] $(-1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * (-4*a*c+b^2)^{1/2} * x^4 * c - (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * x^4 * b * c - c * (-2 * (x^2 * (-4*a*c+b^2)^{1/2} - b * x^2 - 2*a)/a)^{1/2} * ((x^2 * (-4*a*c+b^2)^{1/2} + b * x^2 + 2*a)/a)^{1/2} * a * x * \text{EllipticF}(1/2 * x^2^{1/2} * (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2 * 2^{1/2} * ((b * (-4*a*c+b^2)^{1/2} - 2*a*c+b^2)/a/c)^{1/2}) + c * (-2 * (x^2 * (-4*a*c+b^2)^{1/2} - b * x^2 - 2*a)/a)^{1/2} * ((x^2 * (-4*a*c+b^2)^{1/2} + b * x^2 + 2*a)/a)^{1/2} * a * x * \text{EllipticE}(1/2 * x^2^{1/2} * (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2}, 1/2 * 2^{1/2} * ((b * (-4*a*c+b^2)^{1/2} - 2*a*c+b^2)/a/c)^{1/2}) - (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * (-4*a*c+b^2)^{1/2} * x^2 * b - (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * x^2 * b^2 - (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * (-4*a*c+b^2)^{1/2} * a - (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} * a * b) / x^{3/2} * (x * (c * x^4 + b * x^2 + a))^{1/2} / (c * x^4 + b * x^2 + a) / (1/a(-b+(-4*a*c+b^2)^{1/2}))^{1/2} / (b+(-4*a*c+b^2)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^5 + b*x^3 + a*x))*x^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{cx^5 + bx^3 + ax} \sqrt{x}}{cx^7 + bx^5 + ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} \sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5 + bx^3 + ax} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)

$$3.117 \quad \int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=391

$$\frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right), \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b - 2\sqrt{a}\sqrt{c})\sqrt{ax + bx^3 + cx^5}} + \frac{b\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\right)}{a^{3/4}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}}$$

[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.248059, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1953, 1197, 1103, 1195}

$$\frac{b\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} E\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}(\sqrt{a} + \sqrt{cx^2}) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{cx}}{\sqrt[4]{a}}\right)\middle|\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b - 2\sqrt{a}\sqrt{c})\sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a*x + b*x^3 + c*x^5])

rt[c])*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1924

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && LtQ[m + p*q + 1, n - q]

Rule 1953

Int[((x_)^(m_)*((A_) + (B_)*(x_)^(j_)))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1103

Int[1/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]

Rule 1195

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -

$4ac, 0$ && $\text{PosQ}[c/a]$

Rubi steps

$$\begin{aligned} \int \frac{x^{3/2}}{(ax + bx^3 + cx^5)^{3/2}} dx &= \frac{x^{3/2}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\int \frac{\sqrt{x}(2ac + bcx^2)}{\sqrt{ax + bx^3 + cx^5}} dx}{a(b^2 - 4ac)} \\ &= \frac{x^{3/2}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{(\sqrt{x}\sqrt{a + bx^2 + cx^4}) \int \frac{2ac + bcx^2}{\sqrt{a + bx^2 + cx^4}} dx}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} \\ &= \frac{x^{3/2}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} + \frac{(b\sqrt{c}\sqrt{x}\sqrt{a + bx^2 + cx^4}) \int \frac{1 - \frac{\sqrt{cx^2}}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}} dx}{\sqrt{a}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{((b + 2\sqrt{a}\sqrt{c}))}{\sqrt{a}} \\ &= \frac{x^{3/2}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{b\sqrt{cx^{3/2}}(a + bx^2 + cx^4)}{a(b^2 - 4ac)(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} + \frac{b^4\sqrt{c}\sqrt{x}(\sqrt{a} - \sqrt{c})}{4a(b^2 - 4ac)} \end{aligned}$$

Mathematica [C] time = 1.08292, size = 463, normalized size = 1.18

$$\frac{\sqrt{x} \left(-i \left(b\sqrt{b^2 - 4ac} + 4ac - b^2 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left(i \sinh^{-1} \left(\sqrt{2x} \sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}} \right), \frac{\sqrt{b^2 - 4ac} + b}{b - \sqrt{b^2 - 4ac}} \right) \right)}{4a(b^2 - 4ac)}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x]

[Out] $-(\text{Sqrt}[x] * (-4 * \text{Sqrt}[c / (b + \text{Sqrt}[b^2 - 4 * a * c])]) * x * (b^2 - 2 * a * c + b * c * x^2) + I * b * (-b + \text{Sqrt}[b^2 - 4 * a * c]) * \text{Sqrt}[(b + \text{Sqrt}[b^2 - 4 * a * c] + 2 * c * x^2) / (b + \text{Sqrt}[b^2 - 4 * a * c])] * \text{Sqrt}[(2 * b - 2 * \text{Sqrt}[b^2 - 4 * a * c] + 4 * c * x^2) / (b - \text{Sqrt}[b^2 - 4 * a * c])]) * \text{EllipticE}[I * \text{ArcSinh}[\text{Sqrt}[2] * \text{Sqrt}[c / (b + \text{Sqrt}[b^2 - 4 * a * c])]] * x], (b + \text{Sqrt}[b^2 - 4 * a * c]) / (b - \text{Sqrt}[b^2 - 4 * a * c])]) - I * (-b^2 + 4 * a * c + b * \text{Sqrt}[b^2 - 4 * a * c]) * \text{Sqrt}[(b + \text{Sqrt}[b^2 - 4 * a * c] + 2 * c * x^2) / (b + \text{Sqrt}[b^2 - 4 * a * c])] * \text{Sqrt}[(2 * b - 2 * \text{Sqrt}[b^2 - 4 * a * c] + 4 * c * x^2) / (b - \text{Sqrt}[b^2 - 4 * a * c])] * \text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[2] * \text{Sqrt}[c / (b + \text{Sqrt}[b^2 - 4 * a * c])]] * x], (b + \text{Sqrt}[b^2 - 4 * a * c]) / (b - \text{Sqrt}[b^2 - 4 * a * c])])]) / (4 * a * (b^2 - 4 * a * c) * \text{Sqrt}[c / (b + \text{Sqrt}[b^2 - 4 * a * c])]) * \text{Sqrt}[x * (a + b * x^2 + c * x^4)])$

Maple [A] time = 0.027, size = 533, normalized size = 1.4

$$\frac{1}{(cx^4 + bx^2 + a)a(4ac - b^2)} \sqrt{x(cx^4 + bx^2 + a)} \left(-\sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} x^3 b^2 c - \sqrt{\frac{1}{a}(-b + \sqrt{-4ac + b^2})} \sqrt{-4ac + b^2} x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)

[Out] (x*(c*x^4+b*x^2+a))^(1/2)*(-1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x^3*b^2*c-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b*c+c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x^2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2)+b*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*EllipticE(1/2*x^2^(1/2)*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*a*b*c+2*(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*c-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*x*b^3-(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-4*a*c+b^2)^(1/2)*x*b^2)/x^(1/2)/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)/(1/a*(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/(b+(-4*a*c+b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^5 + bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}}{c^2x^9 + 2bcx^7 + (b^2 + 2ac)x^5 + 2abx^3 + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^9 + 2*b*c*x^7 + (b^2 + 2*a*c)*x^5 + 2*a*b*x^3 + a^2*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(x(a + bx^2 + cx^4))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)

[Out] Integral(x**(3/2)/(x*(a + b*x**2 + c*x**4))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^5 + bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)

$$3.118 \quad \int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=103

$$\frac{\sqrt{x}(-2ac + b^2 + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*a^(3/2))

Rubi [A] time = 0.072923, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1922, 1913, 206}

$$\frac{\sqrt{x}(-2ac + b^2 + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]

[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*a^(3/2))

Rule 1922

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, -((n - q)*(2*p + 3))]

Rule 1913

```
Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)]
, x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{(ax + bx^3 + cx^5)^{3/2}} dx &= \frac{\sqrt{x}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} dx}{a} \\ &= \frac{\sqrt{x}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{\sqrt{x}(2a + bx^2)}{\sqrt{ax + bx^3 + cx^5}}\right)}{a} \\ &= \frac{\sqrt{x}(b^2 - 2ac + bcx^2)}{a(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a + bx^2)}{2\sqrt{a}\sqrt{ax + bx^3 + cx^5}}\right)}{2a^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0829563, size = 126, normalized size = 1.22

$$\frac{\sqrt{x} \left((b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \tanh^{-1} \left(\frac{2a + bx^2}{2\sqrt{a}\sqrt{a + bx^2 + cx^4}} \right) - 2\sqrt{a}(-2ac + b^2 + bcx^2) \right)}{2a^{3/2} (4ac - b^2) \sqrt{x} (a + bx^2 + cx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2), x]
```

```
[Out] (Sqrt[x]*(-2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
+ c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])))/(2*a
^(3/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.019, size = 179, normalized size = 1.7

$$-\frac{1}{(2cx^4 + 2bx^2 + 2a)(4ac - b^2)} \sqrt{x(cx^4 + bx^2 + a)} \left(2x^2bc\sqrt{a} + 4 \ln \left(\frac{2a + bx^2 + 2\sqrt{a}\sqrt{cx^4 + bx^2 + a}}{x^2} \right) \right) ac\sqrt{cx^4 + bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x)

[Out] $-1/2*(x*(c*x^4+b*x^2+a))^{1/2}/a^{3/2}*(2*x^2*b*c*a^{1/2}+4*\ln((2*a+b*x^2+2*a^{1/2}*(c*x^4+b*x^2+a)^{1/2})/x^2)*a*c*(c*x^4+b*x^2+a)^{1/2}-\ln((2*a+b*x^2+2*a^{1/2}*(c*x^4+b*x^2+a)^{1/2})/x^2)*b^2*(c*x^4+b*x^2+a)^{1/2}-4*a^{3/2}*c+2*b^2*a^{1/2})/x^{1/2}/(c*x^4+b*x^2+a)/(4*a*c-b^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(cx^5 + bx^3 + ax)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x, algorithm="maxima")

[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)

Fricas [B] time = 1.7769, size = 918, normalized size = 8.91

$$\left[\frac{\left((b^2c - 4ac^2)x^5 + (b^3 - 4abc)x^3 + (ab^2 - 4a^2c)x \right) \sqrt{a} \log \left(-\frac{(b^2+4ac)x^5 + 8abx^3 + 8a^2x - 4\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5} \right) + 4\sqrt{cx^5+bx^3+ax}}{4 \left((a^2b^2c - 4a^3c^2)x^5 + (a^2b^3 - 4a^3bc)x^3 + (a^3b^2 - 4a^4c)x \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x, algorithm="fricas")

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x(a + bx^2 + cx^4)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2), x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2), x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

$$3.119 \quad \int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=468

$$\frac{\sqrt[4]{c}\sqrt{x}(\sqrt{ab}\sqrt{c}-6ac+2b^2)(\sqrt{a}+\sqrt{cx^2})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{cx^2})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{2\sqrt{cx^{3/2}}(b^2-3ac)}{a^2(b^2-4ac)(\sqrt{a}+\sqrt{c}}$$

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]) + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt[a*x + b*x^3 + c*x^5]/(a^2*(b^2 - 4*a*c)*x^(3/2))) - (2*c^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.408659, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1924, 1951, 1953, 1197, 1103, 1195}

$$\frac{2\sqrt{cx^{3/2}}(b^2-3ac)(a+bx^2+cx^4)}{a^2(b^2-4ac)(\sqrt{a}+\sqrt{cx^2})\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}(b^2-4ac)} + \frac{\sqrt[4]{c}\sqrt{x}(\sqrt{ab}\sqrt{c}-6ac+2b^2)(\sqrt{a}+\sqrt{cx^2})}{2a^{7/4}(b^2-4ac)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]) + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt[a*x + b*x^3 + c*x^5]/(a^2*(b^2 - 4*a*c)*x^(3/2))) - (2*c^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2

+ Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4)]/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1924

Int[(x_)^(m_)*((b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_))^(p_), x_Symbol] := -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q] && LtQ[m + p*q + 1, n - q]

Rule 1951

Int[(x_)^(m_)*((c_)*(x_)^(j_) + (b_)*(x_)^(n_) + (a_)*(x_)^(q_))^(p_)*(A_) + (B_)*(x_)^(r_)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]

Rule 1953

Int[((x_)^(m_)*((A_) + (B_)*(x_)^(j_)))/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] := Dist[(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))])/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)], Int[(x^(m - q/2)*(A + B*x^(n - q)))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]

Rule 1197

Int[((d_) + (e_)*(x_)^2)/Sqrt[(a_) + (b_)*(x_)^2 + (c_)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; NeQ[e + d*q, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[

c/a]

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] := With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c*x^4]), x] /; EqQ[e + d*q^2, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}(ax + bx^3 + cx^5)^{3/2}} dx &= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\sqrt{x}\sqrt{ax + bx^3 + cx^5}} - \frac{\int \frac{-2b^2 + 6ac - bcx^2}{x^{3/2}\sqrt{ax + bx^3 + cx^5}} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\sqrt{x}\sqrt{ax + bx^3 + cx^5}} - \frac{2(b^2 - 3ac)\sqrt{ax + bx^3 + cx^5}}{a^2(b^2 - 4ac)x^{3/2}} + \frac{\int \frac{\sqrt{x}(abc + 2c(b^2 - 3ac)x^2)}{\sqrt{ax + bx^3 + cx^5}} dx}{a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\sqrt{x}\sqrt{ax + bx^3 + cx^5}} - \frac{2(b^2 - 3ac)\sqrt{ax + bx^3 + cx^5}}{a^2(b^2 - 4ac)x^{3/2}} + \frac{(\sqrt{x}\sqrt{a + bx^2 + cx^4})}{a^2(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\sqrt{x}\sqrt{ax + bx^3 + cx^5}} - \frac{2(b^2 - 3ac)\sqrt{ax + bx^3 + cx^5}}{a^2(b^2 - 4ac)x^{3/2}} - \frac{(2\sqrt{c}(b^2 - 3ac)\sqrt{x}\sqrt{a + bx^2 + cx^4})}{a^{3/2}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)\sqrt{x}\sqrt{ax + bx^3 + cx^5}} + \frac{2\sqrt{c}(b^2 - 3ac)x^{3/2}(a + bx^2 + cx^4)}{a^2(b^2 - 4ac)(\sqrt{a} + \sqrt{cx^2})\sqrt{ax + bx^3 + cx^5}} - \frac{2(b^2 - 3ac)\sqrt{x}\sqrt{a + bx^2 + cx^4}}{a^{3/2}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}}
\end{aligned}$$

Mathematica [C] time = 1.35104, size = 519, normalized size = 1.11

$$ix \left(b^2 \sqrt{b^2 - 4ac} - 3ac \sqrt{b^2 - 4ac} + 4abc - b^3 \right) \sqrt{\frac{\sqrt{b^2 - 4ac} + b + 2cx^2}{\sqrt{b^2 - 4ac} + b}} \sqrt{\frac{-2\sqrt{b^2 - 4ac} + 2b + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left(i \sinh^{-1} \left(\sqrt{2} x \sqrt{\frac{c}{\sqrt{b^2 - 4ac} + b}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]

[Out] $-(2\sqrt{c/(b + \sqrt{b^2 - 4ac})}) * (-4a^2c + 2b^2x^2(b + cx^2) + a(b^2 - 7bcx^2 - 6c^2x^4)) - I(b^2 - 3ac) * (-b + \sqrt{b^2 - 4ac}) * x * \sqrt{(b + \sqrt{b^2 - 4ac} + 2cx^2)/(b + \sqrt{b^2 - 4ac})} * \sqrt{(2b - 2\sqrt{b^2 - 4ac} + 4cx^2)/(b - \sqrt{b^2 - 4ac})} * \text{EllipticE}[I * \text{ArcSinh}[\sqrt{2} * \sqrt{c/(b + \sqrt{b^2 - 4ac})}] * x], (b + \sqrt{b^2 - 4ac})/(b - \sqrt{b^2 - 4ac})] + I(-b^3 + 4ab^2c + b^2 * \sqrt{b^2 - 4ac} - 3ac * \sqrt{b^2 - 4ac}) * x * \sqrt{(b + \sqrt{b^2 - 4ac} + 2cx^2)/(b + \sqrt{b^2 - 4ac})} * \sqrt{(2b - 2\sqrt{b^2 - 4ac} + 4cx^2)/(b - \sqrt{b^2 - 4ac})} * \text{EllipticF}[I * \text{ArcSinh}[\sqrt{2} * \sqrt{c/(b + \sqrt{b^2 - 4ac})}] * x], (b + \sqrt{b^2 - 4ac})/(b - \sqrt{b^2 - 4ac})] / (2a^2 * (b^2 - 4ac) * \sqrt{c/(b + \sqrt{b^2 - 4ac})}) * \sqrt{x} * \sqrt{x * (a + b * x^2 + c * x^4)})$

Maple [B] time = 0.029, size = 1136, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)

[Out] $-1/2 * (x * (c * x^4 + b * x^2 + a))^{1/2} / x^{3/2} * (12 * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2})^{1/2} * (-4 * a * c + b^2)^{1/2} * x^4 * a * c^2 - 4 * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2} * (-4 * a * c + b^2)^{1/2} * x^4 * b^2 * c + 12 * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2} * x^4 * a * b * c^2 - 4 * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2} * x^4 * b^3 * c + a * b * c * (-2 * (x^2 * (-4 * a * c + b^2)^{1/2}) - b * x^2 - 2 * a) / a)^{1/2} * ((x^2 * (-4 * a * c + b^2)^{1/2} + b * x^2 + 2 * a) / a)^{1/2} * \text{EllipticF}(1/2 * x^2^{1/2} * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2}, 1/2 * 2^{1/2} * ((b * (-4 * a * c + b^2)^{1/2} - 2 * a * c + b^2) / a / c)^{1/2}) * x * (-4 * a * c + b^2)^{1/2} + 12 * (-2 * (x^2 * (-4 * a * c + b^2)^{1/2}) - b * x^2 - 2 * a) / a)^{1/2} * ((x^2 * (-4 * a * c + b^2)^{1/2} + b * x^2 + 2 * a) / a)^{1/2} * \text{EllipticF}(1/2 * x^2^{1/2} * (1/a * (-b + (-4 * a * c + b^2)^{1/2}))^{1/2}, 1/2 * 2^{1/2} * ((b * (-4 * a * c + b^2)^{1/2} - 2 * a * c + b^2) / a / c)^{1/2}) * x * a^2 * c^2 - 3 * a * b^2 * c * (-2 * (x^2 * (-4 * a * c + b^2)^{1/2}) - b * x^2 - 2 * a) / a)^{1/2} * ((x^2 * (-4 * a * c + b^2)^{1/2} + b * x^2 + 2 * a) / a)^{1/2}$

$$2*a)/a)^{(1/2)}*EllipticF(1/2*x*2^{(1/2)}*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}, 1/2*2^{(1/2)}*((b*(-4*a*c+b^2)^{(1/2)}-2*a*c+b^2)/a/c)^{(1/2)})*x-12*(-2*(x^2*(-4*a*c+b^2)^{(1/2)}-b*x^2-2*a)/a)^{(1/2)}*((x^2*(-4*a*c+b^2)^{(1/2)}+b*x^2+2*a)/a)^{(1/2)}*EllipticE(1/2*x*2^{(1/2)}*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}, 1/2*2^{(1/2)}*((b*(-4*a*c+b^2)^{(1/2)}-2*a*c+b^2)/a/c)^{(1/2)})*x*a^2*c^2+4*(-2*(x^2*(-4*a*c+b^2)^{(1/2)}-b*x^2-2*a)/a)^{(1/2)}*((x^2*(-4*a*c+b^2)^{(1/2)}+b*x^2+2*a)/a)^{(1/2)}*EllipticE(1/2*x*2^{(1/2)}*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}, 1/2*2^{(1/2)}*((b*(-4*a*c+b^2)^{(1/2)}-2*a*c+b^2)/a/c)^{(1/2)})*x*a*b^2*c+14*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^2*a*b*c-4*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*x^2*b^3+14*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^2*a*b^2*c-4*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*x^2*b^4+8*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*a^2*c-2*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(-4*a*c+b^2)^{(1/2)}*a*b^2+8*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*a^2*b*c-2*(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*a*b^3)/(c*x^4+b*x^2+a)/a^2/(4*a*c-b^2)/(1/a*(-b+(-4*a*c+b^2)^{(1/2)}))^{(1/2)}/(b+(-4*a*c+b^2)^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{\frac{3}{2}} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")

[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}\sqrt{x}}{c^2x^{11} + 2bcx^9 + (b^2 + 2ac)x^7 + 2abx^5 + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x} (x(a + bx^2 + cx^4))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2), x)

[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{\frac{3}{2}} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2), x, algorithm="giac")

[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)

$$3.120 \quad \int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}} dx$$

Optimal. Leaf size=154

$$-\frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2x^{5/2}(b^2 - 4ac)} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}} + \frac{-2ac + b^2 + bcx^2}{ax^{3/2}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}}$$

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]) - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])])/(4*a^(5/2))

Rubi [A] time = 0.174953, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1924, 1951, 12, 1913, 206}

$$-\frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2x^{5/2}(b^2 - 4ac)} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}} + \frac{-2ac + b^2 + bcx^2}{ax^{3/2}(b^2 - 4ac)\sqrt{ax + bx^3 + cx^5}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]

[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]) - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])])/(4*a^(5/2))

Rule 1924

Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_.), x_Symbol] :> -Simp[(x^(m - q + 1)*(b^2 - 2*a*c + b*c*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1))/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m, q]

&& LtQ[m + p*q + 1, n - q]

Rule 1951

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_.)
*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] := Simp[(A*x^(m - q + 1)*(a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1))/(a*(m + p*q + 1)), x] + Dist[1/(a*(m + p*q +
1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1913

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*
(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m,
n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2} (ax + bx^3 + cx^5)^{3/2}} dx &= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)x^{3/2}\sqrt{ax + bx^3 + cx^5}} - \frac{\int \frac{-3b^2 + 8ac - 2bcx^2}{x^{5/2}\sqrt{ax + bx^3 + cx^5}} dx}{a(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)x^{3/2}\sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2(b^2 - 4ac)x^{5/2}} + \frac{\int -\frac{3b(b^2 - 4ac)}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}} dx}{2a^2(b^2 - 4ac)} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)x^{3/2}\sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2(b^2 - 4ac)x^{5/2}} - \frac{(3b) \int \frac{1}{\sqrt{x}\sqrt{ax + bx^3 + cx^5}}}{2a^2} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)x^{3/2}\sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2(b^2 - 4ac)x^{5/2}} + \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{4a - x^2} dx\right)}{2a^2} \\
&= \frac{b^2 - 2ac + bcx^2}{a(b^2 - 4ac)x^{3/2}\sqrt{ax + bx^3 + cx^5}} - \frac{(3b^2 - 8ac)\sqrt{ax + bx^3 + cx^5}}{2a^2(b^2 - 4ac)x^{5/2}} + \frac{3b \tanh^{-1}\left(\frac{\sqrt{x}(2a + bx^2)}{2\sqrt{a}\sqrt{ax + bx^3 + cx^5}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0820778, size = 160, normalized size = 1.04

$$\frac{2\sqrt{a}(-4a^2c + a(b^2 - 10bcx^2 - 8c^2x^4) + 3b^2x^2(b + cx^2)) - 3bx^2(b^2 - 4ac)\sqrt{a + bx^2 + cx^4} \tanh^{-1}\left(\frac{2a + bx^2}{2\sqrt{a}\sqrt{ax + bx^3 + cx^5}}\right)}{4a^{5/2}x^{3/2}(4ac - b^2)\sqrt{x(a + bx^2 + cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x]

[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^4)) - 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.019, size = 220, normalized size = 1.4

$$\frac{1}{(4cx^4 + 4bx^2 + 4a)(4ac - b^2)} \sqrt{x(cx^4 + bx^2 + a)} \left(-16x^4a^{3/2}c^2 + 6x^4b^2c\sqrt{a} + 12 \ln\left(\frac{2a + bx^2 + 2\sqrt{a}\sqrt{cx^4 + bx^2 + a}}{x^2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)`

[Out] $\frac{1}{4}*(x*(c*x^4+b*x^2+a))^{(1/2)}/a^{(5/2)}*(-16*x^4*a^{(3/2)}*c^2+6*x^4*b^2*c*a^{(1/2)}+12*\ln((2*a+b*x^2+2*a^{(1/2)}*(c*x^4+b*x^2+a))^{(1/2)})/x^2)*x^2*a*b*c*(c*x^4+b*x^2+a)^{(1/2)}-3*\ln((2*a+b*x^2+2*a^{(1/2)}*(c*x^4+b*x^2+a))^{(1/2)})/x^2)*x^2*b^3*(c*x^4+b*x^2+a)^{(1/2)}-20*a^{(3/2)}*x^2*b*c+6*x^2*b^3*a^{(1/2)}-8*a^{(5/2)}*c+2*a^{(3/2)}*b^2)/x^{(5/2)}/(c*x^4+b*x^2+a)/(4*a*c-b^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)`

Fricas [A] time = 1.85444, size = 1084, normalized size = 7.04

$$\frac{3\left(\left(b^3c - 4abc^2\right)x^7 + \left(b^4 - 4ab^2c\right)x^5 + \left(ab^3 - 4a^2bc\right)x^3\right)\sqrt{a}\log\left(-\frac{\left(b^2+4ac\right)x^5+8abx^3+8a^2x+4\sqrt{cx^5+bx^3+ax}\left(bx^2+2a\right)\sqrt{a}\sqrt{x}}{x^5}\right) - 4\sqrt{a}\sqrt{x}}{8\left(\left(a^3b^2c - 4a^4c^2\right)x^7 + \left(a^3b^3 - 4a^4bc\right)x^5 + \left(a^4b^2 - 4a^5c\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")`

[Out] $\frac{1}{8}*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*\sqrt{a}*\log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*\sqrt{c*x^5 + b*x^3 + a*x}*(b*x^2 + 2*a)*\sqrt{a}*\sqrt{x}))/x^5) - 4*\sqrt{c*x^5 + b*x^3 + a*x}*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^2)*\sqrt{x})/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*\sqrt{-a}*\arctan(1/2*\sqrt{c*x^5 + b$

```
*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2
*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} \left(x \left(a + bx^2 + cx^4 \right) \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5 + bx^3 + ax \right)^{\frac{3}{2}} x^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

$$3.121 \quad \int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx$$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

[Out] $(-2*x^{((-1+n)/2})*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^{(-1+n)}+b*x^n+c*x^{(1+n)}])$

Rubi [A] time = 0.0499673, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1915}

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{((3*(-1+n))/2)} / (a*x^{(-1+n)} + b*x^n + c*x^{(1+n)})^{3/2}, x]$

[Out] $(-2*x^{((-1+n)/2})*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^{(-1+n)}+b*x^n+c*x^{(1+n)}])$

Rule 1915

$\text{Int}[(x_)^{(m_.)} / ((b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)})^{3/2}, x_Symbol] \rightarrow \text{Simp}[(-2*x^{((n-1)/2})*(b+2*c*x))/((b^2-4*a*c)*\text{Sqrt}[a*x^{(n-1)}+b*x^n+c*x^{(n+1)}]), x] /;$ FreeQ[{a, b, c, n}, x] && EqQ[m, (3*(n-1))/2] && EqQ[q, n-1] && EqQ[r, n+1] && NeQ[b^2-4*a*c, 0]

Rubi steps

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}} dx = -\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A] time = 0.0883845, size = 46, normalized size = 0.9

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{x^{n-1}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2), x]

[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b + c*x))])

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int x^{-\frac{3}{2}+\frac{3n}{2}} (ax^{-1+n} + bx^n + cx^{1+n})^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x)

[Out] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{(cx^{n+1} + ax^{n-1} + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2), x, algorithm="maxima")

[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)

Fricas [A] time = 1.65691, size = 186, normalized size = 3.65

$$\frac{2(2cx^2 + bx)\sqrt{\frac{(cx^2 + bx + a)x^{n+1}}{x^2}}}{(ab^2 - 4a^2c + (b^2c - 4ac^2)x^2 + (b^3 - 4abc)x)x^{\frac{1}{2}n + \frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="fricas")

[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}n - \frac{3}{2}}}{(cx^{n+1} + ax^{n-1} + bx^n)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="giac")

[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)

$$3.122 \quad \int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}} dx$$

Optimal. Leaf size=287

$$\frac{2dx^2 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{3}{4}; \frac{1}{2}; \frac{1}{2}; \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{7}{4}; \frac{1}{2}; \frac{1}{2}; \frac{11}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3 + c*x^5])

Rubi [A] time = 0.399178, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1954, 1335, 1141, 510}

$$\frac{2dx^2 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{3}{4}; \frac{1}{2}; \frac{1}{2}; \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4 \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}} + 1F_1\left(\frac{7}{4}; \frac{1}{2}; \frac{1}{2}; \frac{11}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

[In] Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]])*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]])*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3 + c*x^5])

Rule 1954

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^p_.)*((A_.) + (B_.)*(x_)^(q_.)), x_Symbol] := Dist[(a*x^j + b*x^k + c*x^n)^p/(x^

$(j*p)*(a + b*x^{(k - j)} + c*x^{(2*(k - j))})^p$, $\text{Int}[x^{(m + j*p)}*(A + B*x^{(k - j)})*(a + b*x^{(k - j)} + c*x^{(2*(k - j))})^p, x]$, $x]$ /; $\text{FreeQ}\{a, b, c, A, B, j, k, m, p\}, x]$ && $\text{EqQ}[q, k - j]$ && $\text{EqQ}[n, 2*k - j]$ && $!\text{IntegerQ}[p]$ && $\text{PosQ}[k - j]$

Rule 1335

$\text{Int}[(f_*)*(x_)^{(m_*)}*((d_) + (e_)*(x_)^2)^{(q_*)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_*)}, x_Symbol]$:> $\text{Int}[\text{ExpandIntegrand}[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, m, p, q\}, x]$ && $\text{NeQ}[b^2 - 4*a*c, 0]$ && $(\text{IGtQ}[p, 0] \parallel \text{IGtQ}[q, 0] \parallel \text{IntegersQ}[m, q])$

Rule 1141

$\text{Int}[(d_)*(x_)^{(m_*)}*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_*)}, x_Symbol]$:> $\text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^2 + c*x^4)^{\text{FracPart}[p]})/((1 + (2*c*x^2)/(b + \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]}*(1 + (2*c*x^2)/(b - \text{Rt}[b^2 - 4*a*c, 2]))^{\text{FracPart}[p]})], \text{Int}[(d*x)^m*(1 + (2*c*x^2)/(b + \text{Sqrt}[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b - \text{Sqrt}[b^2 - 4*a*c]))^p, x], x]$ /; $\text{FreeQ}\{a, b, c, d, m, p\}, x]$

Rule 510

$\text{Int}[(e_)*(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_*)})^{(p_*)}*((c_) + (d_)*(x_)^{(n_*)})^{(q_*)}, x_Symbol]$:> $\text{Simp}[(a^p*c^q*(e*x)^{(m + 1)}*\text{AppellF1}[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -(b*x^n)/a, -(d*x^n)/c])/(e*(m + 1)), x]$ /; $\text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x]$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{NeQ}[m, -1]$ && $\text{NeQ}[m, n - 1]$ && $(\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0])$ && $(\text{IntegerQ}[q] \parallel \text{GtQ}[c, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}} dx &= \frac{(\sqrt{x}\sqrt{a+bx^2+cx^4}) \int \frac{\sqrt{x}(d+ex^2)}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{(\sqrt{x}\sqrt{a+bx^2+cx^4}) \int \left(\frac{d\sqrt{x}}{\sqrt{a+bx^2+cx^4}} + \frac{ex^{5/2}}{\sqrt{a+bx^2+cx^4}} \right) dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{(d\sqrt{x}\sqrt{a+bx^2+cx^4}) \int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax+bx^3+cx^5}} + \frac{(e\sqrt{x}\sqrt{a+bx^2+cx^4}) \int \frac{x^{5/2}}{\sqrt{a+bx^2+cx^4}} dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(d\sqrt{x} \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right) \int \frac{\sqrt{x}}{\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}}} dx}{\sqrt{ax+bx^3+cx^5}} + \frac{\left(e\sqrt{x} \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right) \int \frac{x^{5/2}}{\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}}} dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{2dx^2 \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}} F_1 \left(\frac{3}{4}; \frac{1}{2}, \frac{1}{2}; \frac{7}{4}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}} \right)}{3\sqrt{ax+bx^3+cx^5}} + \frac{2ex^4 \sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1 + \frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{3\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A] time = 5.13468, size = 239, normalized size = 0.83

$$\frac{2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(7dx^2F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+3ex^4F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right)}{21\sqrt{x(a+bx^2+cx^4)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5], x]

[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])*(7*d*x^2*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(21*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int x(ex^2 + d) \frac{1}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)`

[Out] `int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)x}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}(ex^2 + d)}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^5 + b*x^3 + a*x)*(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d + ex^2)}{\sqrt{x(a + bx^2 + cx^4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)x}{\sqrt{cx^5 + bx^3 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)

$$3.123 \quad \int \frac{1}{\sqrt{3x^2 - 3x^4 + x^6}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rubi [A] time = 0.0088066, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{3x^2 - 3x^4 + x^6}} dx = -\text{Subst} \left(\int \frac{1}{12 - x^2} dx, x, \frac{x(6 - 3x^2)}{\sqrt{3x^2 - 3x^4 + x^6}} \right)$$

$$= -\frac{\tanh^{-1} \left(\frac{x(6 - 3x^2)}{2\sqrt{3}\sqrt{3x^2 - 3x^4 + x^6}} \right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0182329, size = 73, normalized size = 1.62

$$\frac{x\sqrt{x^4 - 3x^2 + 3} \tanh^{-1} \left(\frac{6 - 3x^2}{2\sqrt{3}\sqrt{x^4 - 3x^2 + 3}} \right)}{2\sqrt{3}\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6], x]

[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0.011, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6} \sqrt{x^4 - 3x^2 + 3} \text{Arctanh} \left(\frac{(x^2 - 2)\sqrt{3}}{2} \frac{1}{\sqrt{x^4 - 3x^2 + 3}} \right) \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^6-3*x^4+3*x^2)^(1/2), x)

[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)

Fricas [A] time = 1.26769, size = 142, normalized size = 3.16

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)

[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

$$3.124 \quad \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rubi [A] time = 0.0122878, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 1996

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\ &= -\text{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0033544, size = 73, normalized size = 1.62

$$\frac{x\sqrt{x^4-3x^2+3} \tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0.008, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\text{Arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^2(x^4-3x^2+3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)

[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

Fricas [A] time = 1.3697, size = 142, normalized size = 3.16

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

$$3.125 \quad \int \frac{1}{\sqrt{1-(1-x^2)^3}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rubi [A] time = 0.0124409, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - (1 - x^2)^3], x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 1996

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1904

Int[1/Sqrt[(a_)*(x_)^2 + (b_)*(x_)^(n_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-(1-x^2)^3}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\ &= -\text{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0039223, size = 73, normalized size = 1.62

$$-\frac{x\sqrt{x^4-3x^2+3}\tanh^{-1}\left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - (1 - x^2)^3], x]

[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0.003, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\text{Arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-(-x^2+1)^3)^(1/2), x)

[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2-1)^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)

Fricas [A] time = 1.28434, size = 142, normalized size = 3.16

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-(-x**2+1)**3)**(1/2),x)

[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - 1)^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

3.126 $\int \sqrt{3x^2 - 3x^4 + x^6} dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

[Out] $-\left(\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x}\right) - \left(\frac{3\sqrt{3x^2 - 3x^4 + x^6} \operatorname{ArcSinh}\left[\frac{3 - 2x^2}{\sqrt{3}}\right]}{16x\sqrt{3 - 3x^2 + x^4}}\right)$

Rubi [A] time = 0.0413086, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\sqrt{3x^2 - 3x^4 + x^6}, x]$

[Out] $-\left(\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x}\right) - \left(\frac{3\sqrt{3x^2 - 3x^4 + x^6} \operatorname{ArcSinh}\left[\frac{3 - 2x^2}{\sqrt{3}}\right]}{16x\sqrt{3 - 3x^2 + x^4}}\right)$

Rule 1903

$\operatorname{Int}[\sqrt{(b_.)(x_)^{(n_.)} + (a_.)(x_)^{(q_.)} + (c_.)(x_)^{(r_.)}}, x_Symbol]$
 $\rightarrow \operatorname{Dist}[\sqrt{a*x^q + b*x^n + c*x^{(2*n - q)}}/(x^{(q/2)}\sqrt{a + b*x^{(n - q)} + c*x^{(2*(n - q))}}), \operatorname{Int}[x^{(q/2)}\sqrt{a + b*x^{(n - q)} + c*x^{(2*(n - q))}], x]$ /; $\text{FreeQ}\{a, b, c, n, q\}, x \ \&\& \ \text{EqQ}[r, 2*n - q] \ \&\& \ \text{PosQ}[n - q]$

Rule 1107

$\operatorname{Int}[(x_*)*((a_) + (b_.)(x_)^2 + (c_.)(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/2, \operatorname{Subst}[\operatorname{Int}[(a + b*x + c*x^2)^p, x], x, x^2], x]$ /; $\text{FreeQ}\{a, b, c, p\}, x]$

Rule 612

$\operatorname{Int}[(a_. + (b_.)(x_) + (c_.)(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b + 2*c*x)*(a + b*x + c*x^2)^p/(2*c*(2*p + 1)), x] - \operatorname{Dist}[(p*(b^2 - 4*a*c))/(2*c*(2$

*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \sqrt{3x^2 - 3x^4 + x^6} dx &= \frac{\sqrt{3x^2 - 3x^4 + x^6} \int x\sqrt{3 - 3x^2 + x^4} dx}{x\sqrt{3 - 3x^2 + x^4}} \\
 &= \frac{\sqrt{3x^2 - 3x^4 + x^6} \operatorname{Subst}\left(\int \sqrt{3 - 3x + x^2} dx, x, x^2\right)}{2x\sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(3\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{3 - 3x + x^2}} dx, x, x^2\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(\sqrt{3}\sqrt{3x^2 - 3x^4 + x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{3}}} dx, x, -3 + 2x^2\right)}{16x\sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2)\sqrt{3x^2 - 3x^4 + x^6}}{8x} - \frac{3\sqrt{3x^2 - 3x^4 + x^6} \sinh^{-1}\left(\frac{3 - 2x^2}{\sqrt{3}}\right)}{16x\sqrt{3 - 3x^2 + x^4}}
 \end{aligned}$$

Mathematica [A] time = 0.034346, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1}\left(\frac{2x^2 - 3}{\sqrt{3}}\right) - 18\right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6], x]

[Out] $(x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*\text{Sqrt}[3 - 3*x^2 + x^4]*\text{ArcSinh}[(-3 + 2*x^2)/\text{Sqrt}[3]]))/(16*\text{Sqrt}[x^2*(3 - 3*x^2 + x^4)])$

Maple [A] time = 0.007, size = 81, normalized size = 0.9

$$\frac{1}{16x} \sqrt{x^6 - 3x^4 + 3x^2} \left(4 \sqrt{x^4 - 3x^2 + 3x^2} - 6 \sqrt{x^4 - 3x^2 + 3} + 3 \text{Arcsinh} \left(\frac{1}{3} \sqrt{3} (2x^2 - 3) \right) \right) \frac{1}{\sqrt{x^4 - 3x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^6-3*x^4+3*x^2)^(1/2),x)`

[Out] $1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(1/2)+3*\text{arcsinh}(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^6 - 3x^4 + 3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)`

Fricas [A] time = 1.29569, size = 157, normalized size = 1.83

$$\frac{12x \log\left(-\frac{2x^3-3x-2\sqrt{x^6-3x^4+3x^2}}{x}\right) - 8\sqrt{x^6-3x^4+3x^2}(2x^2-3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")`

[Out] $-1/64*(12*x*\log(-(2*x^3 - 3*x - 2*\text{sqrt}(x^6 - 3*x^4 + 3*x^2))/x) - 8*\text{sqrt}(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^6 - 3x^4 + 3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)

[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)

Giac [A] time = 1.08852, size = 93, normalized size = 1.08

$$\frac{1}{16} \left(2 \sqrt{x^4 - 3x^2 + 3} (2x^2 - 3) - 3 \log(-2x^2 + 2\sqrt{x^4 - 3x^2 + 3} + 3) \right) \operatorname{sgn}(x) + \frac{3}{16} \left(2\sqrt{3} + \log(2\sqrt{3} + 3) \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")

[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)

3.127 $\int \sqrt{x^2 (3 - 3x^2 + x^4)} dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

[Out] $-\left(\left(3 - 2x^2\right)\sqrt{3x^2 - 3x^4 + x^6}\right)/\left(8x\right) - \left(3\sqrt{3x^2 - 3x^4 + x^6}\right)\text{ArcSinh}\left[\left(3 - 2x^2\right)/\sqrt{3}\right]/\left(16x\sqrt{3 - 3x^2 + x^4}\right)$

Rubi [A] time = 0.0420436, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] $\text{Int}\left[\sqrt{x^2(3 - 3x^2 + x^4)}, x\right]$

[Out] $-\left(\left(3 - 2x^2\right)\sqrt{3x^2 - 3x^4 + x^6}\right)/\left(8x\right) - \left(3\sqrt{3x^2 - 3x^4 + x^6}\right)\text{ArcSinh}\left[\left(3 - 2x^2\right)/\sqrt{3}\right]/\left(16x\sqrt{3 - 3x^2 + x^4}\right)$

Rule 1996

$\text{Int}\left[\left(u\right)^{\left(p\right)}, x_{\text{Symbol}}\right] \rightarrow \text{Int}\left[\text{ExpandToSum}\left[u, x\right]^{\left(p\right)}, x\right] \text{ ; FreeQ}\left[p, x\right] \ \&\& \ \text{GeneralizedTrinomialQ}\left[u, x\right] \ \&\& \ \text{!GeneralizedTrinomialMatchQ}\left[u, x\right]$

Rule 1903

$\text{Int}\left[\sqrt{\left(b\right)\left(x\right)^{\left(n\right)} + \left(a\right)\left(x\right)^{\left(q\right)} + \left(c\right)\left(x\right)^{\left(r\right)}\right], x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\sqrt{\left[a\left(x\right)^{\left(q\right)} + b\left(x\right)^{\left(n\right)} + c\left(x\right)^{\left(2n - q\right)}\right]}/\left(x^{\left(q/2\right)}\sqrt{\left[a + b\left(x\right)^{\left(n - q\right)} + c\left(x\right)^{\left(2\left(n - q\right)\right)}\right]}\right), \text{Int}\left[x^{\left(q/2\right)}\sqrt{\left[a + b\left(x\right)^{\left(n - q\right)} + c\left(x\right)^{\left(2\left(n - q\right)\right)}\right]}, x\right], x\right] \text{ ; FreeQ}\left[\left\{a, b, c, n, q\right\}, x\right] \ \&\& \ \text{EqQ}\left[r, 2n - q\right] \ \&\& \ \text{PosQ}\left[n - q\right]$

Rule 1107

$\text{Int}\left[\left(x\right)\left(\left(a\right) + \left(b\right)\left(x\right)^2 + \left(c\right)\left(x\right)^4\right)^{\left(p\right)}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[1/2, \text{Subst}\left[\text{Int}\left[\left(a + b\left(x\right) + c\left(x\right)^2\right)^{\left(p\right)}, x\right], x, x^2\right], x\right] \text{ ; FreeQ}\left[\left\{a, b, c, p\right\}, x\right]$

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x^2(3-3x^2+x^4)} dx &= \int \sqrt{3x^2-3x^4+x^6} dx \\
&= \frac{\sqrt{3x^2-3x^4+x^6} \int x\sqrt{3-3x^2+x^4} dx}{x\sqrt{3-3x^2+x^4}} \\
&= \frac{\sqrt{3x^2-3x^4+x^6} \operatorname{Subst}\left(\int \sqrt{3-3x+x^2} dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{(3\sqrt{3x^2-3x^4+x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}} dx, x, x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{(\sqrt{3}\sqrt{3x^2-3x^4+x^6}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, -3+2x^2\right)}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0094265, size = 70, normalized size = 0.81

$$\frac{x \left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1} \left(\frac{2x^2 - 3}{\sqrt{3}} \right) - 18 \right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{1}{16x} \sqrt{x^2(x^4 - 3x^2 + 3)} \left(4\sqrt{x^4 - 3x^2 + 3}x^2 - 6\sqrt{x^4 - 3x^2 + 3} + 3 \operatorname{Arcsinh} \left(\frac{1}{3}\sqrt{3}(2x^2 - 3) \right) \right) \frac{1}{\sqrt{x^4 - 3x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(x^4-3*x^2+3))^(1/2),x)

[Out] 1/16*(x^2*(x^4-3*x^2+3))^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(1/2)+3*arcsinh(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^4 - 3x^2 + 3)}x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)

Fricas [A] time = 1.28831, size = 157, normalized size = 1.83

$$\frac{12x \log\left(-\frac{2x^3-3x-2\sqrt{x^6-3x^4+3x^2}}{x}\right) - 8\sqrt{x^6-3x^4+3x^2}(2x^2-3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")

[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2*(x**4-3*x**2+3))**(1/2),x)

[Out] Timed out

Giac [A] time = 1.09196, size = 93, normalized size = 1.08

$$\frac{1}{16} \left(2\sqrt{x^4-3x^2+3}(2x^2-3) - 3 \log\left(-2x^2+2\sqrt{x^4-3x^2+3}+3\right) \right) \operatorname{sgn}(x) + \frac{3}{16} \left(2\sqrt{3} + \log(2\sqrt{3}+3) \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")

[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)

$$3.128 \quad \int \sqrt{1 - (1 - x^2)^3} dx$$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

[Out] $-\frac{((3 - 2*x^2)*\text{Sqrt}[3*x^2 - 3*x^4 + x^6])}{(8*x)} - \frac{(3*\text{Sqrt}[3*x^2 - 3*x^4 + x^6]*\text{ArcSinh}[(3 - 2*x^2)/\text{Sqrt}[3]])}{(16*x*\text{Sqrt}[3 - 3*x^2 + x^4])}$

Rubi [A] time = 0.0411441, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1996, 1903, 1107, 612, 619, 215}

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2} (3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2} \sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - (1 - x^2)^3], x]

[Out] $-\frac{((3 - 2*x^2)*\text{Sqrt}[3*x^2 - 3*x^4 + x^6])}{(8*x)} - \frac{(3*\text{Sqrt}[3*x^2 - 3*x^4 + x^6]*\text{ArcSinh}[(3 - 2*x^2)/\text{Sqrt}[3]])}{(16*x*\text{Sqrt}[3 - 3*x^2 + x^4])}$

Rule 1996

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1903

Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
 \int \sqrt{1 - (1 - x^2)^3} dx &= \int \sqrt{3x^2 - 3x^4 + x^6} dx \\
 &= \frac{\sqrt{3x^2 - 3x^4 + x^6} \int x \sqrt{3 - 3x^2 + x^4} dx}{x \sqrt{3 - 3x^2 + x^4}} \\
 &= \frac{\sqrt{3x^2 - 3x^4 + x^6} \text{Subst}\left(\int \sqrt{3 - 3x + x^2} dx, x, x^2\right)}{2x \sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2) \sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(3\sqrt{3x^2 - 3x^4 + x^6}) \text{Subst}\left(\int \frac{1}{\sqrt{3 - 3x + x^2}} dx, x, x^2\right)}{16x \sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2) \sqrt{3x^2 - 3x^4 + x^6}}{8x} + \frac{(\sqrt{3} \sqrt{3x^2 - 3x^4 + x^6}) \text{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{3}}} dx, x, -3 + 2x^2\right)}{16x \sqrt{3 - 3x^2 + x^4}} \\
 &= -\frac{(3 - 2x^2) \sqrt{3x^2 - 3x^4 + x^6}}{8x} - \frac{3\sqrt{3x^2 - 3x^4 + x^6} \sinh^{-1}\left(\frac{3 - 2x^2}{\sqrt{3}}\right)}{16x \sqrt{3 - 3x^2 + x^4}}
 \end{aligned}$$

Mathematica [A] time = 0.0033421, size = 70, normalized size = 0.81

$$\frac{x \left(4x^6 - 18x^4 + 30x^2 + 3\sqrt{x^4 - 3x^2 + 3} \sinh^{-1} \left(\frac{2x^2 - 3}{\sqrt{3}} \right) - 18 \right)}{16\sqrt{x^2(x^4 - 3x^2 + 3)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - (1 - x^2)^3], x]

[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSinh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0.003, size = 81, normalized size = 0.9

$$\frac{1}{16x} \sqrt{x^6 - 3x^4 + 3x^2} \left(4\sqrt{x^4 - 3x^2 + 3x^2} - 6\sqrt{x^4 - 3x^2 + 3} + 3 \operatorname{Arcsinh} \left(\frac{1}{3} \sqrt{3} (2x^2 - 3) \right) \right) \frac{1}{\sqrt{x^4 - 3x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-(-x^2+1)^3)^(1/2), x)

[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2-6*(x^4-3*x^2+3)^(1/2)+3*arcsinh(1/3*3^(1/2)*(2*x^2-3)))/x/(x^4-3*x^2+3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(x^2 - 1)^3 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x^2+1)^3)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt((x^2 - 1)^3 + 1), x)

Fricas [A] time = 1.29341, size = 157, normalized size = 1.83

$$\frac{12x \log\left(-\frac{2x^3-3x-2\sqrt{x^6-3x^4+3x^2}}{x}\right) - 8\sqrt{x^6-3x^4+3x^2}(2x^2-3) - 9x}{64x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")

[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1 - (1 - x^2)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x**2+1)**3)**(1/2),x)

[Out] Integral(sqrt(1 - (1 - x**2)**3), x)

Giac [A] time = 1.09643, size = 93, normalized size = 1.08

$$\frac{1}{16} \left(2\sqrt{x^4 - 3x^2 + 3}(2x^2 - 3) - 3 \log\left(-2x^2 + 2\sqrt{x^4 - 3x^2 + 3} + 3\right) \right) \operatorname{sgn}(x) + \frac{3}{16} \left(2\sqrt{3} + \log(2\sqrt{3} + 3) \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")

[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)

$$3.129 \quad \int \frac{1}{x\sqrt{a+bx+cx^2}} dx$$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

[Out] $-(\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2]])/\text{Sqrt}[a])$

Rubi [A] time = 0.0158717, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/(x*\text{Sqrt}[a + b*x + c*x^2]),x]$

[Out] $-(\text{ArcTanh}[(2*a + b*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[a + b*x + c*x^2]])/\text{Sqrt}[a])$

Rule 724

$\text{Int}[1/(((d_.) + (e_.)*(x_.))*\text{Sqrt}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]), x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/\text{Sqrt}[a + b*x + c*x^2]], x] /;$ $\text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0]$

Rule 206

$\text{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ $\text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}} dx = -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\right) \\ = -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0093191, size = 37, normalized size = 0.97

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]/Sqrt[a])

Maple [A] time = 0.003, size = 35, normalized size = 0.9

$$-\ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2+b*x+a)^(1/2),x)

[Out] -1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.36145, size = 273, normalized size = 7.18

$$\left[\frac{\log\left(-\frac{8abx+(b^2+4ac)x^2-4\sqrt{cx^2+bx+a}(bx+2a)\sqrt{a}+8a^2}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2a)\sqrt{-a}}{2(acx^2+abx+a^2)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c))*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a))*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)

Giac [A] time = 1.0966, size = 47, normalized size = 1.24

$$\frac{2 \arctan\left(-\frac{\sqrt{cx-\sqrt{cx^2+bx+a}}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")

[Out] $2 \cdot \arctan\left(\frac{-\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right) / \sqrt{-a}$

$$3.130 \quad \int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx$$

Optimal. Leaf size=45

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

[Out] $-(\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])]/\text{Sqrt}[a])$

Rubi [A] time = 0.0216179, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[x^2*(a + b*x + c*x^2)], x]$

[Out] $-(\text{ArcTanh}[(x*(2*a + b*x))/(2*\text{Sqrt}[a]*\text{Sqrt}[a*x^2 + b*x^3 + c*x^4])]/\text{Sqrt}[a])$

Rule 1996

$\text{Int}[(u_)^(p_), x_Symbol] \rightarrow \text{Int}[\text{ExpandToSum}[u, x]^p, x] /;$ FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1904

$\text{Int}[1/\text{Sqrt}[(a_)*(x_)^2 + (b_)*(x_)^(n_) + (c_)*(x_)^(r_)], x_Symbol] :$
 $> \text{Dist}[-2/(n - 2), \text{Subst}[\text{Int}[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/\text{Sqrt}[a*x^2 + b*x^n + c*x^r]], x] /;$ FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^(-1), x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(a+bx+cx^2)}} dx &= \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right) \\ &\quad \operatorname{tanh}^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right) \\ &= -\frac{\operatorname{tanh}^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0228697, size = 70, normalized size = 1.56

$$-\frac{x\sqrt{a+bx+cx^2} \operatorname{tanh}^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]

[Out] -((x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))]))

Maple [A] time = 0.008, size = 64, normalized size = 1.4

$$-x\sqrt{cx^2+bx+a} \ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right) \frac{1}{\sqrt{x^2(cx^2+bx+a)}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)

[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^2 + bx + a)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)

Fricas [A] time = 1.39367, size = 300, normalized size = 6.67

$$\left[\frac{\log\left(\frac{-8abx^2 + (b^2 + 4ac)x^3 + 8a^2x - 4\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{a}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(bx + 2a)\sqrt{-a}}{2(acx^3 + abx^2 + a^2x)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")

[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)

[Out] Timed out

Giac [A] time = 1.10413, size = 80, normalized size = 1.78

$$-\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{cx-\sqrt{cx^2+bx+a}}}{\sqrt{-a}}\right)}{\sqrt{-a} \operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))

$$3.131 \quad \int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}} dx$$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt[a])

Rubi [A] time = 0.0756258, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]

[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt[a])

Rule 1997

Int[(u_)^(p_)*((d_)*(x_)^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_.)] , x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}} dx &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^2+cx^3}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{\sqrt{x}(2a+bx)}{\sqrt{ax+bx^2+cx^3}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.033077, size = 72, normalized size = 1.53

$$-\frac{\sqrt{x}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a+b*x+c*x^2)]),x]

[Out] -((Sqrt[x]*Sqrt[a+b*x+c*x^2]*ArcTanh[(2*a+b*x)/(2*Sqrt[a]*Sqrt[a+b*x+c*x^2])])/(Sqrt[a]*Sqrt[x*(a+x*(b+c*x))]))

Maple [A] time = 0.011, size = 64, normalized size = 1.4

$$-\sqrt{x}\sqrt{cx^2+bx+a} \ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right) \frac{1}{\sqrt{x}(cx^2+bx+a)} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)

[Out] $-x^{(1/2)}/(x*(c*x^2+b*x+a))^{(1/2)}*(c*x^2+b*x+a)^{(1/2)}/a^{(1/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^2 + bx + a)x}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)`

Fricas [A] time = 1.70371, size = 315, normalized size = 6.7

$$\left[\frac{\log\left(\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2a)\sqrt{-a}\sqrt{x}}{2(acx^3+abx^2+a^2x)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")`

[Out] `[1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)`

[Out] Timed out

Giac [A] time = 1.14507, size = 47, normalized size = 1.

$$\frac{2 \arctan\left(-\frac{\sqrt{cx}-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)

$$3.132 \quad \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx$$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5]])/Sqrt[a])

Rubi [A] time = 0.0899707, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1997, 1913, 206}

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]

[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5]])/Sqrt[a])

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1913

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}} dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0279648, size = 74, normalized size = 1.51

$$-\frac{x^{3/2}\sqrt{a+bx+cx^2} \tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)], x]

[Out] -((x^(3/2)*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))]))

Maple [A] time = 0.006, size = 66, normalized size = 1.4

$$-x^{\frac{3}{2}}\sqrt{cx^2+bx+a} \ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right) \frac{1}{\sqrt{x^3(cx^2+bx+a)}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2), x)

[Out] $-1/(x^3(c*x^2+b*x+a))^{(1/2)}*x^{(3/2)}*(c*x^2+b*x+a)^{(1/2)}/a^{(1/2)}*\ln((2*a+b*x+2*a^{(1/2)}*(c*x^2+b*x+a)^{(1/2)})/x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{(cx^2 + bx + a)}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)`

Fricas [A] time = 1.70724, size = 325, normalized size = 6.63

$$\left[\frac{\log\left(\frac{8abx^3+(b^2+4ac)x^4+8a^2x^2-4\sqrt{cx^5+bx^4+ax^3}(bx+2a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}(bx+2a)\sqrt{-a}\sqrt{x}}{2(acx^4+abx^3+a^2x^2)}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")`

[Out] `[1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 + a^2*x^2))/a]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)`

[Out] Timed out

Giac [A] time = 1.15662, size = 47, normalized size = 0.96

$$\frac{2 \arctan\left(-\frac{\sqrt{cx}-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")

[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)

$$3.133 \quad \int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])

Rubi [A] time = 0.0357734, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1114, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+bx^2+cx^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx+cx^2}} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0122254, size = 44, normalized size = 1.

$$-\frac{\tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a])

Maple [A] time = 0.004, size = 39, normalized size = 0.9

$$-\frac{1}{2} \ln \left(\frac{1}{x^2} \left(2a + bx^2 + 2\sqrt{a}\sqrt{cx^4 + bx^2 + a} \right) \right) \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x)

[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.46687, size = 294, normalized size = 6.68

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a+8a^2}}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{-a}}{2(acx^4+abx^2+a^2)}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

$$3.134 \quad \int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt[a])

Rubi [A] time = 0.0151489, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1996, 1904, 206}

$$\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]

[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/(2*Sqrt[a])

Rule 1996

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}} dx &= \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}} dx \\ &= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{x(2a+bx^2)}{\sqrt{ax^2+bx^4+cx^6}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}} \right)}{2\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0163743, size = 81, normalized size = 1.65

$$-\frac{x\sqrt{a+bx^2+cx^4} \tanh^{-1} \left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}} \right)}{2\sqrt{a}\sqrt{x^2(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]

[Out] -(x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]))/(2*Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.005, size = 72, normalized size = 1.5

$$-\frac{x}{2}\sqrt{cx^4+bx^2+a} \ln \left(\frac{1}{x^2} \left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a} \right) \right) \frac{1}{\sqrt{x^2(cx^4+bx^2+a)}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x)

[Out] $-1/2/(x^2*(c*x^4+b*x^2+a))^{(1/2)}*x*(c*x^4+b*x^2+a)^{(1/2)}/a^{(1/2)}*\ln((2*a+b*x^2+2*a^{(1/2)}*(c*x^4+b*x^2+a)^{(1/2)})/x^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)`

Fricas [A] time = 1.32455, size = 311, normalized size = 6.35

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{a}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{-a}}{2(acx^5+abx^3+a^2x)}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")`

[Out] `[1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)

$$3.135 \quad \int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rubi [A] time = 0.0672684, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]

[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]/(2*Sqrt[a])

Rule 1997

Int[(u_)^(p_)*((d_)*(x_)^(m_)), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)], x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}} dx &= \int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}} dx \\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{\sqrt{x}(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0195306, size = 83, normalized size = 1.63

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]), x]

[Out] -(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.008, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x(cx^4+bx^2+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x)`

[Out] $-1/2*x^{(1/2)}/(x*(c*x^4+b*x^2+a))^{(1/2)}*(c*x^4+b*x^2+a)^{(1/2)}/a^{(1/2)}*\ln((2*a+b*x^2+2*a^{(1/2)}*(c*x^4+b*x^2+a)^{(1/2)})/x^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)`

Fricas [A] time = 1.37217, size = 327, normalized size = 6.41

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^5+bx^3+ax}(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")`

[Out] $[1/4*\log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*\sqrt{c*x^5 + b*x^3 + a*x}*(b*x^2 + 2*a)*\sqrt{a}*\sqrt{x}))/x^5)/\sqrt{a}, 1/2*\sqrt{-a}*\arctan(1/2*\sqrt{c*x^5 + b*x^3 + a*x}*(b*x^2 + 2*a)*\sqrt{-a}*\sqrt{x}/(a*c*x^5 + a*b*x^3 + a^2*x))/a]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

$$3.136 \quad \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx$$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(2*Sqrt[a])

Rubi [A] time = 0.0772904, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1997, 1913, 206}

$$\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]

[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7])]/(2*Sqrt[a])

Rule 1997

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1913

Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}} dx \\ &= -\text{Subst} \left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{3/2}(2a+bx^2)}{\sqrt{ax^3+bx^5+cx^7}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}} \right)}{2\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.0194707, size = 85, normalized size = 1.6

$$-\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{x^3(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)], x]

[Out] -(x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])

Maple [A] time = 0.007, size = 74, normalized size = 1.4

$$-\frac{1}{2}x^{\frac{3}{2}}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x^3(cx^4+bx^2+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x)`

[Out]
$$-1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*\ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{(cx^4 + bx^2 + a)x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)`

Fricas [A] time = 1.36241, size = 338, normalized size = 6.38

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^6+8abx^4+8a^2x^2-4\sqrt{cx^7+bx^5+ax^3}(bx^2+2a)\sqrt{a}\sqrt{x}}{x^6}\right)}{4\sqrt{a}}, \frac{\sqrt{-a} \arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^6+abx^4+a^2x^2)}\right)}{2a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")`

[Out]
$$\left[\frac{1}{4} \log\left(-\frac{(b^2 + 4ac)x^6 + 8abx^4 + 8a^2x^2 - 4\sqrt{cx^7 + bx^5 + ax^3}(bx^2 + 2a)\sqrt{a}\sqrt{x}}{x^6}\right) / \sqrt{a}, \frac{1}{2} \sqrt{-a} \arctan\left(\frac{1}{2} \sqrt{cx^7 + bx^5 + ax^3}(bx^2 + 2a)\sqrt{-a}\sqrt{x} / (acx^6 + abx^4 + a^2x^2)\right) / a \right]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{(cx^4 + bx^2 + a)x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

$$3.137 \quad \int \frac{1}{x\sqrt{3-3x^2+x^4}} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])

Rubi [A] time = 0.0268767, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1114, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])

Rule 1114

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{3-3x^2+x^4}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{3-3x+x^2}} dx, x, x^2 \right) \\ &= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{3(2-x^2)}{\sqrt{3-3x^2+x^4}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{3-3x^2+x^4}} \right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0091794, size = 40, normalized size = 1.

$$-\frac{\tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] -ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3])

Maple [A] time = 0.003, size = 31, normalized size = 0.8

$$-\frac{\sqrt{3}}{6} \text{Artanh} \left(\frac{(-3x^2+6)\sqrt{3}}{6} \frac{1}{\sqrt{x^4-3x^2+3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^4-3*x^2+3)^(1/2),x)

[Out] -1/6*3^(1/2)*arctanh(1/6*(-3*x^2+6)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

Maxima [A] time = 1.6456, size = 27, normalized size = 0.68

$$-\frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")

[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)

Fricas [A] time = 1.31135, size = 131, normalized size = 3.28

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^2+2\sqrt{3}(x^2-2)+2\sqrt{x^4-3x^2+3}(\sqrt{3}+2)-6}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(sqrt(3) + 2) - 6)/x^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^4-3x^2+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x**4-3*x**2+3)**(1/2),x)

[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4-3x^2+3}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x), x)
```

$$3.138 \quad \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx$$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rubi [A] time = 0.0119649, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1996, 1904, 206}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*Sqrt[3])

Rule 1996

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1904

Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, (x*(2*a + b*x^(n - 2)))/Sqrt[a*x^2 + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} dx &= \int \frac{1}{\sqrt{3x^2-3x^4+x^6}} dx \\ &= -\text{Subst} \left(\int \frac{1}{12-x^2} dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}} \right)}{2\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0082086, size = 73, normalized size = 1.62

$$\frac{x\sqrt{x^4-3x^2+3} \tanh^{-1} \left(\frac{6-3x^2}{2\sqrt{3}\sqrt{x^4-3x^2+3}} \right)}{2\sqrt{3}\sqrt{x^2(x^4-3x^2+3)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -(x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(6 - 3*x^2)/(2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4])])/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])

Maple [A] time = 0., size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3x^2+3}\text{Arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3x^2+3}}\right)\frac{1}{\sqrt{x^2(x^4-3x^2+3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)

[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

Fricas [A] time = 1.28909, size = 142, normalized size = 3.16

$$\frac{1}{6} \sqrt{3} \log \left(-\frac{3x^3 + 2\sqrt{3}(x^3 - 2x) + 2\sqrt{x^6 - 3x^4 + 3x^2}(\sqrt{3} + 2) - 6x}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^2)*(sqrt(3) + 2) - 6*x)/x^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4 - 3x^2 + 3)x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)

$$3.139 \quad \int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}} dx$$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])

Rubi [A] time = 0.0474514, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$, Rules used = {1997, 1913, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]

[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])

Rule 1997

Int[(u_)^(p_)*((d_)*(x_))^(m_), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]

Rule 1913

Int[(x_)^(m_)/Sqrt[(b_)*(x_)^(n_) + (a_)*(x_)^(q_) + (c_)*(x_)^(r_)] , x_Symbol] := Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, (x^(m + 1)*(2*a + b*x^(n - q)))/Sqrt[a*x^q + b*x^n + c*x^r]], x] /; FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}} dx &= \int \frac{1}{\sqrt{x}\sqrt{3x-3x^2+x^3}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{12-x^2} dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0197291, size = 62, normalized size = 1.44

$$\frac{\sqrt{x}\sqrt{x^2-3x+3} \tanh^{-1}\left(\frac{\sqrt{3}(x-2)}{2\sqrt{x^2-3x+3}}\right)}{\sqrt{3}\sqrt{x}(x^2-3x+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3-3*x+x^2)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[3-3*x+x^2]*ArcTanh[(Sqrt[3]*(-2+x))/(2*Sqrt[3-3*x+x
^2])])/(Sqrt[3]*Sqrt[x*(3-3*x+x^2)])
```

Maple [A] time = 0.011, size = 50, normalized size = 1.2

$$\frac{\sqrt{3}}{3} \sqrt{x}\sqrt{x^2-3x+3} \operatorname{Artanh}\left(\frac{(-2+x)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right) \frac{1}{\sqrt{x}(x^2-3x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x)
```


[Out] $1/3*x^{(1/2)}/(x*(x^2-3*x+3))^{(1/2)}*(x^2-3*x+3)^{(1/2)}*3^{(1/2)}*\operatorname{arctanh}(1/2*(-2+x)*3^{(1/2)}/(x^2-3*x+3)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^2 - 3x + 3)x}\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)`

Fricas [A] time = 1.39325, size = 132, normalized size = 3.07

$$\frac{1}{6}\sqrt{3}\log\left(\frac{7x^3 + 4\sqrt{3}\sqrt{x^3 - 3x^2 + 3x}(x-2)\sqrt{x} - 24x^2 + 24x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")`

[Out] `1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x) - 24*x^2 + 24*x)/x^3)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)`

[Out] Timed out

Giac [A] time = 1.14413, size = 69, normalized size = 1.6

$$-\frac{1}{3}\sqrt{3}\log\left(\left| -x + \sqrt{3} + \sqrt{x^2 - 3x + 3} \right|\right) + \frac{1}{3}\sqrt{3}\log\left(\left| -x - \sqrt{3} + \sqrt{x^2 - 3x + 3} \right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")

[Out] -1/3*sqrt(3)*log(abs(-x + sqrt(3) + sqrt(x^2 - 3*x + 3))) + 1/3*sqrt(3)*log(abs(-x - sqrt(3) + sqrt(x^2 - 3*x + 3)))

$$3.140 \quad \int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}} dx$$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

[Out] $-(\text{ArcTanh}[(x^{(q/2)}*(2*a + b*x^{(n - q)}))/(2*\text{Sqrt}[a]*\text{Sqrt}[b*x^n + c*x^{(2*n - q)} + a*x^q])]/(\text{Sqrt}[a]*(n - q)))$

Rubi [A] time = 0.0666428, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1913, 206}

$$\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^{(-1 + q/2)}/\text{Sqrt}[b*x^n + c*x^{(2*n - q)} + a*x^q], x]$

[Out] $-(\text{ArcTanh}[(x^{(q/2)}*(2*a + b*x^{(n - q)}))/(2*\text{Sqrt}[a]*\text{Sqrt}[b*x^n + c*x^{(2*n - q)} + a*x^q])]/(\text{Sqrt}[a]*(n - q)))$

Rule 1913

$\text{Int}[(x_)^{(m_.)}/\text{Sqrt}[(b_.)*(x_)^{(n_.)} + (a_.)*(x_)^{(q_.)} + (c_.)*(x_)^{(r_.)}], x_Symbol] := \text{Dist}[-2/(n - q), \text{Subst}[\text{Int}[1/(4*a - x^2), x], x, (x^{(m + 1)}*(2*a + b*x^{(n - q)}))/\text{Sqrt}[a*x^q + b*x^n + c*x^r]], x] /;$ FreeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && EqQ[m, q/2 - 1]

Rule 206

$\text{Int}[(a_. + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx = -\frac{2 \operatorname{Subst}\left(\int \frac{1}{4a-x^2} dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\right)}{n-q}$$

$$= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n + cx^{2n-q} + ax^q}}\right)}{\sqrt{a}(n-q)}$$

Mathematica [F] time = 0.392503, size = 0, normalized size = 0.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx$$

Verification is Not applicable to the result.

[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int x^{-1+\frac{q}{2}} \frac{1}{\sqrt{bx^n + cx^{2n-q} + ax^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2), x)

[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```



```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157 ElementaryFunctionQ := proc(func)
158     member(func,[
159         exp,log,ln,
160         sin,cos,tan,cot,sec,csc,
161         arcsin,arccos,arctan,arccot,arcsec,arccsc,
162         sinh,cosh,tanh,coth,sech,csch,
163         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
164 end proc:
165
166 SpecialFunctionQ := proc(func)
167     member(func,[
168         erf,erfc,erfi,
169         FresnelS,FresnelC,
170         Ei,Ei,Li,Si,Ci,Shi,Chi,
171         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
172         EllipticF,EllipticE,EllipticPi])
173 end proc:
174
175 HypergeometricFunctionQ := proc(func)
176     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
177 end proc:
178
179 AppellFunctionQ := proc(func)
180     member(func,[AppellF1])
181 end proc:
182
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                   asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                   asinh,acosh,atanh,acoth,asech,acsch
25                   ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                   fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                   gamma,loggamma,digamma,zeta,polylog,LambertW,
31                   elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                   ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by


```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```